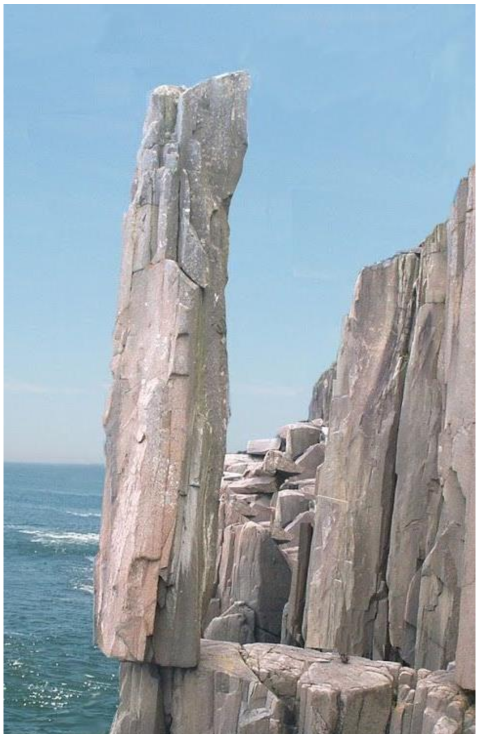


Hardware Store Science Curriculum

Mechanical Equilibrium

ICP Maker-STEM Content Document

Lesson 1 – Causes of Motion: This module begins by engaging with Newton's 1st Law of motion and identifying change in motion causes. We will identify balanced and unbalanced forces, develop free-body diagrams as a means of visualizing the forces acting on an object, and apply this understanding to everyday phenomena. *(ICP.3.2)*

Lesson 2 – Action Reaction Pairs: Next, we will explore a simple lever and identify three classes of levers based on fulcrum position, applied input force, and the resulting output force. This will include looking at mechanical advantage and breaking down mechanical advantage into its input and output forces. *(ICP.4.4; POE-3.3, 6.1)*

Lesson 3 – Modeling Action Reaction Using Levers: Armed with and understanding of levers we will be ready to take on the task of engineering a model that will allow us to determine the equilibrium rule for a simple lever when the fulcrum is located at its midpoint. *(ICP.3.5, 3.7; IED-0.1, 2.6, 6.10; POE-3.2, 3.4, 3.7, 6.1, 6.6)*

Lesson 4 – Defining Work: Finally, after collecting experimental data for determining mass distance ratios for a balanced lever arm at equilibrium we will define work. Work is an important scientific concept involving force and displacement. *(ICP.3.2, 4.4; POE-3.2, 6.1, 6.4)*

Module 4 Guiding Question

How does the action of a lever demonstrate the mechanical equilibrium?

Key Terms

Balanced Force – two forces acting in opposite directions on an object, and equal in size
Equilibrium – The condition of equal balance between opposing forces
Force – any interaction that, when unopposed, will change the motion of an object $F = ma$
Inertia – the resistance an object has to changes in its state of motion
Unbalanced Force – two forces acting in opposite directions on an object, and not equal in size
Work – a measure of energy transfer that occurs when an object is moved over a distance by an external force.

Equilibrium and Conservation

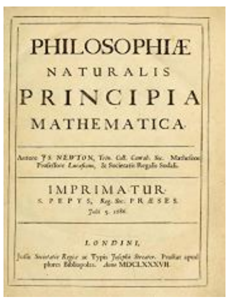
One of the major themes that runs through physical science is the **Conservation Laws**. These laws tell us that an isolated system in equilibrium will have identifiable properties that cannot change. These are often referred to as constants. These constants of an object/system are said to be conserved; and the resulting conservation laws can be considered the most fundamental principle of the system. In mechanics, examples of conserved quantities include energy and momentum. Studying motion from the basis of the law of conservation provides a unique advantage when compared to other methods, the ability to relate that motion to energy. The connection between motion and the conservation laws is possible because all mechanical motion is the result of some form of energy transformation. An understanding that energy is a conserved quantity allows one to follow that energy in all its various forms. Thus, a key question is to ask when studying a particular phenomenon is: Where does the energy of a system come from and where does it go.

Lesson 1: Objectives

- Identify and explain the difference between balanced and unbalanced forces.

Content

- Inertia and Momentum
- Newton's 1st Law of motion
- Balanced vs. unbalanced forces



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

IMPRIMATUR

LONDINI

Philosophiae Naturalis Principia Mathematica (or just the Principia) is perhaps the most important scientific text ever written, where Newton described his three laws of motion, law of gravity and Kepler's law of planetary motion.

Galileo Galilei

One of the major concepts that runs through physical science are the **Conservation Laws**. These laws state that when looking at an isolated system, certain properties of the system do not change as the system evolves over time. In a mechanical system these are often referred to as constants of motion. These constants of an object's motion are said to be conserved, where the resulting conservation laws can (energy, momentum, mass, electrical charge) be considered the most fundamental characteristic of the system.

One important advantage of studying motion from the basis of the law of conservation is that it allows one to relate that motion to energy. The connection between motion and the conservation laws allows is one of the most important foundation principles to understand. Since energy is a conserved quantity, then motion do to that energy can be followed in all its various forms; speed, velocity, acceleration, momentum, and inertia are a few examples.

In this module we will introduce the concept of work, which relates gain/loss of energy to a specific type of motion. We will investigate (i) how motion causes work, (ii) how work is conserved for a system without friction and without energy storage and (iii) mechanical equilibrium – a concept that is transferable to other major physical processes throughout this course. We will also learn more about forces, both balanced and unbalanced, and how forces that move an object require work.

You may already be familiar with the idea of a lever, as used by cartoon characters when tipping over large rocks. In these lessons we will develop a deeper understanding of how a lever operates as well as the concept of work. Work is one of the key concepts used throughout physical science, including mechanics, chemistry and electricity. Understanding of work is foundational, because it will be used in subsequent modules of this course.

Lesson 1: Causes of Motion

Newton's first law of motion states that an object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force. In fact, all objects tend to resist changes in their state of motion. This tendency to resist changes in their state of motion is described as **inertia**.

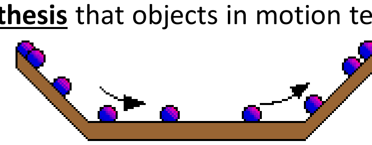
> "Every object persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state by forces impressed on it."
>
> *Sir Isaac Newton's 1st Law of Motion, 1686*

Inertia and Momentum

In the 17th century Galileo developed the concept of inertia. Galileo's conception of inertia stood in direct opposition to popular thinking about motion. In his day the dominant thought was that objects come to a state of rest (no motion), an idea that came from watching moving objects in everyday life, i.e. a basketball rolling across the gym floor will eventually stop.

Galileo did an interesting experiment to test the **hypothesis** that objects in motion tend to come to rest. In his experiment he had a pair of opposing inclined planes as shown in diagram A below. Galileo placed the opposing inclined planes facing one another and released

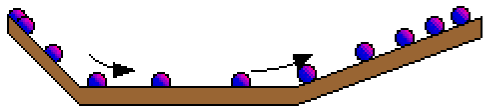
A



a ball from the top of the left-hand inclined plane. The ball then rolled down the plane gathering speed and rolled up the right-hand plane to nearly the same height.

