Classical Probability Model for an Arbitrary Experimental Setup

Andrei Khrennikov

Center Math Modeling in Physics and Cognitive Sciences, Linnaeus University, Växjö , Sweden

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D. Avis, P. Fischer, A. Hilbert, A. Khrennikov, Single, Complete, Probability Spaces Consistent With EPR-Bohm-Bell Experimental Data. In: *Foundations of Probability and Physics-5*, AIP Conference Proceedings, 1101, 294-301 (2009).

A. Khrennikov, CHSH inequality: quantum probabilities as classical conditional probabilities *Found. Phys.* 45, N 7, 711-725 (2015).

Dzhafarov, E. N., & Kon, M. (2018). On universality of classical probability with contextually labeled random variables. *Journal of Mathematical Psychology, 85*, 17-24.

EPR-Bohm-Bell experiment

There are considered four observables A_1, A_2, B_1, B_2 taking values $\pm 1.$ It is assumed that the pairs of observables $(A_i, B_j), i, j = 1, 2,$ can be measured jointly, i.e., A-observables are compatible with B-observables. Probability distributions $p_{A_i B_j}$ can be verified experimentally.

Observables in pairs A_1, A_2 and B_1, B_2 are incompatible.

This is the standard presentation of the EPR-Bohm-Bell experiment.

One tries to map this observational scheme onto a CP-model by representing observables A_1, A_2, B_1, B_2 by random variables (RVs) $a_1, a_2, b_1, b_2 = \pm 1.$.

This correspondence is based on identification of observational probabilities $p_{A_i B_j}$ with jpds $p_{a_i b_j}$ of RVs.

In CP, RVs a_1, a_2, b_1, b_2 have the jpd, $p(\alpha_1, \alpha_2, \beta_1, \beta_2)$.

In particular, jpds for pairs a_i, a_j and b_i, b_j also should exist. This should be surprising! Since these observables are incompatible!

If one's aim is not simply **confrontation with the principle of complementarity**, then he should assume that jpds p_{a_i,a_j} and p_{b_i,b_j} are simply mathematical quantities.

In any event, by assuming the CP-representation of observables with identification $p_{A_iB_j} = p_{a_ib_j}$, one comes to contradiction: CP-correlations satisfy the CHSH-inequality, but observational correlations violate it.

In the CP-model one can form the CHSH linear combination of correlations for pairs of RVs $\boldsymbol{a_i, b_j}$

$$(1) \qquad \boldsymbol{B = \langle a_1 b_1 \rangle - \langle a_1 b_2 \rangle + \langle a_2 b_1 \rangle + \langle a_2 b_2 \rangle}$$

and prove the CHSH-inequality:

$$(2) \qquad \boldsymbol{|B| \leq 2.}$$

Here

(3)

$$\boldsymbol{\langle a_i b_j \rangle \equiv E(a_i b_j) = \int_\Lambda a_i(\lambda) b_j(\lambda) dP(\lambda) = \sum_{\alpha,\beta} \alpha\beta p_{a_i b_j}(\alpha, \beta).}$$

where, e.g., $\boldsymbol{p_{a_1 b_1}(\alpha, \beta) = \sum_{x,y} p(\alpha, x, \beta, y).}$

The crucial point is the straightforward identification of observational and CP probabilities and hence correlations:

$$\boldsymbol{\langle a_i b_j \rangle = \langle A_i B_j \rangle.}$$

Identification of observational and CP probabilities is not so trivial as CHSH did.

It is a complex problem. Moreover, there is a crucial difference between justification of this identification in the original Bell inequality and in CHSH inequality.

We shall be back to this problem.

Here I just remark that De Broglie claimed that there is no reason for this identification and this is the main counter-argument against the common interpretation of the Bell-type inequalities, see

A. Khrennikov, After Bell. *Fortschritte der Physik (Progress in Physics)* **65, N 6-8, 1600014 (2017).**

The contradiction implied by a violation of the CHSH-inequality by observational probabilities could be expected from the very beginning by paying attention to the incompatibility issue. I recall that the EPR-paper was directed against the complementarity principle...

