Quantum Measurements and Contextuality

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Overview

□ John Bell introduced a notion of "contextual" and argued that quantum mechanics is *contextual*.

– Measurement results depend on their *context*

- I claim Bell was *mistaken*:

– Quantum mechanics is *not* Bell-contextual

- Need to understand *quantum measurements*

– Measurement outcomes (1st measurement problem)

– Measured properties (2d measurement problem)

- Need *probabilities* to discuss measurements

– Proper way to introduce probabilities in QM

□ Current use of "contextual" $\neq$ Bell's "contextual"

- But its application to QM is not satisfactory

– Again, the issue is *quantum measurements*

Bell Quantum Contextuality I

□ Quantum physical quantities $\leftrightarrow$ *operators*, not numbers

• Energy, momentum, angular momentum, etc., all represented by operators

□ Operators may *commute*, $AB = BA$, then
∘ A and B are *compatible*, can be measured *simultaneously*

□ If operators do *not commute*, $BC \neq CB$:

∘ B and C are *incompatible*, *cannot* be measured simultaneously
Must be measured *separately* in *different* runs

∘ Suppose B was measured. What *would have been* the value of C *if instead* C had been measured?
– Counterfactual question. No answer if $BC \neq CB$.

Bell Quantum Contextuality II

□ Let A, B, C be operators for three physical quantities

$$AB = BA; \quad AC = CA; \quad BC \neq CB$$

• A compatible with both B and C, but B and C are incompatible.

∘ Example: Spherically-symmetrical potential. $A = H$ the energy; $B = J_x$, $C = J_y$, the x and y components of angular momentum.

□ Does it make a difference if A measured with B or measured with C?

• A measured with $B \rightarrow A = a$. Would outcome $A = a$ have been the same if *in this run* A had been measured with C instead of B?

∘ Bell: No, or not necessarily. QM is contextual

∘ Griffiths: Yes, as I will show you. QM is *not* contextual

□ The counterfactual question does *NOT* refer to the *probability distribution* $\Pr(a)$ of outcomes of the A measurement

∘ Everyone agrees $\Pr(a)$ same if A measured with B or with C

Quantum Measurements I

☐ Physical quantity $A = \sum_j a_j P_j$,

– a_j are eigenvalues (possible values) of A; the P_j are projectors

☐ The $\{P_j\}$ form a *projective decomposition of the identity* (PDI) $=$ *quantum sample space* $=$ *framework*:

$$I = \text{ Identity } = \sum_j P_j, \quad P_j = P_j^\dagger = P_j^2, \quad P_j P_k = P_k P_j = \delta_{jk} P_j$$

- Sample space: Mutually exclusive possibilities, one of which is true.

– Classical coin: HEADS or TAILS. Quantum spin half: UP or DOWN

- If eigenvalues a_j of A are nondegenerate, $P_j = |a_j\rangle\langle a_j|$

- Geometry: Each P_j projects on a subspace of Hilbert space *orthogonal* to the other subspaces

☐ (Ideal) *measurement* of A gives: some $a_j \leftrightarrow$ this P_j is *true*

☐ If $A = \sum_j a_j P_j$ *commutes* with $B = \sum_k b_k Q_k$, then $P_j Q_k = Q_k P_j$

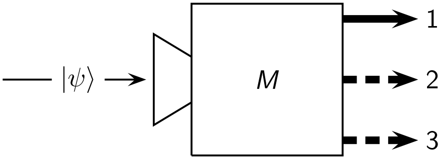
- Joint measurement of A *and* B: PDI $\leftrightarrow$ nonzero $\{P_j Q_k\}$.

Quantum Measurements II

□ Schematic measurement apparatus M

◦ Particle to be measured arrives from left

◦ Apparatus pointer gives measurement outcome



□ To measure $A = \sum_j a_j |a_j\rangle\langle a_j|$:

◦ When $|\psi\rangle = |a_j\rangle$ enters, apparatus $\to |\Phi_j\rangle$ (unitary time development of particle + apparatus)

- Macroscopic pointer PDI $\{M_k\}$: $M_k|\Phi_j\rangle = \delta_{jk}|\Phi_j\rangle$
  – M_k subspace (macroscopic) $\leftrightarrow$ pointer points at symbol k.
- So if $|\psi\rangle = |a_j\rangle$ enters apparatus, pointer will point at symbol j.

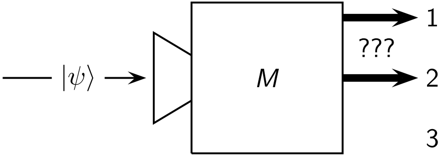
□ If $|\psi\rangle = c_1|a_1\rangle + c_2|a_2\rangle \to c_1|\Phi_1\rangle + c_2|\Phi_2\rangle$ (Schrödinger cat)

◦ Will pointer point at 1? at 2? at both? neither?