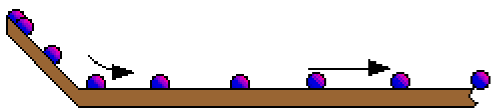
In a second experiment (B, below), Galileo set up the incline planes where the plane on the right-hand side was at a lower angle but much longer. Again, the ball was released from the top of the left-hand side, gained speed going down the left-hand incline plane, and went up the right-hand incline plane to roughly the same height at which it was released, even though it traveled an even longer distance. This experiment could be repeated with the right-hand ramp having even less angle and traveling even a longer distance.

B



Finally, think about the situation where the angle is very, very small (nearly flat). This would require a very, very long right-hand incline plane for the ball to reach the same height (C, below). As the incline plane approaches the limit of zero angle the ball would roll on indefinitely as if attempting to reach the height from which it started.

C



We now understand that the ball would stop due to friction between the ball and the ramp but imagine the idealized case of no friction. This experiment shows that once the ball is put into motion it will continue to stay in motion unless there is an opposing force, e.g. frication and/or gravity if the right-hand ramp goes up. Thus, a careful, well designed experiment showed that the original hypothesis was false. This tendency for an object to remain in motion is referred to as **inertia**.

Demonstration Experiment: Galileo Ramp

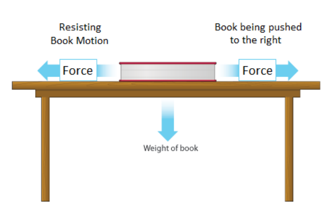
A ramp like Galileo's can readily be constructed from metal channel with wood supports. Using a steel ball there is minimal friction If the ball diameter is approximately two times the channel width. This an excellent way to have students make a hypothesis like was done in Galileo's time, have them look at the data from the Galileo Ramp, and then have them examine the validity of their hypothesis coming to the conclusion that 'an object in motion tends to remain in motion'.

Newton built on Galileo's thoughts about motion with his first law of motion. He declared that a **force** is not needed to keep an object in motion, rather a force is required to change that motion. As an example, slide a book across a table and watch it come to a rest position. Newton postulated that the book in motion on the tabletop does not come to a rest position because of the *absence* of a force; rather, it is the *presence* of the opposing frictional force that brings the book to rest. In the absence of a friction to slow down its

The rolling ball experiment is an excellent example of a well formulated hypothesis:

1. **A clearly stated hypothesis: an object in motion goes towards a resting state**
2. **A well-designed experiment shown in A and B above**
3. **A logical extension (by thinking, not doing) of the experiments in A and B to a ramp that does not go up on the right hand -side shown in C**
4. **Conclusion from the data that the hypothesis is wrong.**

Detailed construction plans for a Galileo Ramp are in the Levers additional resources at hardwarestorescience.org and video sources can be found using the YouTube search tool



Postulate: *(verb)* to assume or claim as true, existent, or necessary

motion, the book would continue this motion with the same speed and direction indefinitely.

This is nearly the case of an air hockey table, where the puck slides across the table on a cushion of air with very little friction – even here the puck will eventually slow down due to the friction, albeit small, between the puck and the table.

Inertia is defined as the resistance all physical objects have to changes in their velocity. Inertia explains why an object at rest continues at rest while an object in uniform motion in a straight line continues moving along that straight line at the same speed. All objects resist changes in their state of motion. An object's velocity, i.e. the speed and direction that the object is moving, defines the state of motion of that object. Thus, inertia could also be defined as the tendency of an object to resist changes in its state of motion.

The vector nature of velocity and acceleration were discussed when reviewing speed, velocity, and acceleration.

An object's ability to resist changes in its state of motion tends to vary with mass. Consider a rock that is rolling down a hill. The force to stop the rock will be greater if the mass of the rock is large. At the same time, if the rock is moving down the hill at a rapid speed, it will take more force to stop the rock. The mathematical relationship of velocity times mass is called **momentum** and is given by the equation

$$p = mv$$

where p is the momentum, m is the mass of the object and v is the velocity of the object. Since velocity is a vector, then momentum is also a vector. Note that if the object is at rest, then the velocity $v = 0$, and the momentum p is also 0.

It is important to remember that although inertia and momentum are related they have different meanings. Inertia is an underlying characteristic of matter related to its mass and describes how much force it will take to cause an acceleration of the object. In contrast, momentum is derived from the mass and velocity of an object and is a measure of the kinetic energy of the object. Conservation laws state that the momentum of an object is constant, unless there are outside forces acting on the object. If an object is at rest, it has zero velocity and if momentum is conserved (i.e. it stays the same in the absence of any external forces) the object will continue to have zero velocity. This is an important scientific principal, relating the conservation laws and momentum. There is no conservation of inertia principal.

Newton's 2nd Law is a precise statement of the relationship between the change in momentum and force. Newton's 2nd Law will be developed during a discussion on projectile motion.

Student Activity: Momentum of Common Objects

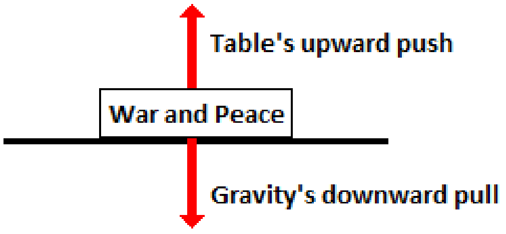
In order to gain a more intuitive understanding of the momentum of various objects determine the momentum of various objects that you are familiar with. For example, determine the momentum of various players on a football team – remember a lineman may weight more than a wide receiver, but he will not run as fast. Or, a box truck going at 60 mph or an Indy race car going at 220 mph – you will need to estimate the mass of both vehicles. A falcon diving at maximum speed or a slow moving duck. Use your imagination to come up with other comparisons. Use Google to estimate mass and speed of objects.

Balanced and Unbalanced Forces

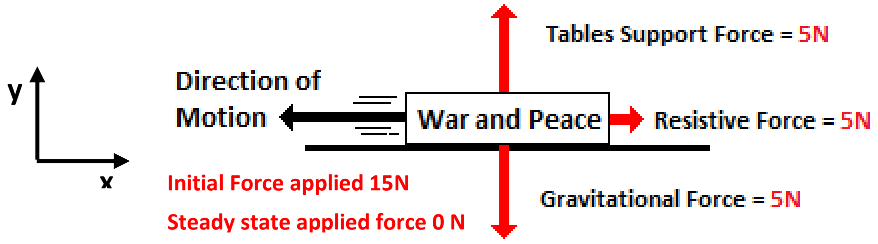
The momentum (and hence the velocity) of an object can be changed by the application of an external force. Consider a baseball that has been thrown towards home plate by the pitcher. When the bat makes contacts with the ball, the direction of the ball changes. It is the force provided by the bat that changes the velocity of the ball, where the direction of the ball's motion dramatically changes upon being hit with the bat *(velocity is a vector – a quantity with both magnitude and direction)*. The change in the ball's momentum, in this case velocity (v), is the result of an application of an **unbalanced force** on the object.

What is the difference between a **balanced force** and an unbalanced force? Consider a tug-of-war competition. Both teams pull hard on the rope applying a lot of force. If the force by the first team is exactly balanced by the force from the second team there is no net motion of the center of the rope – a balanced force. However, if one team is stronger, perhaps having contestants that have larger masses (tug-of-rope competitors are usually large individuals), there is an unbalanced force, where the team applying the larger force moves the center of the rope in their direction.

