Random Utility without Regularity

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Work w. J. Dana, C. Davis-Stober, J. Müller-Trede, M.Robinson

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Outline

1. Context (✓)
2. Random Utility & Random Preference
3. Context-Dependent Random Utility & Random Preference
4. Random Utility without Regularity
5. Conclusions

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Rationality of decision making.

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Huge amounts of heterogeneity within and across decision makers.

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They are subject to genuine qualitative variation.

I only use classical probability theory.

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Random Utility Model

Finite set $\mathcal{A}$

Noncoincident RVs: $\forall a, b \in \mathcal{A}, a \neq b, \Pr(\mathbf{U}_a = \mathbf{U}_b) = 0$

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Random Utility Model for Best-Choice

$$P_X(x) = \Pr(\mathbf{U}_x = \max_{y \in X} \mathbf{U}_y), \qquad (x \in X \subseteq \mathcal{A}).$$

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Random Utility Model for Best-Worst-Choice

$$P_X(x, y) = \Pr(\mathbf{U}_x = \max_{v \in X} \mathbf{U}_v, \mathbf{U}_y = \min_{w \in X} \mathbf{U}_w), \qquad (x \neq y \in X \subseteq \mathcal{A}),$$

Random Utility $\leftrightarrow$ Random Preference

Every joint realization of noncoincident RVs $(\mathbf{U}_x)_{x \in \mathcal{A}}$ generates a linear order $\succ$ on $\mathcal{A}$.

Linear Order: Transitive, Asymmetric, Complete.

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Linear Order: Transitive, Asymmetric, Complete.

Every probability distribution on linear orders on $\mathcal{A}$ can be represented with noncoincident RVs $(\mathbf{U}_x)_{x \in \mathcal{A}}$.

Random Preference Model

$\mathcal{L}$: the collection of all linear orders on $\mathcal{A}$

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Random Preference Model for Best-Choice

$$P_X(x) = \sum_{\substack{\succ \in \mathcal{L} \\ B_X(\succ)=x}} P(\succ), \qquad (\forall x \in X \subseteq \mathcal{A}).$$

Random Preference Model

$\mathcal{L}$: the collection of all linear orders on $\mathcal{A}$

Random Preference Model for Best-Choice

$$P_X(x) = \sum_{\substack{\succ \in \mathcal{L} \\ B_X(\succ)=x}} P(\succ), \qquad (\forall x \in X \subseteq \mathcal{A}).$$

Random Preference Model for Best-Worst-Choice

$$P_X(x,y) = \sum_{\substack{\succ \in \mathcal{L} \\ BW_X(\succ)=(x,y)}} P(\succ), \qquad (\forall x \neq y \in X \subseteq \mathcal{A}).$$

Random Preference Model for Binary Choice

$\mathcal{L}$: the collection of all linear orders on $\mathcal{A}$

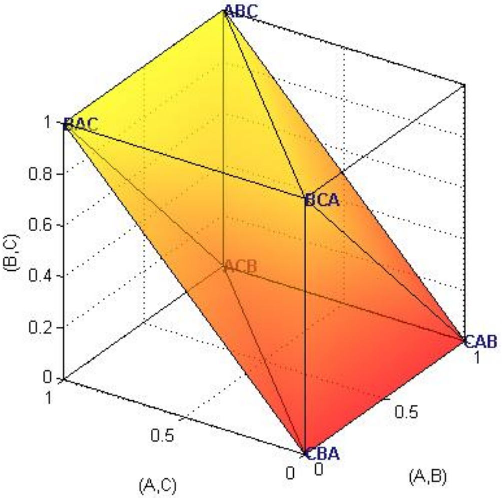
Random Preference Model for Best-Choice

$$P_{\{x,y\}}(x) = \sum_{\substack{\succ \in \mathcal{L} \\ x \succ y}} P(\succ), \qquad (\forall x \neq y \in \mathcal{A}).$$

Random Preference Model for Best-Worst-Choice

$$P_{\{x,y\}}(x, y) = \sum_{\substack{\succ \in \mathcal{L} \\ x \succ y}} P(\succ), \qquad (\forall x \neq y \in \mathcal{A}).$$

Binary Choice & Linear Ordering Polytope



Binary Choice & Linear Ordering Polytope

Triangle Inequalities (Block & Marschak, book, 1960)

$$P_{\{x,y\}}(x) + P_{\{y,z\}}(y) - P_{\{x,z\}}(x) \leq 1 \quad (\forall x, y, z)$$

