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Minimal distance to approximating noncontextual system as a measure of contextuality: Comparison to Contextuality-by-Default

Let random vectors $R^c = \left\{R_p^c : p \in P_c\right\}$ represent joint measurements of certain subsets $P_c \subset P$ of properties $p \in P$ in different contexts $c \in C$. Such a system is traditionally called noncontextual if there exists a jointly distributed set $\{Q_p : p \in P\}$ of random variables such that R^c has the same distribution as $\{Q_p : p \in P_c\}$ for all $c \in C$. A trivial necessary condition for noncontextuality and a precondition for many measures of contextuality is that the system is *consistently connected*, i.e., all $R_p^c, R_p^{c'}, \ldots$ measuring the same property $p \in P$ have the same distribution. The Contextuality-by-Default (CbD) approach allows defining more general measures of contextuality that apply to inconsistently connected systems as well, but at a higher computational cost.

In Kujala (Foundations of Physics 47, 911–932, 2017) a novel measure of contextuality is proposed that shares the generality of the CbD approach and the computational benefits of the previously proposed Negative Probability (NP) approach. This approach differs from CbD in that instead of considering all possible joints of the double-indexed random variables R_p^c, it considers all possible approximating single-indexed systems $\{Q_p : p \in P\}$. The degree of contextuality is defined based on the minimum possible probabilistic distance of the actual measurements R^c from $\{Q_p : p \in P_c\}$. This measure, called the optimal approximation (OA) measure, agrees with a certain measure of contextuality of the CbD approach for all systems where each property enters in exactly two contexts. The OA measure can be calculated far more efficiently than the CbD measure and even more efficiently than the NP measure for sufficiently large systems. A variant of the measure, called the OA-NP measure of contextuality, agrees with the NP measure for consistently connected (nonsignaling) systems while extending it to inconsistently connected systems. Thus, the motivation for the OA approach was computational efficiency. Here we compare the OA approach to the CbD approach in terms of what desirable properties can be satisfied in each approach such as preserving noncontextuality in subsystems of noncontextual systems.