It is the presence of unbalanced forces that causes change in momentum, and hence a change in the motion, of an object. Consider a book (a thick, heavy book like War and Peace) sitting on a table. There are two forces acting upon the book: the Earth's gravity is pulling downward on the book, while the desk prevents the downward motion of the book by providing an upwards support pushing support force on the book. Since these two forces are of equal magnitude, and in opposite directions, we say they are balanced and there is no change in the books motion. The book is said to be at equilibrium because there is no unbalanced force acting upon the book and it remains at rest. When all the forces acting on an object balance each other, the object will maintain its state of motion.



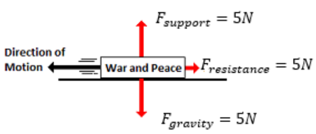
Now consider a book sliding from right to left across a tabletop. If we focus on the motion of the book as it is sliding, where a hand (not seen in the picture) pushes the book to the left initiating the sliding motion and then lets go of the book. We can identify all the forces acting on the book: (i) the gravitational force pulling the book down, (ii) the support force of the table pushing the book up and (iii) friction between the book and the table in the opposite direction of to the sliding motion of the book. Note: we don't consider the force of the initial push, since once the book starts sliding the hand does not continue applying the force that started the motion.



The SI unit of force is the Newton with the symbol N.

The force of gravity pulling downward on the book and the force of the table pushing upwards on the book are still balanced as before; thus, there is no net vertical motion of

Free-body diagrams are a graphical illustration used to visualize the applied forces, moments, and resulting outcomes of a **body** in a given condition



the book. In contrast, there is no force present to balance the resistive force of friction; thus, the <u>unbalanced</u> friction force acts to the right and slows down the book's leftward motion. This unbalanced resistive force is directed opposite the book's motion and the book's state of motion in the horizontal direction changes.

To determine if the forces acting upon an object are balanced or unbalanced, an analysis must first be conducted to determine what forces are acting upon the object and in what direction. Study the force values for the book sliding from right to left across a tabletop with the forces shown in the free-body diagram above. Consider the two directions used to describe the force vectors: the upwards and downwards forces are the vertical direction (or y-coordinate plane), while the left and right forces are the horizontal direction (or x-coordinate plane).

1. <u>Vertical direction</u>. The gravitational force pulls the book downward or in the negative y-direction. The table support force pushes the book upward or in the positive y-direction. The sum of the gravity and table support forces is the net force in the y-direction, which must be zero since there is no vertical motion of the book. Thus

   $$F_{gravity} + F_{support} = F_{net,y} \rightarrow -5N + 5N = F_{net,y} = 0N$$

   where the subscript y indicates in the y-direction (i.e. vertical direction) often labeled F_y.

2. <u>Horizontal direction.</u> Even if there was an initial push of 15N to the left (i.e. in the negative x direction), the only horizontal force present during sliding is the frictional force between the book and the table, which is 5N in the positive x-direction. Thus,

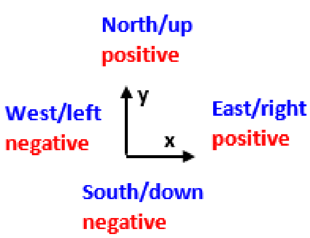
   $$F_{resistance} = F_{net,x} \rightarrow +5N = F_{net,x}$$

   where the subscript x indicates in the x-direction (i.e. horizontal direction) often labeled F_x.

Note the difference between the two components of force. The net vertical force $F_y$is zero and hence the vertical force is *balanced* and there is no change in the vertical velocity (i.e. the book has zero vertical velocity and it doesn't change but remains zero). In contrast, the net horizontal velocity (F_x) is +5N which is an *unbalanced* force that resists the leftward motion of the book and slows it down.

All forces include a specific direction that must be considered when evaluating balanced versus unbalanced forces. By convention forces that are labeled as up/right or North/East are given a positive number while forces labeled as down/left or South/West are given a negative number.

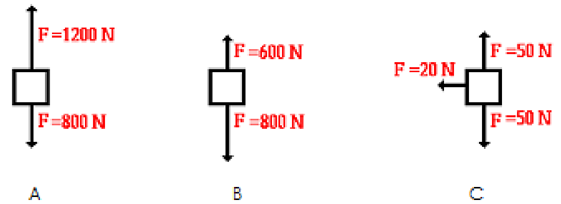
Balanced forces are equal in magnitude and in opposite directions.



To determine if the forces acting upon an object are balanced or unbalanced, an analysis must first be conducted to determine the magnitude of the force acting upon the object and in what direction. If two individual forces are of equal magnitude and opposite direction, then the forces are said to be balanced. An object is said to be *acted upon by an unbalanced force* only when there is an individual force that is <u>not</u> being balanced by a force of equal magnitude and in the opposite direction.

Sample Problem

Describe what the net force acting on objects A, B and C would be. The net force F_{net} (note: F_{net} is a vector and thus has components in the x/y or vertical/horizontal directions) is the result of all of the forces added together. Forces going up or to the right are generally given a positive value while forces going down or to the left are given a negative value.



Object A: vertical direction $1200N + (-800N) = 1200N - 800N = 400N$
horizontal direction no forces applied in horizontal direction
Object B: vertical direction $600N + (-800N) = 600N - 800N = -200N$
horizontal direction no forces applied in horizontal direction
Object C: vertical direction $50N + (-50N) = 50N - 50N = 0N$
horizontal direction $-20N + 0N = -20N$

Object A: results in a $400N$ force pushing up (unbalanced force), net vertical motion
Object B: results in a $200N$ force pulling down (unbalanced force), net vertical motion
Object C: $20N$ force to the left (unbalanced force), net horizontal motion

Lesson 2: Forces, Action-Reaction Pairs and Mechanical Equilibrium

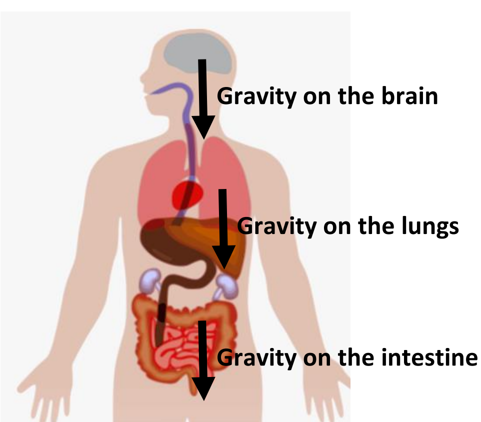
In Lesson 1 the ideas of inertia, momentum and balanced/unbalanced forces were discussed. The key idea was Newton's 1st Law that states an object at rest stays at rest and an object in motion stays in motion, unless there is an external force on the object causing that motion to change. Let us turn our attention to the concept of force. Any discussion on competing forces will eventually lead to the idea of mechanical equilibrium, which is a situation when all forces on a body are in perfect balance. It is the state of mechanical equilibrium that we use to describe bodies that are not undergoing acceleration

Lesson 2: Objectives

- **Demonstrate an understanding of Newton's 3rd law by identifying action and reaction pairs.**

Content

- **Surface and Body Forces**
- **Newton's 3rd Law of Motion**
- **Mechanical Equilibrium**



Surface and Body Forces

There are two types of forces that are qualitatively different: surface forces and body forces. Sometimes surface forces are called contact forces and sometimes body forces are called forces-at-a-distance. You can use either designation, however four our discussion we will utilize the terms surface forces and body forces a more accurate description.