Binary Choice & Linear Ordering Polytope

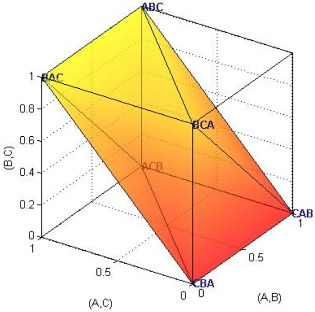
Triangle Inequalities (Block & Marschak, book, 1960)

$$P_{\{x,y\}}(x) + P_{\{y,z\}}(y) - P_{\{x,z\}}(x) \leq 1 \quad (\forall x, y, z)$$

$\lvert\mathcal{A}\rvert$:	3	4	5	6	7	8	9
# FDI's:	2	10	20	910	87,472	$> 4.8 \times 10^8$	unknown

Binary Choice & Linear Ordering Polytope

Test of *Rationality (Transitivity) of Preference*:



Regenwetter, Dana, Davis-Stober (*Psychological Review*, 2011).

Outline

Description-Experience Gap

Binary choice among lotteries

H: Win \$4 with probability .8, otherwise \$0. L: Win \$3 for sure.

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Description-Experience Gap (Hertwig et al., *Psych. Science*, 2004)

Decision makers “overweight” small probabilities in description.
Decision makers “underweight” small probabilities in experience.

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Binary choice among lotteries

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Description-Experience Gap (Hertwig et al., *Psych. Science*, 2004)

Decision makers “overweight” small probabilities in description.
Decision makers “underweight” small probabilities in experience.

How about “context:” Description vs. Experience

Context-Dependent Random Preference for DE

Let $\mathcal{R}^{\{D,E\}}$ denote a finite collection of pairs of binary preference relations of the form $(\succ^D, \succ^E)$, where

$$x \succ^D y \text{ denotes that } x \text{ is preferred to } y \text{ in description}$$
$$x \succ^E y \text{ denotes that } x \text{ is preferred to } y \text{ in experience}$$

according to context-dependent preference pattern $(\succ^D, \succ^E) \in \mathcal{R}^{\{D,E\}}$.

Context-Dependent Random Preference for DE

CONTEXT-DEPENDENT RANDOM-PREFERENCE MODEL

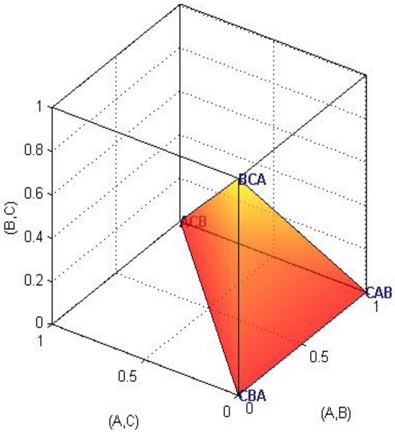
There is a probability distribution over $\mathcal{R}^{\{D,E\}}$ such that

$$P^D_{xy} = \sum_{\substack{(\succ^D,\succ^E)\in\mathcal{R}^{\{D,E\}} s.t. \\ x\succ^D y}} P_{(\succ^D,\succ^E)},$$

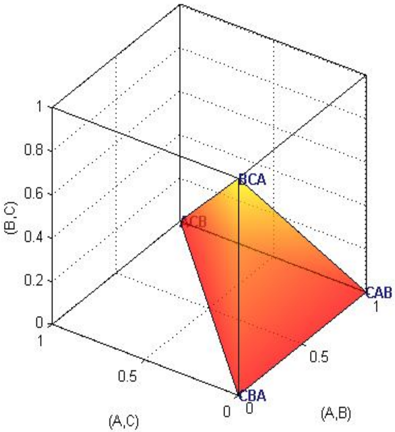
and

$$P^E_{xy} = \sum_{\substack{(\succ^D,\succ^E)\in\mathcal{R}^{\{D,E\}} s.t. \\ x\succ^E y}} P_{(\succ^D,\succ^E)}.$$

Random Preference Model



Random Preference Model



Derived possible preferences from Cumulative Prospect Theory (Tversky & Kahneman, *J. of Risk & Uncertainty*, 1992)

Context-Independent RP of CPT with $\gamma, \delta < 1$

CPT with overweighting and $0 \leq \gamma^D = \gamma^E, \delta^D = \delta^E < 1$.

Context-Independent RP of CPT with $\gamma, \delta < 1$

CPT with overweighting and $0 \leq \gamma^D = \gamma^E, \delta^D = \delta^E < 1$.