- The FIRST measurement problem

Solution to First Measurement Problem

☐ $|\psi\rangle = c_1|a_1\rangle + c_2|a_2\rangle$
$\rightarrow |\Phi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle$
(Recall: $M_k|\Phi_j\rangle = \delta_{jk}|\Phi_j\rangle$)
• Pointer in superposition $|\Phi\rangle$
How shall we interpret it?



☐ Born: Use wavefunction (ket) to calculate probabilities!
• Qm probabilities require PDI = framework = sample space
– No sample space, no probability!

☐ Use PDI $\{M_k\}$; $\Pr(M_k) = \langle\Phi|M_k|\Phi\rangle$ (Born Rule)
∘ Pointer at 1 with probability $|c_1|^2$, at 2 with probability $|c_2|^2$.
∘ $|\Phi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle$ is calculational tool, not physical reality.

☐ Why use PDI $\{M_k\}$ and not some other framework or PDI?
∘ E.g., $\{|\Phi\rangle\langle\Phi|, I - |\Phi\rangle\langle\Phi|\}$ is a PDI, and $\rightarrow \Pr(|\Phi\rangle\langle\Phi| = 1)$
– This PDI incompatible with $\{M_k\}$; if we use it, pointer position makes no sense, cannot be discussed.
– Schrödinger cat is not a cat!

Second Measurement Problem

☐ If we use the $\{M_k\}$ (pointer) PDI or framework, and pointer is at $k = 2$, what can we say about earlier state of the particle?

- Approach of experimental physicist:
  - ∘ Calibration. For various j send in $|a_j\rangle$, leads to M_j?
  - ∘ Once apparatus has passed calibration test,
  - – From pointer position M_j *infer* earlier particle state $|a_j\rangle$ (or, more generally P_j).
- But is this good QM? Maybe earlier state was $|\psi\rangle = c_1|a_1\rangle + c_2|a_2\rangle$

☐ It *is* good QM. For this we need appropriate framework or PDI.

Quantum Histories

□ Sample space for flipping coin twice:

$$\{H,T\}\odot\{H,T\} = H\odot H,\ H\odot T,\ T\odot H,\ T\odot T$$

Possibilities indicated by 'first outcome $\odot$ second outcome'.

,

□ PDI for quantum measurement, input $|\psi\rangle = c_1|a_1\rangle + c_2|a_2\rangle$:

$$\{P_1,P_2\}\odot\{M_1,M_2\} = P_1\odot M_1,\ P_1\odot M_2,\ldots$$

- Probabilities computed using (extended) Born Rule:

$$\Pr(P_1,M_1)=|c_1|^2,\ \Pr(P_2,M_2)=|c_2|^2,\ \Pr(P_1,M_2)=\Pr(P_2,M_1)=0$$

  ◦ These imply conditional probabilities

$$\Pr(P_1|M_1)=1,\quad \Pr(P_2|M_2)=1$$

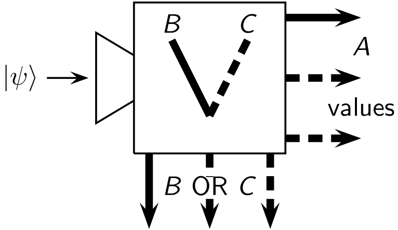
  – Pointer at position $M_1 \Rightarrow$ particle earlier in state $P_1 = |a_1\rangle\langle a_1|$;
  – Similarly $M_2 \Rightarrow P_2 = |a_2\rangle\langle a_2|$ at earlier time.

□ Experimenter's view confirmed using QM and appropriate PDI

Bell Quantum Contextuality III

□ Measure A with B or with C, $AB = BA$; $AC = CA$; $BC \neq CB$

- Lever setting determines if B or C measured with A
  - Pointer on right: A
  - Bottom pointer: B or C



- Lever at B (or C) measured values of A, B (or C) shown by pointers

□ Calibration $\Rightarrow$ Pointer $A \leftrightarrow A$ value BEFORE measurement, BEFORE particle reached apparatus, so NOT influenced by later lever position.

- Value of A measured with B same as had it been measured with C.

□ Conclusion: Quantum Mechanics is *NOT* Bell *CONTEXTUAL*, i.e., it *IS* Bell NONcontextual

Bell Contextuality: Summary

□ $AB = BA,\ AC = CA,\ BC \neq CB$

• A measured with B; would $A = a$ outcome have been the same *in this run* if instead A had been measured with C?