Therefore **I am not sure that the Bell-CHSH "project" can bring something complement to the complementarity principle.**

A. Khrennikov, Bohr against Bell: complementarity versus nonlocality. *Open Physics*, **15, N 1., (2017).**

A. Plotnitsky and A. Khrennikov, Reality without realism: On the ontological and epistemological architecture of quantum mechanics. *Found. Phys.* **45, N 10, 1269-1300 (2015).**

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To resolve this contradiction, one should reject either realism in the form of the above representation of observables by RVs or noncontextuality.

CP-description of contextuality can be presented either in the form of a manifold of Kolmogorov probability spaces coupled with the aid of transition probabilities, see

A. Khrennikov, *Contextual approach to quantum formalism,* **Springer, Berlin-Heidelberg-New York, 2009.**

Another possibility is to proceed withing a single Kolmogorov probability space but reject the possibility of single-index labeling of RVs, see

Dzhafarov, E. N., & Kujala, J. V. (2016). Context-content systems of random variables: The contextuality-by default theory. *Journal of Mathematical Psychology, 74,* **11-33.**

The lost of identity of an observable via its mathematical representations, either in the multi-space or multi-RVs approach, was always disturbing me.

Of course, context dependence is a natural justification for the use of such mathematical representations.

However, it seems that "contextuality" cannot explain why Alice's PBS with the fixed orientation should have different mathematical representations depending whether on the Moon Bob uses one or another orientation of his PBS.

Alice "has the right" to has her own observable with its own concrete mathematical presentation.

Missed component of experimental arrangement

Correlations cannot be jointly measured. The concrete experiment can be performed only for one fixed pair of indexes (i, j), experimental settings.

Generally these settings are selected by using two random generators $R_A, R_B = 1, 2$. taking values $1, 2$. They are a part of the experimental context - two additional observables missed in the standard observational scheme.

Where are these random generators in in the above theoretical considerations? They are absent!

One sort of randomness, namely, generated by R_A, R_B is missed.

The Copenhagen interpretation of QM, Bohr's version: all components of experimental arrangement (context) have to be taken into account.

Experimenters strictly follow the Copenhagen interpretation. Random generators play the fundamental role in the experiments .

However, these generators are not present neither in the standard observational scheme with observables A_1, A_2, B_1, B_2 nor in the CP-model with RVs a_1, a_2, b_1, b_2 and the Bell-CHSH correspondence rule:

$$p_{A_i B_j} = p_{a_i b_j}$$

Thus the commonly told “story” about the the EPR-Bohm-Bell experiment is inadequate to the real experimental situation.

See also:

M. Kupczynski, Can we close the BohrEinstein quantum debate? Phil. Trans. R. Soc. A 375, 20160392.

CP-model adequate to the EPR-Bohm-Bell experiment

Probability space (Λ, F, P)

Observables A_1, A_2, B_1, B_2, are represented by RVs a_1, a_2, b_1, b_2.

Additionally two RVs $r_A, r_B = 1, 2$ are associated with the random generators R_A, R_B.

Besides values ± 1, RVs a_1, a_2, b_1, b_2 can take value zero.

zero-value is used to describe governing of selection of experimental settings by random generators:

- **$a_i = 0$ (with probability one), if the i-setting was not selected, i.e., $r_A \neq i$;**
- **$b_j = 0$ (with probability one), if the j-setting was not selected, i.e., $r_B \neq j$.**

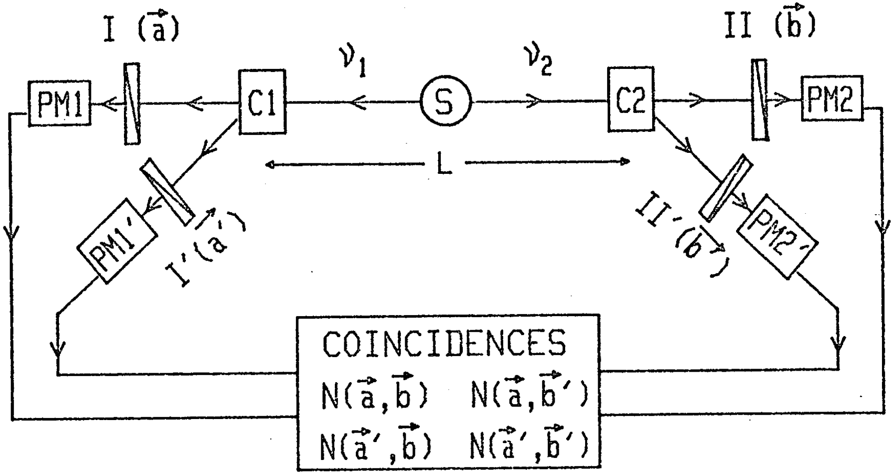

FIGURE 1. The scheme of the pioneer experiment of A. Aspect with four beam splitters [?].