	Description						Experience					
	1	2	3	4	5	6	1	2	3	4	5	6
OV_1	0	0	0	0	0	0	0	0	0	0	0	0
OV_2	0	0	0	0	0	1	0	0	0	0	0	1
OV_3	0	0	0	1	0	0	0	0	0	1	0	0
OV_4	0	0	0	1	0	1	0	0	0	1	0	1
OV_5	0	0	1	0	0	0	0	0	1	0	0	0
OV_6	0	0	1	0	0	1	0	0	1	0	0	1
OV_7	0	0	1	1	0	0	0	0	1	1	0	0
OV_8	0	0	1	1	0	1	0	0	1	1	0	1
OV_9	0	1	0	0	0	0	0	1	0	0	0	0
...												
OV_{32}	1	1	1	1	1	1	1	1	1	1	1	1

Context-Independent RP of CPT with $\gamma, \delta < 1$

CPT with overweighting and $0 \leq \gamma^D = \gamma^E, \delta^D = \delta^E < 1$.

	Description						Experience					
	1	2	3	4	5	6	1	2	3	4	5	6
OV_1	0	0	0	0	0	0	0	0	0	0	0	0
OV_2	0	0	0	0	0	1	0	0	0	0	0	1
OV_3	0	0	0	1	0	0	0	0	0	1	0	0
OV_4	0	0	0	1	0	1	0	0	0	1	0	1
OV_5	0	0	1	0	0	0	0	0	1	0	0	0
OV_6	0	0	1	0	0	1	0	0	1	0	0	1
OV_7	0	0	1	1	0	0	0	0	1	1	0	0
OV_8	0	0	1	1	0	1	0	0	1	1	0	1
OV_9	0	1	0	0	0	0	0	1	0	0	0	0
...												
OV_{32}	1	1	1	1	1	1	1	1	1	1	1	1

$$P_{HL}(D6) = P_{HL}(E6) \geq P_{HL}(D5) = P_{HL}(E5)$$

FDIs Context-Independent RP of CPT with $\gamma, \delta < 1$

Necessary and sufficient conditions for context-independent random preference model of CPT with overweighting.

$$
\begin{aligned}
P_{HL}(D6) = P_{HL}(E6) &\geq P_{HL}(D5) = P_{HL}(E5), \\
P_{HL}(D2) = P_{HL}(E2) &\geq P_{HL}(D1) = P_{HL}(E1), \\
P_{HL}(D2) &\geq P_{HL}(D5), \\
P_{HL}(D3) = P_{HL}(E3), &\quad P_{HL}(D4) = P_{HL}(E4).
\end{aligned}
$$

Context-Dependent RP of CPT with $\gamma^D, \gamma^E, \delta^D, \delta^E < 1$

	Description						Experience					
	1	2	3	4	5	6	1	2	3	4	5	6
OO_1	0	0	1	0	0	0	0	0	1	1	0	1
OO_2	0	0	1	1	0	0	0	0	1	0	0	1
OO_3	0	0	1	0	0	0	0	0	1	0	0	1
OO_4	0	0	1	0	0	0	0	0	0	1	0	1
OO_5	0	0	0	1	0	0	0	0	1	0	0	1
OO_6	0	0	1	0	0	0	0	0	0	0	0	1
OO_7	0	0	0	0	0	0	0	0	1	0	0	1
OO_8	0	0	0	1	0	0	0	0	0	0	0	1
OO_9	0	0	0	0	0	0	0	0	0	1	0	1
OO_{10}	0	0	0	0	0	0	0	0	1	1	0	1
...												
OO_{659}	1	1	0	1	1	1	1	1	0	1	1	1
OO_{660}	1	1	1	1	1	1	1	1	1	1	1	1

Context-Dependent RP of CPT with $\gamma^D, \gamma^E, \delta^D, \delta^E < 1$

	Description						Experience					
	1	2	3	4	5	6	1	2	3	4	5	6
OO_1	0	0	1	0	0	0	0	0	1	1	0	1
OO_2	0	0	1	1	0	0	0	0	1	0	0	1
OO_3	0	0	1	0	0	0	0	0	1	0	0	1
OO_4	0	0	1	0	0	0	0	0	0	1	0	1
OO_5	0	0	0	1	0	0	0	0	1	0	0	1
OO_6	0	0	1	0	0	0	0	0	0	0	0	1
OO_7	0	0	0	0	0	0	0	0	1	0	0	1
OO_8	0	0	0	1	0	0	0	0	0	0	0	1
OO_9	0	0	0	0	0	0	0	0	0	1	0	1
OO_{10}	0	0	0	0	0	0	0	0	1	1	0	1
...												
OO_{659}	1	1	0	1	1	1	1	1	0	1	1	1
OO_{660}	1	1	1	1	1	1	1	1	1	1	1	1

