Purdue Winer Memorial Lectures 2018:
Probability and Contextuality

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Abstracts

Samson Abramsky (Part I) and Giovanni Carù (Part II)

University of Oxford, UK

The sheaf-theoretic approach to contextuality

The goal of this paper is to establish a strong link between two apparently unrelated topics: the study of information in the formal framework of valuation algebras, and the phenomena of non-locality and contextuality in quantum theory. In particular, by adopting Abramsky & Brandenburger's sheaf-theoretic description of contextuality, we show that this highly non-classical phenomenon is in fact mathematically equivalent to an incoherent solution to an inference problem. This result further proves that contextuality is not a phenomenon limited to quantum mechanics, but pervades various domains of mathematics and computer science. Moreover, the connection allows us to apply the well-studied efficient algorithms for single-query inference problems to the task of detecting contextuality in empirical models. Finally, we can use this result to end the quest for a full cohomology invariant for non-locality and contextuality. More specifically, we present a sheaf cohomology complete invariant which is ultimately inspired by the valuation algebra framework and based on network codings.

Part I will introduce the sheaf-theoretic approach, including the use of cohomology to detect contextuality, and the connections to constraint satisfaction and other computational problems.

Part II will develop the connection with valuation algebras, and the results on complete cohomology invariants for contextuality.

Bárbara Lopes Amaral

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Resource theory of contextuality for arbitrary prepare-and-measure experiments

(joint work with Christhiano Duarte)

Contextuality has been identified as a potential resource responsible for the quantum advantage in several tasks. It is then necessary to develop a resource-theoretic framework for contextuality, both in its standard and generalized forms. In this work we provide a formal resource-theoretic approach for generalized contetuality based on a physically motivated set of free operations with an explicit parametrization. Then, using an efficient linear programming characterization for the noncontextual set of prepared-and-measured statistics, we adapt known resource quantifiers for contextuality and nonlocality to obtain natural monotones for generalized contextuality in arbitrary prepare-and-measure experiments.

Costantino Budroni

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Contextuality, memory cost, and nonclassicality for sequential quantum measurements

In this talk, we will review basic notions and results connecting Kochen-Specker original approach to the sequential measurement scenario, together with a more recent extension of such an approach to the general problem of temporal correlations. First, we will motivate the choice of the sequential measurement scenario to overcome the difficulties associated with Kochen-Specker contextuality and imperfect measurements. Moreover, we will discuss the advantages and limitations of such an approach. In the sequential measurement scenario, one can naturally identify memory as the key resource to classically simulate correlations. We will review basic results on the "memory cost" of simulating contextual correlations. Finally, motivated by the above results, we will discuss a possible extension of this approach that takes the notion of memory as fundamental to define a notion of nonclassicality beyond that of quantum contextuality. We will present recent results on temporal correlations, in particular, inequalities able to distinguish classical and quantum theories with different amounts of memory available.

Jerome Busemeyer

Indiana University, USA

Contextuality in strategic games

(joint work with Zheng Joyce Wang)

Does contextuality occur in strategic game situations? First, we present empirical evidence for violations of marginal invariance from our recent experiments using a new design of the centipede game. In the experiments, under a predict-act condition, a player first predicts an opponent's move (defect or cooperate) and then takes his or her own action (defect or cooperate). In comparison, under an act-alone condition, the player simply takes the action without making any explicit prediction about the opponent. Contrary to what is expected by traditional game theory, we observed significant differences between the total probability of defecting obtained from the predict-act condition as compared to that obtained from the act-alone condition. Second, we propose a game of chicken in which two players, A and B, are racing toward each other before they enter a narrow one-lane bridge. If neither player yields, they crash into each other in the narrow one lane bridge (payoff -1 million). If both yield before the bridge, there is no crash, but they both look like chickens (-10 payoff for each player). If B yields first, then A is the winner and B is the chicken ($+10$ for A, -10 for B). Likewise, if A yields first, then B is the winner and A is the chicken (-10 for A, $+10$ for B). There are two equally attractive pure equilibrium in this game (one yields and the other does not). Now suppose we collect predictions by participants made for games involving each of the possible pairs of three players A, B, C. This produces three contingency tables: a 2 by 2 table containing predictions for choice pairs for players A, B; another for players B, C; and a third for players A, C. These tables are expected to violate the " context by default " contextually criterion. What do these violations (marginal invariance, context by default criterion) imply for the psychology of strategic decision making? We will discuss these implications.