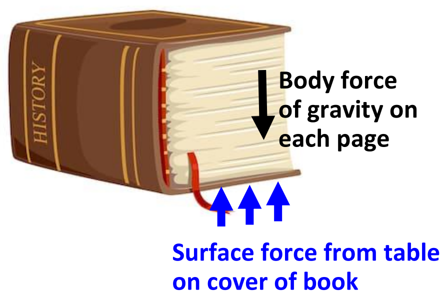
Consider the human body. One type of force that the body experiences is due to gravity. Gravity reaches inside the body and exerts a downward force on the heart, liver, spleen and the rest of the internal body parts. Gravity travels through the skin, muscles and bones to reach these internal parts, simultaneously exerting its downward pull on the skin, muscles and bones as well. Every cell in the body (as well as every internal component of the cell like the nucleus, ribosomes, etc.) experiences the force of gravity. You can now see why body forces is a more accurate description of this force

Simultaneously - *(adverb)* used to describe actions that occur at the same time

than calling it a forces-at-a-distance. The gravitational forces we feel on Earth are due to the mass of the earth. This force originates at Earth's center and travels through all the intervening mass of Earth to your own internal organs.

The gravitational body force does not only apply to the human body, but also to other objects. In an automobile gravity acts on all of the internal car parts including the engine, transmission, tires, seats and even the driver in the car. In an airplane the gravitational body force acts on the wings, fuselage, engine, jet fuel, pilot and the passengers. The body forces in any object act on all the internal components of that object.

Now consider a surface force on your body. If one of your friends gives you a hug, that is also a force on the body, but one that comes through the surface of the body. Also, when you are standing the bottoms of your feet transmit a force up into the body – it is this surface force that is measured when you step onto a scale. Surface forces are exerted on the exterior surface, which for the human body example is your skin.



Consider a second example of body and surface forces. Imagine a thick book that is sitting on a table. The gravitational body force is acting on every page in the book, pulling each one of the pages down. If there is nothing to counteract gravity the book falls. You can observe his when you hold a book above your head and let it go, the book accelerates downward until it hits the floor with a bang. However, when the book is sitting on a table, the table exerts an upward force on the surface of the book that is exactly equal to the gravitational body force. Because this for is in the opposite direction of the force due to gravity it, stops the book from hitting the floor. Thus, for the book on the table the body force (gravity) pulls the downward and a surface force (exerted by the table) cancels out the pull of gravity and there is no movement of the book.

A body force goes through space and pulls/pushes every molecule in the object – force-at-a-distance. A surface force only acts on the surface of the body – contact force.

One of the most common types of body forces is the force due to gravity. On the earth the force of gravity pulls an object towards the center of the earth. The gravitational force depends upon the mass of the object. You are able to toss your notebook up into the air; if you are really strong you may be able to toss your backpack filled with books into the air (although not very high); however, there is no way you can toss a bookcase filled with books into the air, there is just too much mass. The gravitational force $\boldsymbol{F_g}$ is calculated using the equation

$$F_g = m \times \boldsymbol{g}$$

where m is the mass of the object and $\boldsymbol{g}$ is the gravitational constant. The gravitational constant $\boldsymbol{g}$ is a vector quantity containing a magnitude and direction and so the force due to gravity F_g must also be a vector. On earth the gravitational vector $\boldsymbol{g}$ points in the direction of the center of the earth, while on the moon it points towards the center of the moon. The magnitude of the gravitational vector is a scalar and is called the gravitational constant g. On the surface of the earth g is $9.807 m/s^2$; while on the moon it is $1.62 m/s^2$. The gravitational constant on the moon is approximately 1/6th of that on the earth so you could jump six times higher – you would be an NBA star. The units for gravitational force $m\boldsymbol{g}$ is $Kg \cdot m/s^2$, also called a Newton, because we are combining a unit of mass (kg) and a unit of acceleration (m/s^2).

Action-Reaction Pairs

When one considers a force acting on a surface, this force is always accompanied by an opposing force. These opposing forces are known as action-reaction pairs. As an example, consider a weightlifter and focus on the surface of the hands of the weightlifter, where they are in contact with the barbell. There are two opposing forces acting between the barbell and the weightlifter's hands.

Scalar - a quantity that is fully described by a magnitude (or numerical value) alone

Vector - a quantity that is fully describe by both a magnitude and a direction

The downward force of the barbell is paired with the upward force that the weightlifter is exerting on the bar of the barbell. As the weightlifter holds the barbell overhead there is no vertical motion of the barbell; thus, the two surface forces where the palms of the weightlifter touch the barbell are exactly balance. The action force of the barbell being pulled downward by the force of gravity is counter acted by the reaction force of the weightlifter pushing upwards on the barbell. This is called an action-reaction pair, because for every action (i.e. force) there is an equal and opposite reaction (i.e. an opposing force).

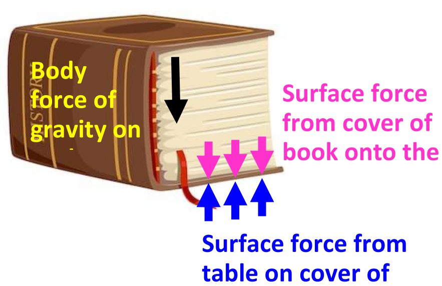
Newton Units

$$\mathbf{1N = 1kg \cdot m/s^2}$$

As a second example of action-reaction pairs consider a strong man that is between two horses pulling in opposite directions. Since there is no motion if the strong man doesn't let go of the ropes, all the forces must be balanced. Can you identify the six action-reaction pairs?

"To every action, there is an equal and opposite re-action."
Sir Isaac newton's 3rd Law of Motion, 1686

Like the weightlifter example, there is one action-reaction pair at the man's left hand, where the rope is the *action force* pulling to the left in opposition to the *reaction force* of the man's left hand pulling to the right. There is a similar action-reaction pair between the man's right hand and the rope pulling to the right. Consider where the rope is attached to the harness on the horse on the man's left-hand side. The force on the rope is the action force that is exactly balanced by the reaction force on the harness in the opposite direction. Just like the action-reaction pair of forces between the rope and harness on the man's left-hand side, there is a similar rope-harness action-reaction pair between the rope and horse on the man's right-hand side. We now have four action-reaction pairs, what about the last two? For the 5th and 6th example consider the forces between the horses' hooves and the ground – they again form an action-reaction pair – actually eight more action-reaction pairs, since there are two horses with four hooves each.



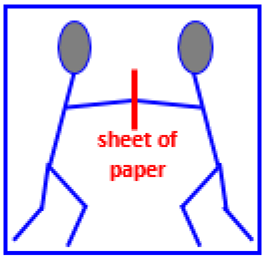
Let us re-examine the book example discussed earlier. The body force due to gravity, acting on every page of the book, was balanced by the surface force from the table. If we just focus on the cover of the book that rests on the table, the surface force from the table (in this case the action force) will be exactly balanced by the surface force from the cover of the book (the reaction force). If there was a scale on the table, and the book was placed on the scale, the scale would register a weight, i.e. a force. If one also put a micro-scale in the cover of the book (there are such pressure sensors) it would also measure a force that would be of exactly the same magnitude as that measured by the scale on the table, but in the opposite direction.