$$\max\Big(P_{HL}(E1), P_{HL}(D1), P_{HL}(D5)\Big) \leq P_{HL}(D2)$$

FDIs Context-Dep. RP of CPT with $\gamma^D, \gamma^E, \delta^D, \delta^E < 1$

Necessary and sufficient conditions for context-dependent random preference model of CPT with overweighting.

$$
\begin{aligned}
\max\left(P_{HL}(E1), P_{HL}(D1), P_{HL}(D5)\right) &\leq P_{HL}(D2),\\
P_{HL}(E1) + P_{HL}(E5) &\leq P_{HL}(E2) + P_{HL}(D1) + P_{HL}(D6),\\
P_{HL}(D6) + P_{HL}(E5) &\leq 1 + P_{HL}(D2),\\
P_{HL}(E1) + P_{HL}(E6) &\leq 1 + P_{HL}(D1), + P_{HL}(D6),\\
P_{HL}(D6) + P_{HL}(E2) + P_{HL}(E5) &\leq 1 + P_{HL}(D2) + P_{HL}(E6),\\
P_{HL}(E1) + P_{HL}(E6) + P_{HL}(D2) &\leq 1 + P_{HL}(E2) + P_{HL}(D1) + P_{HL}(D6),\\
P_{HL}(D3) + P_{HL}(D4) &\leq 1 + P_{HL}(E3) + P_{HL}(E4),
\end{aligned}
$$

and first 7 Conditions hold with the labels E and D swapped.

Context-Dependent RP $\gamma^D, \delta^D < 1 < \gamma^E, \delta^E$

	Description						Experience					
	1	2	3	4	5	6	1	2	3	4	5	6
OU_1	0	0	1	0	0	0	0	0	1	1	0	0
OU_2	0	0	1	0	0	1	0	0	1	1	0	0
OU_3	0	1	1	0	0	0	0	0	1	1	0	0
OU_4	0	1	1	0	0	1	0	0	1	1	0	0
OU_5	0	1	1	0	1	1	0	0	1	1	0	0
OU_6	0	0	1	0	0	0	1	0	1	1	0	0
OU_7	**1**	1	1	0	0	0	**0**	0	1	1	0	0
OU_8	0	1	1	0	0	0	1	0	1	1	0	0
OU_9	0	1	1	0	0	0	1	1	1	1	0	0
OU_{10}	0	1	1	0	0	1	1	0	1	1	0	0
...												
OU_{249}	1	1	0	0	1	1	1	1	0	0	1	0
OU_{250}	1	1	0	0	1	1	1	1	0	0	1	1

Context-Dependent RP $\gamma^D, \delta^D < 1 < \gamma^E, \delta^E$

	Description						Experience					
	1	2	3	4	5	6	1	2	3	4	5	6
OU_1	0	0	1	0	0	0	0	0	1	1	0	0
OU_2	0	0	1	0	0	1	0	0	1	1	0	0
OU_3	0	1	1	0	0	0	0	0	1	1	0	0
OU_4	0	1	1	0	0	1	0	0	1	1	0	0
OU_5	0	1	1	0	1	1	0	0	1	1	0	0
OU_6	0	0	1	0	0	0	1	0	1	1	0	0
OU_7	**1**	1	1	0	0	0	**0**	0	1	1	0	0
OU_8	0	1	1	0	0	0	1	0	1	1	0	0
OU_9	0	1	1	0	0	0	1	1	1	1	0	0
OU_{10}	0	1	1	0	0	1	1	0	1	1	0	0
...												
OU_{249}	1	1	0	0	1	1	1	1	0	0	1	0
OU_{250}	1	1	0	0	1	1	1	1	0	0	1	1

$$\max\Big(P_{HL}(E1), P_{HL}(D1), P_{HL}(D5)\Big) \leq P_{HL}(D2)$$

FDIs Context-Dependent RP $\gamma^D, \delta^D < 1 < \gamma^E, \delta^E$

Necessary and sufficient conditions for context-dependent random preference model of CPT with overweighting.