Adán Cabello

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The totalitarian principle explains quantum correlations

Wheeler conjectured that the laws of physics follow from an ultimate law which cannot be itself a law of physics. Hence the expression "law without law." In elementary-particle physics, Gell-Mann observed that any process which is not forbidden by a conservation law actually does take place. This is called "Gell-Mann's totalitarian principle." A long-standing problem in physics is what is the property of nature that singles out the set of quantum correlations for each measurement scenario. Here we show that the "no-law/totalitarian principle," stating that nature lacks of laws governing the outcomes of some experiments and, as a consequence, every outcome probability distribution that is not inconsistent must take place as a result of some reproducible experimental procedure, is enough to single out the set of quantum correlations for any Bell and contextuality scenario.

Víctor Hernando Cervantes

Purdue University, USA

Direct influences vs contextuality in human choices

(joint work with Ehtibar Dzhafarov)

A system of random variables exhibits true contextuality when different contexts force measurements of the same property (in psychology, e.g., responses to the same question) to be more dissimilar than required by the differences of their distributions. The difference in distributions is itself a form of context-dependence, but it is attributed to direct information-carrying influences exerted by contexts upon the random variables. We discuss and clarify the difference between direct influences and contextual differences. In particular, we address the issue of "hidden direct influences," those not reflected in distributional differences. We argue that hidden differences are immaterial as they cannot carry information, and that if they could somehow be observed, it would change the system of random variables and lead to different conclusions. While in quantum mechanics there are situations when measurements of the same property under different conditions have the same probability distribution, in psychology some form of direct influences is always present. Analysis of many previous attempts to demonstrate contextuality in human judgments has shown that they were unsuccessful because direct influences in them were all that was needed to account for context-dependence. However, in our "Snow Queen" experiment (Decision 5, 193-204, 2018) we unequivocally demonstrated true contextuality in human decision making. The formal structure of the experiment was that of a cyclic system of rank 4 (also used to depict the Einstein-Podolsky-Rosen paradigm in the Bohm-Bell version, and the double-slit experiment). The experiment laid down a template by which contextuality could be explored in a variety of empirical situations. We present a series of experiments conducted with I. Basieva and A. Khrennikov (arXiv:1807.05684) that follow this template using the formal structure of cyclic systems of ranks 3 and 4, and that confirm the presence of true contextuality in human decision making.

Hans Colonius

Carl von Ossietzky Universität Oldenburg (Carl von Ossietzky University of Oldenburg), Germany

Copulas for neural and behavioral parallel systems

An *n-copula* is an n-variate distribution function with univariate margins uniformly distributed on $[0, 1]$. Thus, a copula is a function that joins a multivariate distribution to its one-dimensional margins. The concept has stirred a lot of interest in recent years in several areas of statistics mainly for the following reasons: it allows one (i) to study the structure of stochastic dependency in a " scale-free " manner, i.e., independent of the specific marginal distributions, and (ii) to construct families of multivariate distributions with specified properties. First, we demonstrate how specific copulas with negative stochastic dependencies can be used in models of multisensory integration deriving quantitative measures in a given context for both neural (spike frequency) and behavioral (reaction time) data. Second, we show how a copula defining perfect negative dependence allows to solve a paradox between neural and behavioral measurements in a paradigm of inhibitory control.