The relationship between action and reaction forces is formally given by Newton's 3^{rd} Law of motion: for every reaction there is an equal and opposite reaction. Implicit in this statement is that there are two opposing forces.

Implicit - *(adjective)* when something is understood but clearly stated

Advanced Topic

What is an intuitive reason for Newton's 3^{rd} Law. Consider two individuals pushing against each other on a thin sheet of paper. From the paper's perspective it sees two opposing forces. If the forces are balanced there is no movement, but if there is more force from let's say the individual on the right, the paper begins to move to the left. In fact, if the force is unbalanced it will change the momentum of the paper (momentum is the combination of mass m and velocity v).



Since the mass of the paper is constant, a change in the momentum means there is a change in the velocity and a change in velocity is acceleration. The relationship between an unbalanced force and motion is given by Newton's 2^{nd} Law; thus, an unbalanced force on the paper will cause an acceleration. However, the magnitude of the acceleration depends upon the mass, where if the mass decreases the acceleration must increase so that the product ***ma*** equals the unbalanced force.

Now perform a thought experiment: allow the thickness of the paper (and hence its mass) to decease and decrease, which would then require the acceleration to increase and increase so that *ma* remains equal to the unbalanced force *F*. In the limit that the paper mass goes to zero the acceleration must go to infinity – which makes no sense, because the paper does not fly off at an infinite rate. The only way out of this conundrum is for the net force on the infinitely thin paper (i.e. a mathematical surface) to be zero; the surface forces are balanced; *for each action force there is a reaction forces that is equal and opposite*.

Mechanical Equilibrium

Thought experiments are often used in physical science. They are used to take a situation to an extreme that may not be physically achievable, but that provides insight.

In developing the theory of relativity Einstein thought about flashing a light while riding on a train moving at the speed of light. A weird, but informative

Mechanical equilibrium is a state where the sum of all forces acting on an object of interest is zero. For this to happen all of the forces are required to be balanced. As was discussed in Lesson 1, when there are no net forces acting on an object to change in the motion of that object – objects at rest remain at rest and objects in motion will continue their motion indefinitely. This is the basic idea behind inertia.

In addition, when a system is in mechanical equilibrium all the internal parts of the system are in mechanical equilibrium. Looking at the tightrope walker we notice that she is balanced on the tightrope, not swinging from left-to-right, and the pots on her head are also in mechanical equilibrium, not topple off. Mechanical equilibrium is a way to say that all of the forces in a system of interest are balanced – this concept is used in the analysis of experiments and investigations. Though the forces acting on an object may be called by different names a state of mechanical equilibrium always requires that the net forces acting on the object of interest add up to zero.

Lesson 3: Objectives

- **Utilize hand tools to construct a fulcrum for investigating equilibrium and mechanical advantage of a lever.**
- **Investigate lever action and use collected data to support the equilibrium rule.**

Content

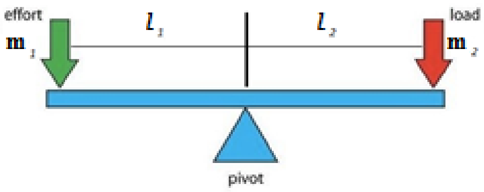
- **Recording Experimental Data**
- **Mechanical Equilibrium Applications**
- **Lever Rule**

Lesson 3: Levers – A Mechanical Equilibrium Experiment

Armed with the knowledge of inertia, balanced and unbalanced forces, and mechanical equilibrium we are ready to discuss one of the most important concepts in physical science, the concept of work.

Work is the way scientists and engineers relate force and motion. It is from the idea of work that we will subsequently develop ideas about energy and power. Work allows us to discuss concepts like energy storage, potential energy, kinetic energy, and energy loss through processes like friction.

A lever is a simple machine consisting of a beam or solid rod and a fulcrum or pivot point. The beam is placed so that some part of it rests on top of the fulcrum. In a traditional lever, the fulcrum remains in a stationary position, while an input force is applied somewhere along the length of the beam. The beam then pivots on (or around) the fulcrum, exerting the output force on some sort of object that needs to be moved.



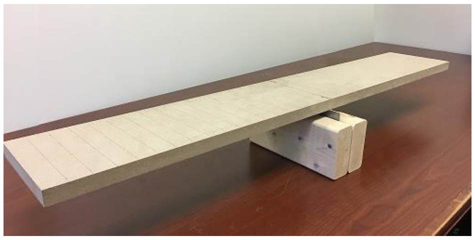
The fundamental piece of knowledge to be discovered when working with levers is the relationship between the input force and the output force. In its simplest form this can be expressed as the length l_1 times the mass m_1 on the left side of fulcrum equals the length l_2 times the mass m_2 on the right side of fulcrum. When a lever is balanced this equation can be written as

$m_1 \times l_1 = m_2 \times l_2$ where m is the mass of the object and l is the distance

When the distances from the fulcrum are the same ($l_1 = l_2$) then the weights on either side of the fulcrum must be the same ($m_1 = m_2$) if the lever is going to balance. The situation gets much more interesting when m_1 does not equal m_2, and when l_1 does not equal l_2. In these situations, manipulation of this mathematical relationship between either side of the fulcrum allows us to determine unknown components of the input (effort) or output (load) forces.

Lever Investigation

Modeling work can be accomplished by building an inexpensive teeter-totter. Objects of different mass can be placed on the support beam; and by measuring the masses of the objects and the distances they are placed from the fulcrum one can calculate the work required to balance the teeter totter.



When constructing your investigation model, you should create a fulcrum on which to balance a straight, 36-in to 48-in board (beam). A rigid piece of MDF board is ideal for this short beam. Other materials could be used instead of MDF; however, the material may not balance when centered from end to end. MDF in manufactured to be uniform

In this experiment students will need to use the following tools:

1. **Handsaw**
2. **Power drill**
3. **Screwdriver**
4. **Etc.**

A description of how to use these tools is available under the Supplemental Resources tab at hardwarestorescience.org

throughout the length of the board and therefore is a better choice than many other options due to cost and availability.

The fulcrum, like the one in the image above, can be constructed by placing a short piece of 1/8 a flat aluminum (or similar thin plate of metal) between two pieces of 2x4 lumber and securing it in place with three 2-½ inch screws. If you allow the aluminum to extend above the surface of the 2x4s you have effectively created a narrow platform on which to balance you MDF board and will obtain very accurate data.

During your investigation, you can analyze how a simple lever performs work on an object. Data collected during the investigation will allow you to explore the mathematical rule for equilibrium. This can be accomplished by manipulating the placement of known masses on either side of the MDF board, so that it is balanced on the fulcrum.

In principle, this experiment can be completed, with a reasonable amount of accuracy, using a single piece of 1x4 as a fulcrum. However, using the thin piece of metal as the fulcrum increases the accuracy of the measurements and provides a clearer picture of the mathematical rule for equilibrium, enabling you to accurately assess the work done when taking advantage of a lever to do work on an object.

See the additional resources associated with each module at hardwarestorescience.org for examples of good lab notebook procedures. These can be found on the Educational Materials tab under List of Experiments

Safe Use of Tools is of Critical Importance

The safe use of tools is important. Safe use of a handsaw and simple power tools is described in the Maker Skills Document available at hardwarestorescience.org. In addition, there are many additional resources on the safe use of tools on the web. It is important that a responsible adult monitor student safe use of hand tools and the "making" processes in your classroom or "Maker Space."