$$
\begin{aligned}
P_{HL}(E6) \leq P_{HL}(E5) \leq P_{HL}(E2) &\leq P_{HL}(D2) \\
\max\left(P_{HL}(D1), P_{HL}(D5)\right) &\leq P_{HL}(D2) \\
\max\left(P_{HL}(D5), P_{HL}(E5)\right) &\leq P_{HL}(D6) \\
P_{HL}(E2) &\leq P_{HL}(E1) \\
P_{HL}(E3) &\leq P_{HL}(E4) \\
P_{HL}(D1) + P_{HL}(E5) &\leq P_{HL}(D2) + P_{HL}(D5) \\
P_{HL}(D1) + P_{HL}(D5) &\leq P_{HL}(D2) + P_{HL}(E1) \\
P_{HL}(D6) + P_{HL}(E1) &\leq 1 + P_{HL}(D2) \\
P_{HL}(D4) + P_{HL}(E3) &\leq 1 + P_{HL}(D3) \\
P_{HL}(D3) + P_{HL}(D4) &\leq 1 + P_{HL}(E4) \\
P_{HL}(D1) + P_{HL}(D6) &\leq 1 + P_{HL}(E1) \\
P_{HL}(D1) + P_{HL}(D6) + P_{HL}(E5) &\leq 1 + P_{HL}(D5) + P_{HL}(E1)
\end{aligned}
$$

Statistical Analysis

Bayes Factors on Hertwig et al. (2004) data.

Context-independent overweighting: $\sim 10^{-8}$

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Bayes Factors on Hertwig et al. (2004) data.

Context-independent overweighting:	$\sim 10^{-8}$
Context-dependent overweighting:	0.002

Statistical Analysis

Bayes Factors on Hertwig et al. (2004) data.

Context-independent overweighting:	$\sim 10^{-8}$
Context-dependent overweighting:	0.002
Overweighting in description, underweighting in experience:	300

Regenwetter & Robinson (*Psychological Review*, 2017).

Outline

Asymmetric Dominance & Regularity

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Jim shops for a new TV.

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Option t (the “target”) has better picture quality.

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Jim shops for a new TV.
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Option t (the “target”) has better picture quality.
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Asymmetric Dominance & Regularity

Asymmetric Dominance

Jim shops for a new TV.
Faced with two options, he is unsure which one to buy.
Option t (the “target”) has better picture quality.
Option c (the “competitor”) has better reliability.
Only when Jim is shown a “decoy” option d that resembles t but is slightly worse, he feels inclined to choose t over both c and d.

Asymmetric Dominance & Regularity

Asymmetric Dominance

Jim shops for a new TV.
Faced with two options, he is unsure which one to buy.
Option t (the "target") has better picture quality.
Option c (the "competitor") has better reliability.
Only when Jim is shown a "decoy" option d that resembles t but is slightly worse, he feels inclined to choose t over both c and d.

Regularity:

$$X \subseteq Y \quad \Rightarrow \quad P_X(x) \geq P_Y(x) \quad (\forall x \in X).$$

Asymmetric Dominance & Regularity

Asymmetric Dominance

Jim shops for a new TV.
Faced with two options, he is unsure which one to buy.
Option *t* (the "target") has better picture quality.
Option *c* (the "competitor") has better reliability.
Only when Jim is shown a "decoy" option *d* that resembles *t* but is slightly worse, he feels inclined to choose *t* over both *c* and *d*.

Regularity:

$$X \subseteq Y \;\Rightarrow\; P_X(x) \geq P_Y(x) \quad (\forall x \in X).$$

Violations of regularity are broadly viewed as violations of random utility models and random preference models in general.

Best-Choice

Random Utility Model for Best-Choice

$$P_X(x) = \Pr(\mathbf{U}_x = \max_{y \in X} \mathbf{U}_y), \qquad (x \in X \subseteq \mathcal{A}).$$

Random Preference Model for Best-Choice

$$P_X(x) = \sum_{\substack{\succ \in \mathcal{L} \\ B_X(\succ) = x}} P(\succ), \qquad (\forall x \in X \subseteq \mathcal{A}).$$

Best-Choice

Falmagne (*JMP*,1978); Barberá & Pattanaik (*Econometrica*, 1986):

Necessary and sufficient conditions regardless of $|\mathcal{A}|$.

Best-Choice

Falmagne (*JMP*,1978); Barberá & Pattanaik (*Econometrica*, 1986):

Necessary and sufficient conditions regardless of $|\mathcal{A}|$.