Marcelo Terra Cunha

Universidade Estadual de Campinas (University of Campinas), Brazil

On measures and measurements

We start from Kolmogorovian notions of probability spaces and present some generalisations of this notion, in order to allow for contextuality. The first generalisation is strongly related to the contextuality by default program; the second to ontological theories with epistemic constraints; while the third brings with it the topological view of contextuality, possibly in a shape more comprehensible for the community of probability theory.

José Acacio de Barros

San Francisco State University, USA

Indinstinguishability, properties, and signed probabilities

One of the fundamental aspects of quantum particles is that they may be indistinguishable. This indistinguishability leads to some novel aspects of quantum statistical mechanics, when compared to classical statistical mechanics. Another fundamental aspect of quantum particles is that their properties are contextual. This contextuality can manifest even when experiments are space-like separated. In this talk I will discuss the relationship between indistinguishability, contextuality of properties, and signed probabilities.

Adele Diederich

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A dynamic stochastic framework for dual process models

Many phenomena in judgment and decision-making are often attributed to the interaction of two systems of reasoning. In neurosciences these systems are sometimes even related to distinct brain areas. While these so-called dual process theories can explain many types of behavior, they are rarely formalized as mathematical or computational models. Rather, dual process models are typically conceptual frameworks and verbal theories, which are difficult to conclusively evaluate or test. That is, unlike conceptual frameworks, computational frameworks guarantee logically valid predictions and allow the derivation of precise quantitative predictions from the underlying assumptions, thereby enabling empirical falsification of these assumptions. In the cases where formal dual process models have been proposed, they are often silent when it comes to the timing of the two systems. In this talk, I present a dynamic dual process model framework of risky decision-making that provides an account of the timing and interaction of the two systems and can explain both choice and response time data. I outline several predictions of the model including how changes in the timing of the two systems as well as time pressure can influence behavior. The purposed framework also allows the user to explore different assumptions about how preferences are constructed by the two systems as well as the dynamic interaction of the two systems. In particular, I examine various different possible functional forms of the two systems and two possible ways the systems can interact (simultaneously or serially). The model versions are fit to existing data sets.

Ehtibar Dzhafarov

Purdue University, USA

Miscellaneous comments on contextuality

I will address some foundational aspects of contextuality. I will argue that two founding principles of the Contextuality-by-Default (CbD) approach, use of the content–contextual double-indexation of random variables and use of probabilistic couplings subject to constraints, are logically necessary if one describes the results of an experiments in terms of classically understood random variables. I will discuss the relationship of CbD to the notions of contextuality-through-hidden-variables and contextuality-through-counterfactual-values, showing that CbD can be viewed as a rigorous way of describing either of the two notions. I will discuss two measures of the degree of contextuality stemming from the CbD approach, one based on the maximal sum of the probabilities with which two content-sharing variables can be equal within an overall coupling of the system, and one based on the minimal total variation in a quasicoupling with signed probabilities.

Robert B. Griffiths

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Quantum measurements and contextuality

Quantum contextuality has to do with measurements of a collection of quantum observables, not all of which commute with each other, and hence cannot all be measured simultaneously. Bell (Rev. Mod. Phys. 38, 447, 1966) posed the following question: If A commutes with B and also with C, but B and C do not commute, will the value of A measured along with B differ from its value if measured along with C? If "yes", quantum mechanics is contextual in that the measured value of A depends on the context, A, B or A, C; if "no", quantum mechanics is noncontextual. Using an analysis of quantum measurements not available to Bell I will argue that quantum mechanics is noncontextual. To be precise, if a measurement of A with B yields a value $A = a$, then had the same experiment been carried out with apparatus modified to measure A along with C, the same value $A = a$ would have been obtained. More recent literature employs "contextual" with a different meaning from that used by Bell. Thus in a proposal by Abramsky et al. (Phys. Rev. Lett. 119, 050504, 2017) noncontextuality is associated with the existence of a joint probability distribution for outcomes of measurements of a collection of observables, not all of which commute. While this definition poses some interesting mathematical problems, I will argue that it is not related in any simple way to the physics of quantum measurements.