∘ Answer: "Yes." QM is *non*contextual. Demonstration requires:

– Solve 1st measurement problem: pointer in definite position.

– Solve 2d measurement problem: pointer position $\Rightarrow$ prior value

∘ Tools:

– Quantum sample space = PDI (projective decomposition of identity)

– Single framework rule: incompatible PDIs cannot be combined

– Choice of appropriate PDI(s) or framework(s)

– Measurement outcome $\leftrightarrow$ property of particle *before* measurement

– Plausible counterfactual construction

Non-Bell Definitions of Contextuality

☐ "QM is contextual"' claims often reference Bell, Kochen & Specker

◦ Bell had a clear definition

– By that definition QM is *NOT* contextual

◦ Kochen & Specker: paper does not mention 'contextual'

☐ Abramsky et al., Phys. Rev. Lett. 119 (2017) 050504

◦ Collection of measurements, some compatible, some incompatible

◦ "Context" = subcollection of *compatible* measurements

◦ Empirical model → joint probabilities for measurement outcomes for each context

◦ Marginals agree on overlaps of contexts

• "An empirical model is said to be *contextual* if this family of distributions *cannot* itself be obtained as the marginals of a single probability distribution on global assignments of outcomes to all measurements"

◦ Let's look at an example

Example of (Non)Contextuality

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

- $AB = BA$, $AC = CA$, $BC \neq CB$. Two contexts: $\{A, B\}$ and $\{A, C\}$
  - Eigenvalues a of A, b of B, and c of C are $+1$, -1
- Empirical model: Density operator $\rho = \begin{pmatrix} p & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{pmatrix}$; $p + 2r = 1$
- Probability tables: $\{A, B\}$ & $\{A, C\}$ using density operator

$\{A, B\}$

$b =$	1	-1
$a = 1$	r	r
$= -1$	p	0

$\{A, C\}$

$c =$	1	-1
$a = 1$	r	r
$= -1$	p	0

$\{A, B, C\}$

$b = c =$	1	-1	$b \neq c$
$a = 1$	r	r	0
$= -1$	p	0	0

- The $\{A, B, C\}$ distribution $\rightarrow$ $\{A, B\}$, $\{A.C\}$ as marginals
- So this empirical model is *noncontextual*

Example of (Non)Contextuality (continued)

- $AB = BA$, $AC = CA$, $BC \neq CB$. Two contexts: $\{A, B\}$ and $\{A, C\}$

$\{A, B\}$

$b =$	1	-1
$a = 1$	r	r
$= -1$	p	0

$\{A, C\}$

$c =$	1	-1
$a = 1$	r	r
$= -1$	p	0

$\{A, B, C\}$

$b = c =$	1	-1	$b \neq c$
$a = 1$	r	r	0
$= -1$	p	0	0

- The $\{A, B, C\}$ distribution $\rightarrow$ $\{A, B\}$, $\{A.C\}$ as marginals

☐ This empirical model is *noncontextual*? I consider it *nonsense*

- $BC \neq CB \Rightarrow$ the $\{A, B, C\}$ distribution is a fairy tale!
  - Need a *sample space* (PDI) to assign probabilities!
  - $AB = BA$, so a common PDI. Likewise $AC = CA$.
  - But $BC \neq CB \leftrightarrow$ no common PDI; cannot assign probabilities!

☐ Consider $\{A, B, C\} \leftrightarrow$ macroscopic measurement outcomes?

- But given $BC \neq CB$, *WHAT did these measurements measure?*
  - Can measurements that measure nothing be good quantum physics?

Conclusion

□ Making sense of 'contextual' (Bell or later) requires *understanding quantum measurements* and what it is that measurements measure

• Bell's (almost) last paper: "Against Measurement". Discussions of quantum measurements known to him (in 1990) were unsatisfactory

□ My Consistent Histories (CH) approach: Compatible (projective) measurements $\leftrightarrow$ microscopic quantum properties: quantum sample space, or framework, or projective decomposition of the identity (PDI)

∘ CH has a formulation of quantum measurements [1,2] that handles both measurement problems. It was not available to Bell.

∘ If you don't like it you are in good company (d'Espagnat, Ghirardi, Kent, Maudlin, Mermin, . . .)

– But then you need to come up with something better!

[1] Chs. 17, 18 of R. B. Griffiths, Consistent Quantum Theory (Cambridge 2002) http://quantum.phys.cmu.edu/CQT/

[2] R. B. Griffiths, "What Quantum Measurements Measure", Phys. Rev. A 96 (2017) 032110. arXiv:1704.08725