Equality constraints such as $\sum_{x \in X} P_X(x) = 1$.
Inequality constraints

$$\sum_{Y:\ X \subseteq Y \subseteq \mathcal{A}} (-1)^{|Y \setminus X|} P_Y(x) \geq 0, \qquad \text{(for all possible } x \in X \subseteq \mathcal{A}\text{)}.$$

Block-Marschak Polynomials

Best-Choice

Block-Marschak Polynomials for $\mathcal{A} = \{a, b, c\}$

$$\begin{aligned} P_{\mathcal{A}}(x) &\geq 0, \qquad (\forall x \in \mathcal{A}), \\ P_X(x) &\geq P_{\mathcal{A}}(x), \quad (\forall X \subset \mathcal{A}, |X| = 2), \\ 1 - P_{\{x,y\}}(x) - P_{\{x,z\}}(x) + P_{\mathcal{A}}(x) &\geq 0, \quad (\forall \{x, y, z\} = \{a, b, c\}), \end{aligned}$$

(using $P_{\{x\}}(x) = 1$)

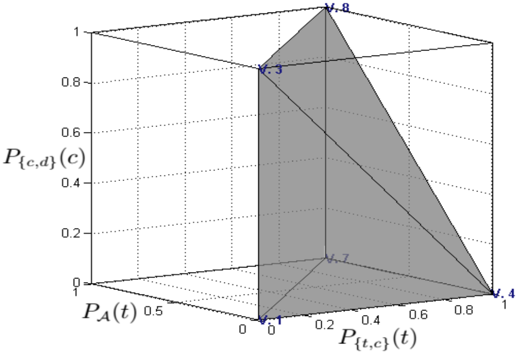
Best-Choice

Block-Marschak Polynomials for $\mathcal{A} = \{a, b, c\}$

$$P_X(x) \quad \geq \quad P_{\mathcal{A}}(x), \quad (\forall X \subset \mathcal{A}, |X| = 2),$$

Linear Ordering Polytope & Regularity

Fiorini (*JMP*, 2004) gave FDI's of Linear Ordering Polytope.



V_1 : *dct*; V_3 : *cdt*, *ctd*; V_4 : *dtc*; V_7 : *tdc*; V_8 : *tcd*

Context-Dependence with Dominance

$t \succ d, t \triangleright d$

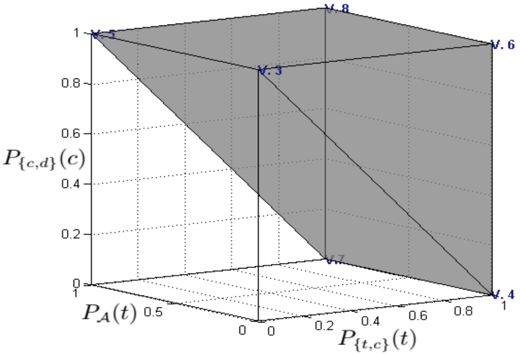
Context-Dependence with Dominance

$t \succ d, t \rhd d$

i	Binary choice	Best ch. from $\mathcal{A}$		joint random utility in binary and best choice from $\mathcal{A}$
1	$t \succ d \succ c$	$t \rhd d \rhd c$	p_1	$[\mathbf{U}_t > \mathbf{U}_d > \mathbf{U}_c] \cap [\mathbf{V}_t > \mathbf{V}_d > \mathbf{V}_c]$
2	$t \succ d \succ c$	$t \rhd c \rhd d$	p_2	$[\mathbf{U}_t > \mathbf{U}_d > \mathbf{U}_c] \cap [\mathbf{V}_t > \mathbf{V}_c > \mathbf{V}_d]$
3	$t \succ d \succ c$	$c \rhd t \rhd d$	p_3	$[\mathbf{U}_t > \mathbf{U}_d > \mathbf{U}_c] \cap [\mathbf{V}_c > \mathbf{V}_t > \mathbf{V}_d]$
4	$t \succ c \succ d$	$t \rhd d \rhd c$	p_4	$[\mathbf{U}_t > \mathbf{U}_c > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_d > \mathbf{V}_c]$
5	$t \succ c \succ d$	$t \rhd c \rhd d$	p_5	$[\mathbf{U}_t > \mathbf{U}_c > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_c > \mathbf{V}_d]$
6	$t \succ c \succ d$	$c \rhd t \rhd d$	p_6	$[\mathbf{U}_t > \mathbf{U}_c > \mathbf{U}_d] \cap [\mathbf{V}_c > \mathbf{V}_t > \mathbf{V}_d]$
7	$c \succ t \succ d$	$t \rhd d \rhd c$	p_7	$[\mathbf{U}_c > \mathbf{U}_t > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_d > \mathbf{V}_c]$
8	$c \succ t \succ d$	$t \rhd c \rhd d$	p_8	$[\mathbf{U}_c > \mathbf{U}_t > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_c > \mathbf{V}_d]$
9	$c \succ t \succ d$	$c \rhd t \rhd d$	p_9	$[\mathbf{U}_c > \mathbf{U}_t > \mathbf{U}_d] \cap [\mathbf{V}_c > \mathbf{V}_t > \mathbf{V}_d]$