Emmanuel Haven

Memorial University of Newfoundland, Canada

Probability as input or output of a social science model?

In this talk we attempt to first discuss why the separation of state and measurement in economics and finance may (or may not) have reason of existence. What can be a palatable meaning of a state function in economics and finance? Those questions revolve around a more general (and very important) issue: does the model inform the probability, or can probability be part of the model? As an example, say we want to investigate a certain phenomenon in economics. We build a model to come up with a probability value (which obviously must have relevance to the phenomenon studied). The model gives us a probability value, and probability itself does not explain anything. Rather the model does explain why the probability value is x or y. Can we argue for cases in economics or finance, where the probability formalism is an intricate part of the model? Finally, how much formalism (or little formalism) from physics does one need in economics or finance, to declare the whole 'physics-social science' endeavour defunct or not? Or is that question completely irrelevant?

Federico Holik

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Interpretation and informational aspects of non-Kolmogorovian probability theory

We discuss generalized pobabilistic models for which states not necessarily obey Kolmogorov's axioms of probability. We analyze the particular cases of probabilities appearing in classical and quantum mechanics. We argue for considering quantum probabilities as the natural probabilistic assignments for rational agents dealing with contextual probabilistic models [1,2]. In this way, the formal structure of quantum probabilities as a non-Boolean probabilistic calculus is endowed with a natural interpretation [3]. In this setting, we explore the possibility of defining information measures [4,5] and discuss the status of information theory [4] and information-based principles, such as MaxEnt [6,7].

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Gregg Jaeger

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The conceptual and mathematical bases of quantum potentiality and chance

Quantum potentiality, a notion by Werner Heisenberg in his definitive statement of the Copenhagen Interpretation of quantum mechanics, is closely associated with chance and probability. Heisenberg's introduction into modern physics of this notion, which it is associated with the Aristotelian tradition, represents a break with a long development in natural philosophy, and so raises a number of deeply intriguing questions regarding causation and chance in this realm, several of which I have been addressing recently. My lecture will lay out these questions and explore answers to them and their pertinence to explanation in physics.

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Matt Jones

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EPR with people

A number of recent experiments in psychology have attempted to demonstrate contextuality in human behavior. Whereas analogous experiments in physics carry theoretical assurance of no-signaling from properties like spacelike separation, these human studies have no such guarantees, and in fact signaling is highly like to be present. This situation has been a motivation for the development of the contextuality-by-default approach of Dzhafarov and colleagues, which seeks to define contextuality in the presence of signaling. The present experiment takes a different approach, creating a situation with two human subjects in which signaling theoretically should not occur. Two subjects are placed in a closed room and asked to converse until a timer goes off. They then proceed into separate sealed rooms (connected to the first), each to answer one of two randomly selected binary questions on a computer. Each question takes the form of a 2×2 coordination game, a structure that is paradoxical for rational game theory but that humans routinely succeed at. Once data are collected, they will be subjected to the classical CHSH criterion for contextuality.

Andrei Khrennikov

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Classical probability model for an arbitrary experimental setup

Nowadays it is commonly believed that classical probability (CP) theory, see Kolmogorov, cannot be used to represent quantum observables. This belief is based on the Bell attempt to proceed with the CP-description of the EPR-Bohm-Bell type experiments. This approach is known as the hidden variables description. Since it is very difficult to test experimentally the original Bell inequality, Clauser, Horne, Shimony, & Holt (CHSH) modified Bell's approach on the basis of the CHSH-inequality. We denote the CP-model proposed by CHSH by the symbol $\mathcal{M}_{\text{BCHSH}}$.

Bell emphasized the role of locality. However, Fine showed that to prove the CHSH-inequality, one needs to assume only the existence of the joint probability distribution (jpd). The latter is equivalent to using CP. Therefore a violation of the CHSH-inequality inequality by quantum probabilities (theoretical and experimental) implies inapplicability of CP. Erroneously inapplicability of one concrete CP-model, namely, $\mathcal{M}_{\text{BCHSH}}$, to describe the EPR-Bohm-Bell type experiments was commonly treated as inapplicability of CP in general.

