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Classical probability model for an arbitrary experimental setup

Nowadays it is commonly believed that classical probability (CP) theory, see Kolmogorov, cannot be used to represent quantum observables. This belief is based on the Bell attempt to proceed with the CP-description of the EPR-Bohm-Bell type experiments. This approach is known as the hidden variables description. Since it is very difficult to test experimentally the original Bell inequality, Clauser, Horne, Shimony, & Holt (CHSH) modified Bell's approach on the basis of the CHSH-inequality. We denote the CP-model proposed by CHSH by the symbol $\mathcal{M}_{\mathrm{BCHSH}}$.

Bell emphasized the role of locality. However, Fine showed that to prove the CHSH-inequality, one needs to assume only the existence of the joint probability distribution (jpd). The latter is equivalent to using CP. Therefore a violation of the CHSH-inequality inequality by quantum probabilities (theoretical and experimental) implies inapplicability of CP. Erroneously inapplicability of one concrete CP-model, namely, $\mathcal{M}_{\mathrm{BCHSH}}$, to describe the EPR-Bohm-Bell type experiments was commonly treated as inapplicability of CP in general.

Nevertheless, as was shown by Khrennikov and coauthors (2009, 2015) and by Dzhafarov and coauthors (2012, 2015, 2016, 2018), the EPR-Bohm- Bell type experiments can be modeled with the aid of the CP-representation of quantum observables. However, such CP-models are not so straightforward as $\mathcal{M}_{\mathrm{BCHSH}}$. Denote the models developed by Khrennikov and coauthors and by Dzhafarov and coauthors by the symbols $\mathcal{M}_{KH}$ and $\mathcal{M}_{DZ}$, respectively.

The basic distinguishing feature of $\mathcal{M}_{KH}$ is taking into account the *random generators for selecting experimental settings*. They are represented as random variables (RVs) r_a, r_b additional to the "basic" RVs a_1, a_2, b_1, b_2. These generators' RVs are absent in $\mathcal{M}_{\mathrm{BCHSH}}$. At the same time the random generators play the crucial role in the real experimental design of such experiments. We remark that Bohr emphasized that in modeling quantum phenomena all components of the experimental arrangement should be taken into account. Thus ignoring the random generators makes a model without them (as, e.g., $\mathcal{M}_{\mathrm{BCHSH}}$) inadequate to the real physical situation.

Model $\mathcal{M}_{DZ}$ does not contain explicit counterparts of the random generators for setting selection. It is based on contextual coupling of random variables corresponding to the choice of experimental settings. In spite of different mathematical structures, both models, $\mathcal{M}_{KH}$ and $\mathcal{M}_{DZ}$, reflect the procedure of choice of experimental setting: $\mathcal{M}_{KH}$ with the aid of random generators, $\mathcal{M}_{DZ}$ with the aid of contextual indexing of random variables representing observables.

Model $\mathcal{M}_{DZ}$ was applied to study contextuality in the CP-framework with the especial emphasis of the possibility to proceed in the presence of signaling. We remark that signaling is absent in quantum mechanics (QM). Therefore contextuality theory developed by Dzhafarov and coauthors and known as contextuality by default (CbD) is more general than the standard theory of quantum contextuality. This generality provides the possibility to apply CbD outside of physics, especially in psychology, where the condition of no-signaling is generally violated.

Papers of Khrennikov and coauthors were aimed to show the possibility of construction of the CP-representation of quantum observables for the EPR-Bohm-Bell type experiments. Model $\mathcal{M}_{KH}$ was presented in the very concrete framework coupled to classical versus quantum discussion on the CHSH-inequality. This explicit coupling with QM led to ignoring the possibility to use model $\mathcal{M}_{KH}$ even in the presence of signaling. Consistent treatment of contextuality in model $\mathcal{M}_{DZ}$ motivated the authors of this paper to analyze (no-)signaling issue on the basis of $\mathcal{M}_{KH}$. And we found very clear CP-interpretation of no-signaling: *independence of RVs a_1, a_2, r_a representing Alice's observables and random generator from RV r_b representing the random generator for selecting Bob's observables.* Thus no-signaling has clear meaning. The use of the CP-model demystify the role of no-signaling in QM.

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