"We have done a step towards such an ideal experiment by using the modified scheme shown on the figure. In that scheme, each (single-channel) polarizer is replaced by a setup involving a switching device followed by two polarizers in two different orientations: a_1 and a_2 on side I, b_1 and b_2 on side II. The optical switch $C1$ is able to rapidly redirect the incident light either to the polarizer in orientation a_1, or to the polarizer in orientation a_2. This setup is thus equivalent to a variable polarizer switched between the two orientations a_1 and a_2. A similar set up is implemented on the other side, and is equivalent to a polarizer switched between the two orientations b_1 and b_2. "

Observational probabilities as conditional classical probabilities

Consider the observational probabilities p_{A_i,B_j}. They are obtained for the fixed pair of experimental settings (i, j).

CP-counterparts of observational probabilities are obtained by conditioning on the fixed values of random variables r_A and r_B.

Thus coupling between observational and CP-probabilities is based on the following identification:

$$p_{A_iB_j}(\alpha, \beta) = P(a_i = \alpha, b_j = \beta | r_A = i, r_B = j), \tag{4}$$

where $\alpha, \beta = \pm 1$. Thus

$$p_{A_iB_j}(\alpha, \beta) = \frac{P(a_i = \alpha, b_j = \beta, r_A = i, r_B = j)}{P(r_A = i, r_B = j)}. \tag{5}$$

Conditioning on the selection of experimental settings plays the crucial role. The CP-correlations are based on the conditional probabilities

$$\left\langle a_i b_j \right\rangle \equiv E(a_i b_j | r_A = i, r_B = j) \tag{6}$$

$$= \sum_{\alpha,\beta=\pm 1} \alpha\beta P(a_i = \alpha, b_j = \beta | r_A = i, r_B = j).$$

We can form the CHSH linear combination of conditional correlations of RVs:

$$\tilde{B} = \left\langle a_1 b_1 \right\rangle - \left\langle a_1 b_2 \right\rangle + \left\langle a_2 b_2 \right\rangle + \left\langle a_2 b_2 \right\rangle \tag{7}$$

It is possible to find such classical probability spaces that

$$|\tilde{B}| > 2.$$

Since each conditional probability is also a probability measure and since RVs a_i, b_j take values in [-1, +1], the conditional expectations $E(a_i b_j | r_A = i, r_B = j)$ are bounded by 1, so

$$|\tilde{B}| \leq 4.$$

Thus the common claim on mismatching of the CP-description with QM and experimental data was not justified.

Of course, one can consider B composed of correlations $\langle a_1 b_1 \rangle$ which are not conditioned on selection of experimental settings. Such B satisfies the CHSH-inequality. But such correlations cannot be identified with experimental ones, cf. De Broglie.

No-signaling in quantum physics

By definition there is no signaling from the B-side to the A-side if

$$\sum_{\beta} p_{A_i B_j}(\alpha, \beta)$$

does not depend on experimental setting j.

This definition is done at the level of probabilities. Therefore its real experimental meaning is not so clear.

In physics, signaling is often understood as real signaling from the B-side to the A-side and even, what is worse, from the B system to the A-system.

By constructing the CP-model, we can clarify the meaning of (no-)signaling at the level of observations.

No-signaling as condition of independence

Let us fix $r_a = i$. For any value $r_b = j$, consider the quantity

$$\sum_{\beta} P(a_i = \alpha, b_j = \beta | r_a = i, r_b = j) = P(a_i = \alpha | r_a = i, r_b = j)$$

It does not depend on the j-settings governed by r_b iff the following condition holds:

$\mathrm{I}_{\mathrm{a_i}}$ The pair of random variables a_i, r_a does not depend on r_b.