Nevertheless, as was shown by Khrennikov and coauthors (2009, 2015) and by Dzhafarov and coauthors (2012, 2015, 2016, 2018), the EPR-Bohm- Bell type experiments can be modeled with the aid of the CP-representation of quantum observables. However, such CP-models are not so straightforward as $\mathcal{M}_{\text{BCHSH}}$. Denote the models developed by Khrennikov and coauthors and by Dzhafarov and coauthors by the symbols $\mathcal{M}_{KH}$ and $\mathcal{M}_{DZ}$, respectively.

The basic distinguishing feature of $\mathcal{M}_{KH}$ is taking into account the *random generators for selecting experimental settings*. They are represented as random variables (RVs) r_a, r_b additional to the "basic" RVs a_1, a_2, b_1, b_2. These generators' RVs are absent in $\mathcal{M}_{\text{BCHSH}}$. At the same time the random generators play the crucial role in the real experimental design of such experiments. We remark that Bohr emphasized that in modeling quantum phenomena all components of the experimental arrangement should be taken into account. Thus ignoring the random generators makes a model without them (as, e.g., $\mathcal{M}_{\text{BCHSH}}$) inadequate to the real physical situation.

Model $\mathcal{M}_{DZ}$ does not contain explicit counterparts of the random generators for setting selection. It is based on contextual coupling of random variables corresponding to the choice of experimental settings. In spite of different mathematical structures, both models, $\mathcal{M}_{KH}$ and $\mathcal{M}_{DZ}$, reflect the procedure of choice of experimental setting: $\mathcal{M}_{KH}$ with the aid of random generators, $\mathcal{M}_{DZ}$ with the aid of contextual indexing of random variables representing observables.

Model $\mathcal{M}_{DZ}$ was applied to study contextuality in the CP-framework with the especial emphasis of the possibility to proceed in the presence of signaling. We remark that signaling is absent in quantum mechanics (QM). Therefore contextuality theory developed by Dzhafarov and coauthors and known as contextuality by default (CbD) is more general than the standard theory of quantum contextuality. This generality provides the possibility to apply CbD outside of physics, especially in psychology, where the condition of no-signaling is generally violated.

Papers of Khrennikov and coauthors were aimed to show the possibility of construction of the CP-representation of quantum observables for the EPR-Bohm-Bell type experiments. Model $\mathcal{M}_{KH}$ was presented in the very concrete framework coupled to classical versus quantum discussion on the CHSH-inequality. This explicit coupling with QM led to ignoring the possibility to use model $\mathcal{M}_{KH}$ even in the presence of signaling. Consistent treatment of contextuality in model $\mathcal{M}_{DZ}$ motivated the authors of this paper to analyze (no-)signaling issue on the basis of $\mathcal{M}_{KH}$. And we found very clear CP-interpretation of no-signaling: *independence of RVs a_1, a_2, r_a representing Alice's observables and random generator from RV r_b representing the random generator for selecting Bob's observables.* Thus no-signaling has clear meaning. The use of the CP-model demystify the role of no-signaling in QM.

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Janne V. Kujala

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Minimal distance to approximating noncontextual system as a measure of contextuality: Comparison to Contextuality-by-Default

Let random vectors $R^c = \left\{R^c_p : p \in P_c\right\}$ represent joint measurements of certain subsets $P_c \subset P$ of properties $p \in P$ in different contexts $c \in C$. Such a system is traditionally called noncontextual if there exists a jointly distributed set $\{Q_p : p \in P\}$ of random variables such that R^c has the same distribution as $\{Q_p : p \in P_c\}$ for all $c \in C$. A trivial necessary condition for noncontextuality and a precondition for many measures of contextuality is that the system is *consistently connected*, i.e., all $R^c_p, R^{c'}_p, \ldots$ measuring the same property $p \in P$ have the same distribution. The Contextuality-by-Default (CbD) approach allows defining more general measures of contextuality that apply to inconsistently connected systems as well, but at a higher computational cost.