Initial Analysis of Lever

The objective behind doing this investigation is to determine how the locations of various weights must be arranged on the MDF board so that it is perfectly balanced on the fulcrum. Once you have collected data on your model, you will be looking for any observable patterns in the data. What is occurring when the lever is balanced? It is here that a scientist 'plays' with their data, trying to discover something that can unify the quantitative data with the qualitative observations. Try multiplying the distance from the fulcrum to the mass by the mass itself. Compare the value obtained from the right-hand side with that of the left-hand side. Do you see anything interesting?

Quantitative - *(adjective)* expressible in terms of a number value

Qualitative - *(adjective)* expressible in terms of what something is like

It is important to learn effective and efficient ways of recording your experimental data. Whether you utilize a provided lab packet (1.2 Levers Investigation) from your instructor or create you own data table in a science/engineering notebook, it is important to ensure that critical information is included with your collected data.

A typical lab notebook entry for the experiment above could be as follows. Data tables should provide all information necessary for answering questions, accurate analysis, making predictions, and communicating observations clearly. Labeling quantities with appropriate units, throughout the data table, is important to avoid confusion and assist in doing calculations. Writing down formulas used in calculations and aiding analysis is useful when reviewing previous investigations. You may also include diagrams to explain thinking, provide context, and make connections between raw data and calculations.

F = mg
W = F x d

Data Table 1 in = 2.54 cm

Trial using 2 hex nuts	Mass m_1	Distance l_1	m_1 x l_1	Mass m_2	Distance l_2	m_2 x l_2
1	16.5 g	17.25 in 0.438 m	0.0695 Nm	16.5 g	17.125 in 0.435 m	0.0703 Nm
2	16.5 g	13.25 in 0.337 m	0.0545 Nm	16.5 g	13.5 in 0.343 m	0.0555 Nm
3	16.5 g	15.125 in 0.384 m	0.0621 Nm	16.5 g	14.875 in 0.378 m	0.0611 Nm
Trial using 3 hex nuts						
1	33.0 g	8.5 in 0.216 m	0.0699 Nm	16.5 g	17.375 in 0.441 m	0.0713 Nm
2	33.0 g	5.125 in 0.130 m	0.0420 Nm	16.5 g	11.25 in 0.286 m	0.0462 Nm
3	33.0 g	6.625 in 0.168 m	0.0543 Nm	16.5 g	13.375 in 0.340 m	0.0550 Nm

Effort/Load Data Analysis

A comparison of $m_1 \times l_1$ to $m_2 \times l_2$ leads the to following ratios

2 Hex Nuts

$$Trial\ 1 \rightarrow \frac{0.0695Nm}{0.0703Nm} = 0.99$$

$$Trial\ 2 \rightarrow \frac{0.0545Nm}{0.0555Nm} = 0.98$$

$$Trial\ 3 \rightarrow \frac{0.0621Nm}{0.0611Nm} = 1.02$$

3 Hex Nuts

$$Trial\ 1 \rightarrow \frac{0.0699Nm}{0.0713Nm} = 0.98$$

$$Trial\ 2 \rightarrow \frac{0.0420Nm}{0.0462Nm} = 0.91$$

$$Trial\ 3 \rightarrow \frac{0.0543Nm}{0.0550Nm} = 0.99$$

Further Exploration of Lever

The initial analysis in the previous section leads to a working hypothesis: *when a lever is balanced, the product of the distance from the centerline times the mass of the weight is the same on both sides of the lever*. This hypothesis was obtained with limited data. Collecting more data will allow you to see if this hypothesis continues to hold-up for a more extensive data set.

Once you have completed your initial investigation, you will have a working knowledge of the phenomenon for investigating more complex levers and equilibrium questions. Questions that might be interesting to explore regarding levers include:

1. What would happen if two masses with different distances from the fulcrum were placed on the left-hand side of the balance beam; where would the one mass on the right side have to be located?
2. What if the fulcrum was not exactly in the center of the beam so that the unloaded beam tipped to one side or the other? Is it possible to add a weight to one side of the beam so that it comes into equilibrium? After it is in equilibrium, can you add pairs of weights on either side and keep the beam in balance?

3. The current fulcrum is not exactly a knife-edge, but it is very thin as compared to the length of the beam. What happens if the width of the fulcrum is increased (i.e. use a 2"x4" board as the fulcrum or two 2"x4" as the fulcrum)?

These are just a few of the what-if questions that you can ask. As you use your imagination you will come up with many more questions.

Collecting data is only the first step of any investigation. It is important to analyze each data set to determine if it supports or refutes your working hypothesis. Thinking about your data as it is being collected will help you diagnose problems and not waste time. If suddenly, no matter how much weight you add to the lever it is always stuck with the left-hand side on the table; that would be strange. Examination might find that the 1/8-inch piece of metal has dropped below the surface of the 2x4s and changing the pivot point of the beam. There are all sorts of things that can go wrong with an experiment and analysis of data as it is being collected can help reduce frustrating events and misunderstanding.

Lesson 4: Objectives

- **Demonstrate an understanding of work by solving story problems involving work, equilibrium and mechanical advantage**

Content

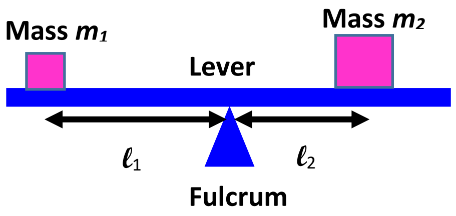
- **The design formula for a Lever**
- **Defining Work**
- **Apply Work to a Lever**
- **Lever as a Simple Machine**

Lesson 4: Work and Simple Machines

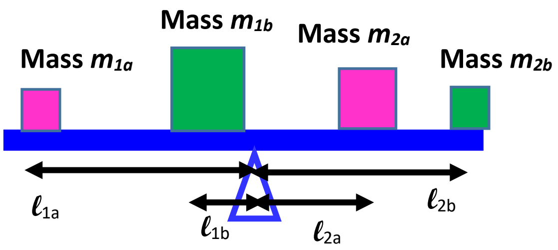
In the previous lesson the basic operation of a lever was discovered using a homemade lever and fulcrum. Based upon data collected in that investigation, the basic design formula for a lever should be discoverable. We now introduce the concept of work, which provides the reasoning behind the lever equilibrium formula. We will discuss the idea of simple machines, with the lever being the first example of a technological device with few or no moving parts that can be used to modify the magnitude and direction of an applied force.

Lever

A schematic of the lever experiment is shown in the figure to the left, where there are two masses m_1 and m_2 balanced on the lever, where l_1 is the distance of a mass (m_1) from the fulcrum on the left-hand side of the lever and l_2 is the distance a mass (m_2) from the fulcrum on the right-hand side of the lever. The investigation discussed in lesson 3 allows one to determine the location of the masses from the centerline when the lever is in balance, i.e. in equilibrium. From the investigation it is possible to observe that if the two masses were equal, i.e. $m_1 = m_2$, then the lever will be in equilibrium when the two distances from the fulcrum were also equal, i.e. $l_1 = l_2$. A second set of experiments would allow one to observe that the lever would be in equilibrium when the product of m times l was the same for both sides. This can be expressed by the equation $m_1 l_1 = m_2 l_2$.