Context-Dependence with Dominance

$t \succ d, t \rhd d$ $\qquad P_{\{d,t\}} = P_{\mathcal{A}}(d) = 0; \qquad P_{\{c,t\}}(t) \geq P_{\{c,d\}}(d).$



$V_4 : t \succ d \succ c, c \rhd t \rhd d; \qquad V_5 : c \succ t \succ d, t \rhd d \wedge t \rhd c$

Context-Dependence with Asymmetric Dominance

$t \succ d, t \triangleright d$.

Context-Dependence with Asymmetric Dominance

$t \succ d, t \rhd d$. Require that $t \succ c \Rightarrow t \rhd c$.

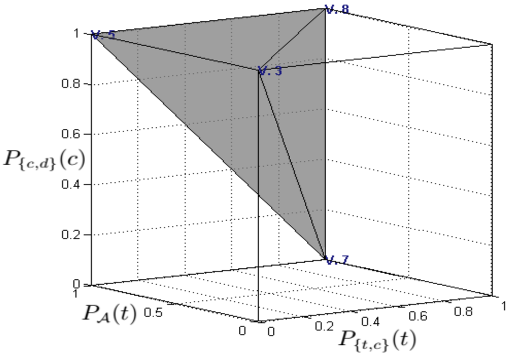
Context-Dependence with Asymmetric Dominance

$t \succ d, t \rhd d$. Require that $t \succ c \Rightarrow t \rhd c$.

i	Binary choice	Best ch. from $\mathcal{A}$		joint random utility in binary and best choice from $\mathcal{A}$
1	$t \succ d \succ c$	$t \rhd d \rhd c$	p_1	$[\mathbf{U}_t > \mathbf{U}_d > \mathbf{U}_c] \cap [\mathbf{V}_t > \mathbf{V}_d > \mathbf{V}_c]$
2	$t \succ d \succ c$	$t \rhd c \rhd d$	p_2	$[\mathbf{U}_t > \mathbf{U}_d > \mathbf{U}_c] \cap [\mathbf{V}_t > \mathbf{V}_c > \mathbf{V}_d]$
4	$t \succ c \succ d$	$t \rhd d \rhd c$	p_4	$[\mathbf{U}_t > \mathbf{U}_c > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_d > \mathbf{V}_c]$
5	$t \succ c \succ d$	$t \rhd c \rhd d$	p_5	$[\mathbf{U}_t > \mathbf{U}_c > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_c > \mathbf{V}_d]$
7	$c \succ t \succ d$	$t \rhd d \rhd c$	p_7	$[\mathbf{U}_c > \mathbf{U}_t > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_d > \mathbf{V}_c]$
8	$c \succ t \succ d$	$t \rhd c \rhd d$	p_8	$[\mathbf{U}_c > \mathbf{U}_t > \mathbf{U}_d] \cap [\mathbf{V}_t > \mathbf{V}_c > \mathbf{V}_d]$
9	$c \succ t \succ d$	$c \rhd t \rhd d$	p_9	$[\mathbf{U}_c > \mathbf{U}_t > \mathbf{U}_d] \cap [\mathbf{V}_c > \mathbf{V}_t > \mathbf{V}_d]$

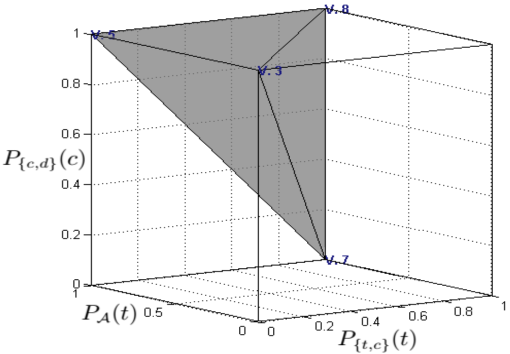
Context-Dependence with Asymmetric Dominance

$t \succ d, t \rhd d$. Require that $t \succ c \Rightarrow t \rhd c$.