In Kujala (Foundations of Physics 47, 911–932, 2017) a novel measure of contextuality is proposed that shares the generality of the CbD approach and the computational benefits of the previously proposed Negative Probability (NP) approach. This approach differs from CbD in that instead of considering all possible joints of the double-indexed random variables R^c_p, it considers all possible approximating single-indexed systems $\{Q_p : p \in P\}$. The degree of contextuality is defined based on the minimum possible probabilistic distance of the actual measurements R^c from $\{Q_p : p \in P_c\}$. This measure, called the optimal approximation (OA) measure, agrees with a certain measure of contextuality of the CbD approach for all systems where each property enters in exactly two contexts. The OA measure can be calculated far more efficiently than the CbD measure and even more efficiently than the NP measure for sufficiently large systems. A variant of the measure, called the OA-NP measure of contextuality, agrees with the NP measure for consistently connected (nonsignaling) systems while extending it to inconsistently connected systems. Thus, the motivation for the OA approach was computational efficiency. Here we compare the OA approach to the CbD approach in terms of what desirable properties can be satisfied in each approach such as preserving noncontextuality in subsystems of noncontextual systems.

Ravi Kunjwal

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Hypergraph framework for Spekkens contextuality applied to Kochen-Specker scenarios

I will introduce some recent work [1, 2] on formulating contextuality within the Spekkens framework [3] for scenarios that are traditionally considered in Kochen-Specker (KS) contextuality [4, 5]. I will follow Ref. [1] in motivating the discussion and then present a hypergraph framework that applies to KS-colourable contextuality scenarios, obtaining noise-robust noncontextuality inequalities that generalize the KS-noncontextuality bounds from Ref. [4]. The framework for Ref. [1] relies on the introduction of a new hypergraph invariant in obtaining the noncontextuality inequalities. We will see that this hypergraph invariant is, in fact, the only relevant invariant when it comes to obtaining noncontextuality inequalities for KS-uncolourable scenarios and we will outline a complementary framework for the latter following Ref. [2]. Taken together, Refs. [1, 2] complete the project of turning proofs of Kochen-Specker theorem – whether statistical or logical – into noise-robust noncontextuality inequalities following the Spekkens framework. Along the way, time permitting, we will also comment on the status of Specker's principle [6] in these considerations and outline applications of the framework to quantum information protocols.

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Paweł Kurzyński

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Evolution of negative probability distributions

(joint work with Marcin Karczewski)

There is an intriguing idea that quantum theory would be recovered if standard probabilities were replaced by negative probabilities and some events were deemed unobservable. However, such approach would be able to recover only one half of quantum theory-state description and measurement. The other half of the theory describes how states change in time. In this presentation I will discuss which evolutions of negative probability distributions are allowed. It is known that the evolution of standard probability distributions is determined by stochastic matrices, which generate either simple reversible permutations, or fundamentally irreversible dynamics. On the other hand, the evolution of negative probability distributions can be described by pseudo-stochastic matrices, i.e., matrices whose entries are given by negative probabilities. These matrices give rise to a much richer dynamics with nontrivial reversible transformations. In addition, pseudo-stochastic matrices may allow to reverse some irreversible stochastic transformations, i.e., by allowing probabilities to go negative we get a possibility to reverse some processes.

