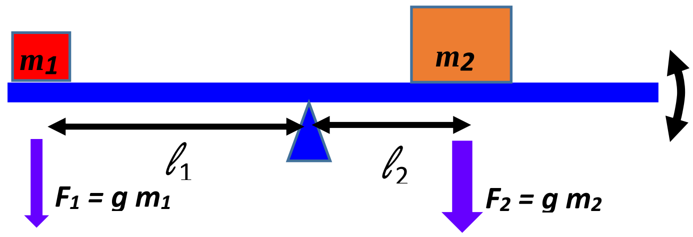
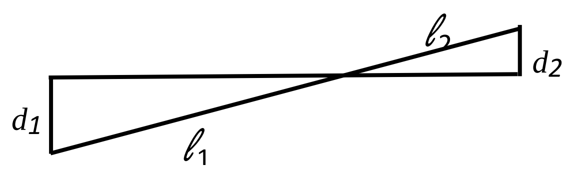
Origin of Design Equation for Levers

The equation for describing the operation of a lever is $m_1\ell_1 = m_2\ell_2$, where ℓ_1 is the distance of mass (m_1) from the fulcrum on side 1 of the lever and ℓ_2 is the distance of mass (m_2) from the fulcrum on side 2 of the lever. This equation expresses the simple idea that if a see-saw is to be balanced a smaller mass must be located further from the pivot point than a larger mass.



Where does the $m_1\ell_1 = m_2\ell_2$ equation come from? The answer lies in determining the force required to lift the mass on one side of the see-saw. The force acting on either mass is the gravitational force (F_{grav}), which is simply the mass times the acceleration due to gravity (9.8m/s^2) or $F_{grav} = mg$. The unit of force in SI units (i.e. the International System of units that is based upon meter, kilogram and second) is a Newton, where a 1 kg mass on the surface of the earth exerts a force due to gravity of 9.8 Newtons. If one were on the moon a 1 kg mass would only exert a force of 1.62 Newtons. Thus, $F = mg$ where the unit of force is a Newton. The abbreviation for a Newton is N.

The key physical idea about the operation of a see-saw is conservation of energy. If the see-saw is to freely go up and down it cannot require any extra work to make this motion happen. As we have already learned, work W is force F times distance d; thus, $W = Fd = mg\,d$, where we have replaced the force (F) with its individual components discussed in the previous paragraph. Now if the see-saw is to move freely with no additional work then



$$W_1 + W_2 = 0 \quad \text{or equivalently} \quad gm_1d_1 - gm_2d_2 = 0 \qquad (1)$$

where d_1 is the vertical distance that m_1 moves and d_2 is the vertical distance that m_2 moves. There is a positive sign in front of gm_1d_1 because it requires work to increase the height of m_1; in contrast, there is a negative sign in front of gm_2d_2 because the height of m_2 decreases by a distance d_2. Examining the figure, we notice that the two triangles are similar; thus, geometrically $d_1/\ell_1 = d_2/\ell_2$ or $d_1 = d_2\ell_1/\ell_2$. Replacing this expression for d_1 in Eqn. 1, we obtain

$$gm_1d_2\ell_1/\ell_2 - gm_2d_2 = 0$$

And simplifying

$$m_1\ell_1/\ell_2 - m_2 = 0 \quad \text{or equivalently} \quad \boxed{m_1\ell_1 = m_2\ell_2} \qquad (2)$$

This is the equation for a lever, where the key physical idea that led to the lever equation was that the work done in raising the mass on one side of the see-saw was exactly balanced by the work recovered on lowering the mass on the other side of the teeter-totter. The balancing of work on the two sides of the lever is just one specific example of conservation of energy.