Under this condition we have

$$\sum_{\beta} P(a_i = \alpha, b_j = \beta | r_a = i, r_b = j) = P(a_i = \alpha | r_a = i).$$

This is the conditional version of no-signaling for a_i.

In the same way,

$\mathrm{I_{b_j}}$ The pair of random variables b_j, r_b does not depend on r_a.

Under this condition we have

$$\sum_{\alpha} P(a_i = \alpha, b_j = \beta | r_a = i, r_b = j) = P(b_j = \beta | r_b = j).$$

This is the conditional version of no-signaling for random variable b_j.

The CP-presentation of no-signaling in terms of conditions $\mathrm{I_a}, \mathrm{I_b}$ explains the meaning of signaling.

For example, $b \to a$ signaling means either interdependence of random generators, or dependence of a-variables on random generator r_b.

Under the condition of independence of r_a and r_b, $b \to a$ signaling has the meaning dependence of a-variables on random generator r_b, i.e., the latter governs not only b-variables, but even the a-variables. And nothing more!

Interrelation of (no-)signaling for observables and random variables

By using the correspondence rule between the observational and CP-probabilities we can lift the CP-interpretation of signaling to the level of observables:

(8)
$$\sum_\beta P(a_i = \alpha, b_j = \beta | r_a = i, r_b = j) = \sum_\beta p_{A_i B_j}(\alpha, \beta).$$

The absence of $B \to A$ signaling for observables, i.e., independence of the right-hand side of index j, is equivalent to the absence of $b \to a$ signaling RVs. At observational level $B \to A$ no-signaling is as independence of A-observables from selection of experimental settings governed by random generator R_B.

Thus, no-signaling and signaling for quantum observables have very natural explanation.

Signaling in CHSH-experiments

It seems that before our study with Guillaume Adenier the data from Aspect's and Weih's experiments, this topic was not present in experimental papers at all. People were happy that they violate the CHSH-inequality and as much as possible. They did not pay attention that their data contains statistically significant signaling patterns, see

G. Adenier and A. Khrennikov, Is the fair sampling assumption supported by EPR experiments? *Journal of Physics B: Atomic, Molecular and Optical Physics*, **40, 131-141 (2007).**

In fact, even the first loophole free experiment performed in Delf in 2015 also suffers of the signaling loophole:

G. Adenier and A. Khrennikov, Test of the no-signaling principle in the Hensen loophole-free CHSH experiment. *Fortschritte der Physik (Progress in Physics)*, **65, N. 9, 1600096.**

It happened that all experiments which we checked for signaling demonstrated signaling. We did not check data from 2015-experiments in Vienna and NIST, but it would be interesting to do, independently from Jan-ke Larsson's analysis (as one of coauthros).

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Signaling in psychology

As was found, see

Dzhafarov, E. N., Zhang, R., & Kujala, J.V. (2015). Is there contextuality in behavioral and social systems? *Philosophical Transactions of the Royal Society: A, 374*, **20150099.**

signaling is present in all known data.

Question: It is the fundamental feature of cognitive systems or just "badly performed" experiments?

Complementarity versus contextuality

Bohr's position on the meaning of complementarity (incompatibility);

- **(B1): An output of any observable is composed of contributions from a system under measurement and the measurement device.**
- **(B2): Therefore the whole experimental arrangement (context) has to be taken into account.**
- **(B3): There is no reason to expect that all experimental contexts can be combined. Therefore there is no reason to expect that all observables can be measured jointly. This is the essence of** *the principle of complementarity.* **Hence, there can exist incompatible observables. Their existence is proved by interference experiments.**

(B1): Bohr's contextuality of QM.

My works on contextuality represented mathematically as a manifold of Kolmogorov probability spaces were done in Bohr's framework. Here contextuality means non-Kolmogorovness.

No contextuality without complementarity: otherwise all observables can be measured in the same context and there is no meaning to consider context dependent Kolmogorov spaces.

Now, turn to quantum mechanics: here we have the notion of Bell's contextuality expressed in violation of the Bell type inequalities.

Is contextuality reduced to complementarity?