Note that the results from the first investigation can be described by this Lever Equation, where if $m_1 = m_2$ then the Lever Equation gives the result $l_1 = l_2$.



Now consider a more complex situation with multiple masses on each side of the fulcrum. The design of formula for a lever in mechanical equilibrium is similar in structure, but a little more complicated as it takes in account several masses at various locations.

$$m_{1a}l_{1a} + m_{1b}l_{1b} = m_{2a}l_{2a} + m_{2b}l_{2b}$$

If there are more than two weights on the left-hand side of the lever, one just adds additional *mass times distance* terms, e.g. $m_{1c}l_{1c}$ and so on, for each weight. It may be helpful to revisit the lever investigation in order to gain a visual understanding of how this formula really works.

Sample Problem

Consider a real-world example of a teeter-totter on the playground. A father is taking his three children to the playground. The oldest boy weighs 80-lbs, the middle daughter weighs 60-lbs, the youngest daughter weighs 50-lbs and the father weighs 180 lbs. The teeter-totter is 12 ft long, where the oldest boy sits 5 ½ ft from the center, the middle daughter sits 5 ft from the center and the youngest daughter sits 4 ½ ft from the center. If all three children sit on the same side, where must the father sit in order for the see-saw to balance?

Remember to keep units in your equations.

Note that both the left-hand and right-hand side of the equation has lbs., which then cancel. The answer for is then in feet

Solution

We can use the lever equilibrium formula to solve this problem. The children are the masses and distances on the left-hand side of the formula and the father is the right-hand side of the formula. Let's put in the numbers.

$$(80lbs \times 5.5ft) + (60lbs \times 5.0ft) + (50lbs \times 4.5ft) \; = \; 180lbs \times l_{Dad}$$

where l_{Dad} is the distance the dad sits from the fulcrum of the teeter-totter? Doing the addition on the left-hand side

$$965lbs \cdot ft = 180lbs \times l_{Dad}$$

And dividing both sides of the equation by 180-lbs gives us an answer of

$$l_{Dad} = 5.36ft$$

Thus, if the dad sits 5.36-ft from the centerline of the see-saw will be exactly in balance, where the slightest push up from either the kids or the dad will cause the see-saw to move up-and-down.

Work

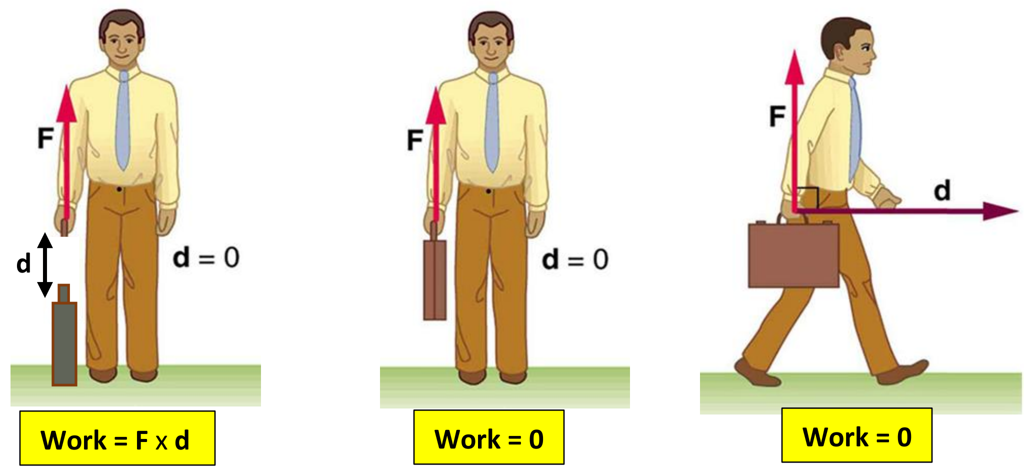
The Lever Formula describes the how various masses must be located, but what is the science behind the formula? The key science concept is work. Work has both a common meaning and a scientific meaning. We often say that 'I worked hard on last night's homework'. Or,' I went to work this morning in order to make enough money for our family to go on vacation this year.' But in science work has a very precise definition. Work is the product of force multiplied by displacement.

Work is a measure of the energy transfer that occurs when an object is moved over a distance by an external force

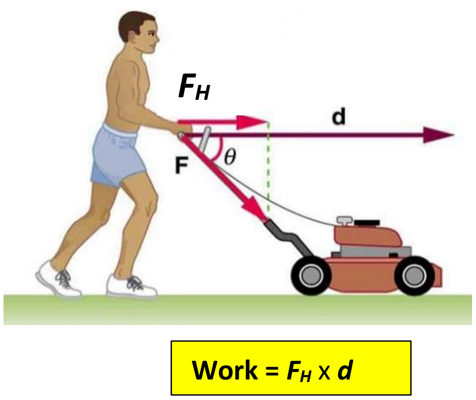
Remember that force has both direction and magnitude, so it is a vector quantity. However, work is scalar. The formal mathematical reasoning behind products of vectors resulting in scalar quantities is rather complicated for our purposes. For This reason, we will use the following definition of work:

$$W = F \times d$$

As an example examine the man with the briefcase below. In the first panel he reaches down and pick up the briefcase exerting a force F to the handle of the briefcase and lifting it a distance d, where the resulting work is $F X \, d$. In the middle panel the man is standing still with the briefcase in his hand. He may be getting tired holding the heavy briefcase filled with school papers, but he is doing no work, since there is no displacement of the briefcase. In the third panel the man is walking holding his briefcase, but the direction of force is up, while the direction of the displacement is to the right. Thus, the upward force on the briefcase handle is not in the direction of the displacement and consequently no work is being done. Work requires that the force and displacement must be in the same direction.



Let's consider a little more complex situation than the man with the briefcase described previously. Consider the man to the right. He pushes on the lawn mower handle, which propels the lawnmower forward, while at the same time pushing the back wheels into the ground. A portion of the force is in the horizontal direction, that is the same direction that the lawnmower is moving. The work done is the product of the horizontal component of the force on the lawn mower handle and the horizontal displacement of the lawnmower. The reason that there is no work on the briefcase for the man walking with briefcase in the third panel was the force is perpendicular to the displacement, i.e. there was no component of the force in the horizontal direction of the displacement.



The units of work in the SI system are Newton-meters, or Nm, which comes directly from the product of force (which has units of Newtons) and distance (which has units of meters). The product Newton-meters has another name called a Joule with the symbol J. As an example, Sarah has a heavy backpack filled with books that has a mass of $12kg$ and she lifts it $1.3m$ to place it on her shoulders.