Context-Dependence with Asymmetric Dominance

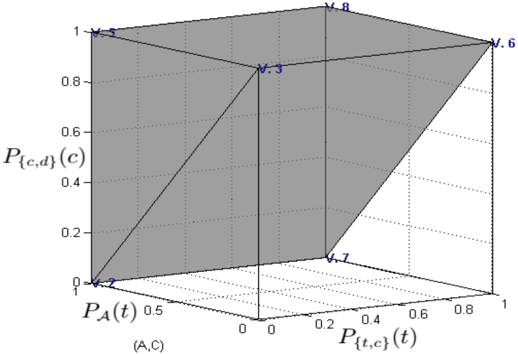
$t \succ d, t \rhd d$. Require that $t \succ c \Rightarrow t \rhd c$.



$$P_{\{t,c\}}(t) \geq P_{\{d,c\}}(d); \quad \mathbf{P_{\{t,c\}}(t) \leq P_{\mathcal{A}}(t)}.$$

Context defined by absence or presence of *d*.

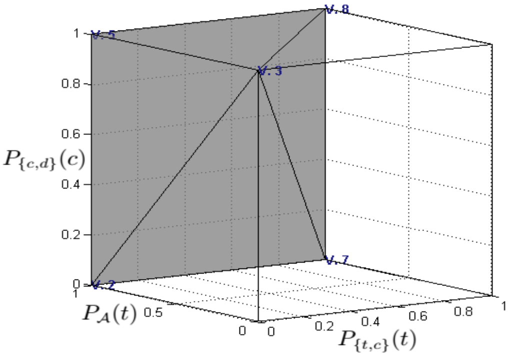
Context defined by absence or presence of *d*.



$$P_{\{c,d\}}(d) \geq P_{\mathcal{A}}(t)$$

Absence or presence of d; Asymm. dom.

Context defined by absence or presence of d. Require $t \succ c \Rightarrow t \rhd c$.



$$P_{\{c,d\}}(d) \geq P_{\mathcal{A}}(t) \quad \mathbf{P}_{\{\mathbf{t},\mathbf{c}\}}(\mathbf{t}) \leq \mathbf{P}_{\mathcal{A}}(\mathbf{t}).$$

Statistical Analysis

Bayes Factors

Context-independent RUM (regularity):	.004
Model 1A:	2.00
Model 1B (reverse regularity):	6.01
Model 2A:	1.99
Model 2B (reverse regularity):	3.00

Work with Johannes Müller-Trede.

Outline

Context-Dependent Random Utility Model

Context-dependent Random Utility Model for Best-Choice

$$P_X^{\Gamma}(x) = \Pr(\mathbf{U}_x^{\Gamma} = \max_{y \in X} \mathbf{U}_y^{\Gamma}), \qquad \text{(for all possible } x \in X \subseteq \mathcal{A}),$$

Context-Dependent Random Utility Model

Context-dependent Random Utility Model for Best-Choice

$$P_X^\Gamma(x) = \Pr(\mathbf{U}_x^\Gamma = \max_{y \in X} \mathbf{U}_y^\Gamma), \qquad \text{(for all possible } x \in X \subseteq \mathcal{A}),$$

Context-dependent Random Utility Model for Best-Worst-Choice

$$P_X^\Gamma(x, y) = \Pr(\mathbf{U}_x^\Gamma = \max_{v \in X} \mathbf{U}_v^\Gamma, \mathbf{U}_y^\Gamma = \min_{w \in X} \mathbf{U}_w^\Gamma), \qquad (x \neq y \in X \subseteq \mathcal{A}),$$

Conclusions

Building a context-dependent RUM or RP model

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Conclusions

Building a context-dependent RUM or RP model

- List every permissible best (best-worst) choice for every context.

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- List every permissible best (best-worst) choice for every context.
- These patterns define the vertices of a convex polytope.

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Conclusions

Building a context-dependent RUM or RP model

- List every permissible best (best-worst) choice for every context.
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- Use math or software to characterize facet-structure.

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Conclusions

Building a context-dependent RUM or RP model

- List every permissible best (best-worst) choice for every context.
- These patterns define the vertices of a convex polytope.
- Use math or software to characterize facet-structure.
- Use order-constrained freq. or Bayesian inference (e.g., QTEST)

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