Yoshihiro Maruyama

京都大学 (Kyoto University), Japan

Contextuality of reason, contextuality of reality, and the nature of probabilities involved

No-Go theorems in quantum foundations, such as Bell's and Kochen-Specker's, tell us that fundamental reality is inherently contextual (and nonlocal as a particular case); from a theoretical point of view (e.g., Abramsky-Brandenburger, 2011), contextuality is local consistency plus global inconsistency (of contextual information; e.g., probability distributions on measurement contexts). According to recent research in quantum cognitive science (e.g., Cervantes-Dzhafarov, 2017), Bell-type inequalities may be reformulated so as to be applicable in cognitive science, and they are actually violated in certain cognitive experiments. Does it show that human reason is contextual as well as fundamental reality? If so, in what sense? Put another way, do cognitive systems exhibit the same kind of contextuality as quantum systems? And ultimately, do Bell-type results in cognitive science have such a massive impact on our understanding of the world as those in quantum physics indeed had? There are analogies and disanalogies, both mathematically and philosophically, between contextuality of reality and cotenxtuality of reason. In this talk, we shall explain our recent results on contextuality analysis, and articulate the analogies and disnalogies between contextuality of reality and contextuality of reason, especially in light of the nature of probabilities involved. We argue, in particular, that quantum and cognitive systems exhibit the same kind of contextuality at a mathematical level of statistical correlation (apart from the issue of violation of marginal selectivity), and yet physical contextuality differs from cognitive contextuality in terms of how relevant probabilities are interpreted therein. This disagreement about the nature of probabilities, arguably, makes the meaning of Bell-type theorems in cognitive science depart from that in quantum physics in a significant manner. This would also explicate how science is not just about the analysis of statistical correlation (and how the Chomsky versus Norvig debate on statistical AI may be sorted out).

Arkady Plotnitsky

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Complementarity and contextuality in the spirit of Copenhagen

The aim of this talk it to outline Bohr's concept of complementarity in the context of Bohr's epistemology of quantum mechanics. Taking this epistemology into account, I argue, is essential for understanding how complementarity, which can, as a concept, be defined more generally and applies elsewhere, is specifically used by Bohr in quantum mechanics.

Michel Regenwetter

University of Illinois at Urbana-Champaign, USA

Random utility without regularity

Classical random utility models imply a consistency property called regularity. Decision makers who satisfy regularity are more likely to choose an option x from a set X of available options than from any larger set Y that contains X. In light of ample empirical evidence for context-dependent choice that violates regularity, some researchers have questioned the descriptive validity of all random utility models. I show that not all random utility models imply regularity. I propose a general framework for random utility models that accommodate context dependence and may violate regularity. Mathematically, like the classical models, context-dependent random utility models form convex polytopes. They can be leveraged to make parsimonious behavioral predictions, and are empirically testable using contemporary methods of order-constrained inference.

Zheng Joyce Wang

Indiana University, USA

Data fusion using Hilbert space multi-dimensional models

(joint work with Jerome Busemeyer)

General procedures for constructing, estimating, and testing Hilbert space multi-dimensional (HSM) models, built from quantum probability theory, are presented. HSM models can be applied to collections of K different contingency tables obtained from a set of p variables that are measured under different contexts. A context is defined by the measurement of a subset of the p variables that are used to form a table. HSM models provide a representation of the collection of K tables in a low dimensional vector space, even when no single joint probability distribution across the p variables exists. HSM models produce parameter estimates that provide a simple and informative interpretation of the complex collection of tables.

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Probability normalization, marginalization, and maximum entropy inference: An information geometric approach

Information Geometry is the differential geometric study of the manifold of probability models, where each probability distribution is just a point on the manifold. Instead of using metric for measuring distances on such manifolds, these applications often use "divergence functions" for measuring proximity of two points (that do not impose symmetry and triangular inequality), for instance Kullback-Leibler divergence, Bregman divergence, f-divergence, etc. In this talk, I will present an information geometric analysis of probability normalization, marginalization, and maximum entropy inference. Normalization and marginalization of probability measures will be analyzed using the KL divergence function, and discussed in the context of "probability transport" where joint distributions of random variables are order-dependent in general. Maximum entropy inference will be shown to lead to exponential family of probability distributions in the classical case (for Shannon entropy), and to their deformed-exponential family for more general entropy functions. The IG perspective highlights the duality of "natural parameter" (parameter that defines a probability model) and "expectation parameter."

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