How much work has Sarah done? First, the backpack weighs 12 Kg that is acted upon by gravity, where the gravitational force on the earth is mg, with $g = 9.8m/s^2$ as the gravitational constant. The gravitational force F on the backpack is

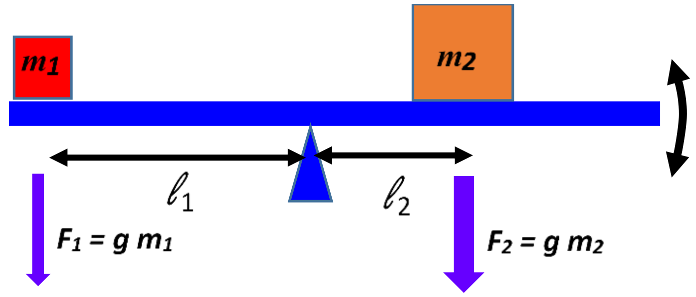
$$12kg \times 9.8m/s^2 = 118kg \cdot m/s^2 = 118N.$$

Because Sarah lifts the backpack 1.3m in the opposite direction of the gravitational force the work is

$W = F \times d = 118N \times 1.3m = 152Nm = 153J.$

Advanced Topic: Origin of the Lever Formula

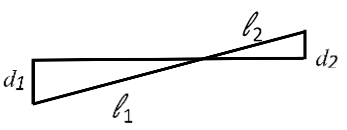
Now that we have the idea of work, i.e. force times displacement, we are ready to revisit the lever. The equation for describing the operation of a lever states that the product of the distance (l_1) and mass (m_1) of an object on side 1 of the lever equals the product of the distance (l_2) and mass (m_2) of an object on side 2 of the lever. This equation expresses the simple idea that if a teeter-totter is to be balanced a smaller mass must be located further from the pivot point than a larger mass.



Where does the equation come from? The answer lies in determining the force required to lift the mass on one side of the teeter-totter. The force acting on each mass is the gravitational force ($F_{gravity}$), which is simply the mass times the acceleration due to gravity ($9.8m/s^2$) or $F_{gravity} = mg$.

The key physical idea about the operation of a teeter-totter is conservation of energy. If the teeter-totter is to freely move up and down it cannot require any extra work to make this motion happen. As we have already learned, work *W* is force *F* times distance *d*. Where if the force is due to gravity, then $W = Fd = mgd$. Now if the teeter-totter is to move freely with no additional work then

$$W_1 + W_2 = 0 \quad \textit{or equivalently} \quad gm_1d_1 - gm_2d_2 = 0$$



Where d_1 is the vertical distance that m_1 moves and d_2 is the vertical distance that m_2 moves. There is a positive sign in front of gm_1d_1 because it requires work to increase the height of m_1; in contrast, there is a negative sign in front of gm_2d_2 because the height of m_2 decreases by a distance d_2. Examining the triangle schematic, we notice that the two triangles are similar; thus, geometrically $d_1/l_1 = d_2/l_2$ *or* $d_1 = d_2l_2/l_1$. Replacing this expression for d_1 in Eqn. 1, we obtain

$$gm_1d_2l_1/l_2 - gm_2d_2 = 0$$

And simplifying

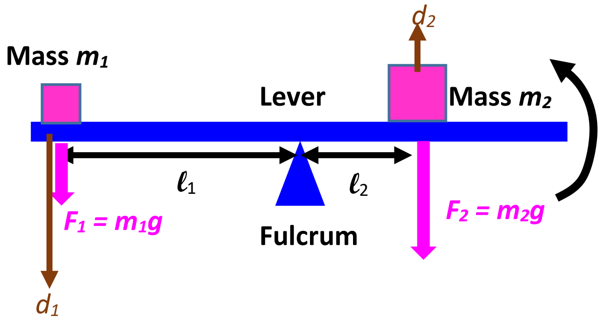
$m_1l_1/l_2 - m_2 = 0$ or equivalently $m_1l_1 = m_2l_2$

This is the equation for a lever, where the key physical idea that led to the lever equation was that the work done in raising the mass on one side of the see-saw was exactly balanced by the work recovered on lowering the mass on the other side of the teeter-totter. The balancing of work on the two sides of the lever is just one specific example of conservation of energy.

Simple Machines

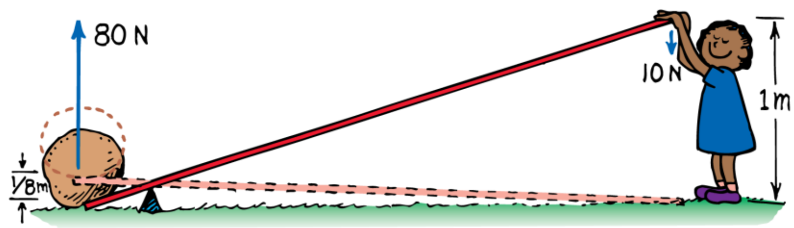
Simple machines are devices that transfer the applied input force into an output force, much greater than the original input force. This is often accomplished by changing the distance through which the force is applied. Levers, gears, and pulleys are well known examples of this type of simple machine. These devices change the amount of effort required to move an object by reducing the amount of input force that is needed to perform a certain job. The ratio of input to output force for any simple machine is called its *mechanical advantage* (MA).

$$MA = \frac{F_{output}}{F_{input}}$$



Consider the total work that is done by a lever. As the left-hand side of the lever goes down, the right hand side of the lever goes up, but if the lever is balanced this motion requires only the slightest force – essentially zero force if the lever is perfectly balanced and there is no friction on the knife edge on the top of the fulcrum. Notice that the force F_1 is less than the force F_2 , but the distance of the downward travel d_1 will be greater than the upward travel d_2. This leads shows us the advantage of the lever as a machine, we can lift a heavy object (mass m_2 with a downward force F_2) a small distance (d_2) using a lever, where on the other end of the lever the one applies a much smaller force F_1 over a much longer distance.

Consider using a lever to lift a very heavy bolder. The force that is applied at one end of the lever is transferred to the other end of the lever. As can be seen in the image below the force being applied is causing one end of the lever to move downward while the other end moves upward. Because there is displacement at the end of the lever, and that displacement is caused by a force, work has been done at that end of the lever. Notice also that the amount of work done on one end of the lever is equal to the amount of work done on the other end of the lever.



$W = F \times d = 80N \times 1/8m = 10Nm$ work in lifting boulder

$W = F \times d = 10N \times 1m = 10Nm$ work applied by girl

What is interesting is that the magnitude of the force exerted by the pry bar, in this case, is much greater than the magnitude of the applied (input) force at the other end, and yet the work is the same. In this example the mechanical advantage is $80N/10N = 8$.

Perhaps the most famous quote concerning using the lever as a simple machine comes from the ancient Greek mathematician scientist and philosopher Archimedes of Syracuse. When considering the properties of a lever he remarked that one could move at object as big as the whole earth if only he had a fulcrum, a long enough lever and a place to stand to apply force to his end of the lever. This is surely an exaggeration, but it does show the basic idea of the lever as a simple machine that can be used to perform useful tasks.

Module 1 Activities and Resources

Lesson 1: Balanced and Unbalanced Forces Student Activity Sheet
Causes of Motion PowerPoint Slide deck

Lesson 2: Action-Reaction Pairs Student Activity Sheet
Levers and Equilibrium PowerPoint Slide deck

Lesson 3: Mechanical Equilibrium *(Lever)* Investigation Data Sheet

Lesson 4: Levers Work and Mechanical Advantage Practice Problems Activity Sheet
Work PowerPoint Slide deck

The following is an incomplete list of referenced resources

https://ips.iat.com
http://static.nsta.org
http://cmse.tamu.edu
http://www.physicsclassroom.com

The Purpose behind all resources within the hardware store science curriculum is to research the effective integration of STEM subjects into a physical science classroom. All material is organized from outside sources and solely intended to provide the researchers a framework for the development of original content based on experimental findings.