ME 270 – Fall 2026 Exam 1 (B) **NAME (Last, First):** SOLUTION **(Please Print)**

PUID #: _____ **(Please Print)**

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: _____

Instructor's Name and Section: (Circle Your Section)

Sections:

West Lafayette Campus (PWL)

J. Osorio, Section 002, MWF 8:30AM-9:20AM
J. Jones, Section 003, MWF 9:30AM-10:20AM
A. McDonald, Section 001, MWF 10:30AM-11:20AM
J. Jones, Section 005, MWF 11:30AM-12:20PM
F. Semperlotti, Section 008, MWF 12:30PM-1:20PM
J. Choi, Section 009, MWF 1:30PM-2:20PM
F. Semperlotti, Section 010, MWF 2:30PM-3:20PM
A. McDonald, Section 032, MWF 3:30PM-4:20PM
T. Ballance, Section 033, MWF 4:30PM-5:20PM
M. Murphy, Section 034, TR 9:00AM-10:15AM
J. Jones, Section Y01, Distance Learning

Indianapolis Campus (PIN)

E. Goff, Section 031, MWF 9:30AM-10:20AM
F. Bipasha, Section 029, MWF 1:30PM-2:20PM
N. Saqib, Section 030, TTh 12:00PM-1:15PM

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

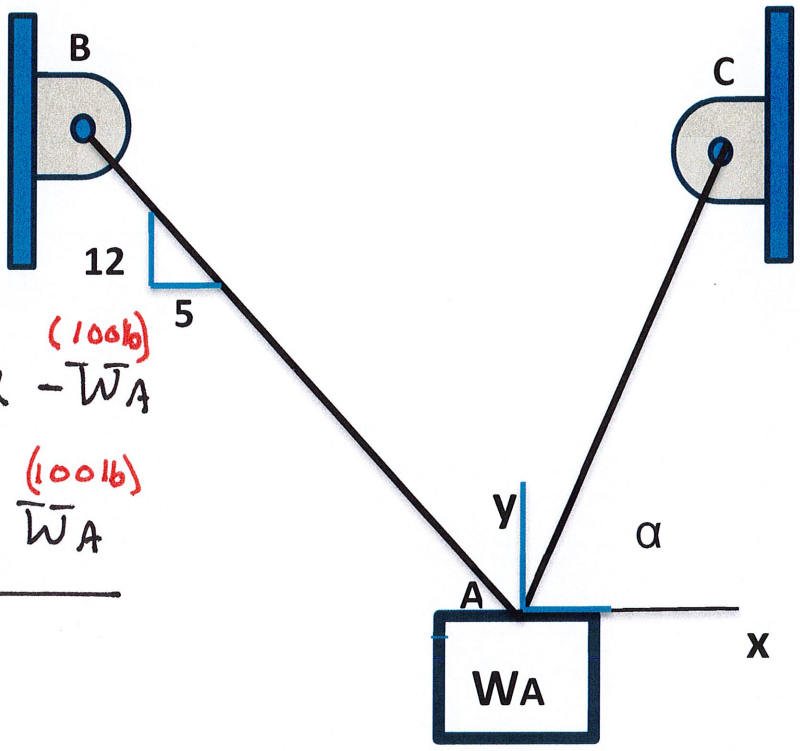
- The allowable exam time for Exam 1 is 90 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.

If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

PROBLEM 1 (20 points)

1A. Block A (W_A) weighs 100 lb and is held in static equilibrium by two cables (T_{AB} and T_{AC}). The magnitude of the tension in Cable AB is a known quantity of T_{AB} = 65 lb. Determine the tension in cable **AC** and the angle alpha (α) that cable **AC** makes with the x-axis.



$$\Sigma F_x = 0 = -\frac{5}{13} T_{AB} + T_{AC}\cos\alpha$$

$$\Sigma F_y = 0 = \frac{12}{13} T_{AB} + T_{AC}\sin\alpha - W_A$$

(65 lb) (100 lb)

$$T_{AC}\sin\alpha = -\frac{12}{13} T_{AB} + W_A$$

$$T_{AC}\cos\alpha = \frac{5}{13} T_{AB}$$

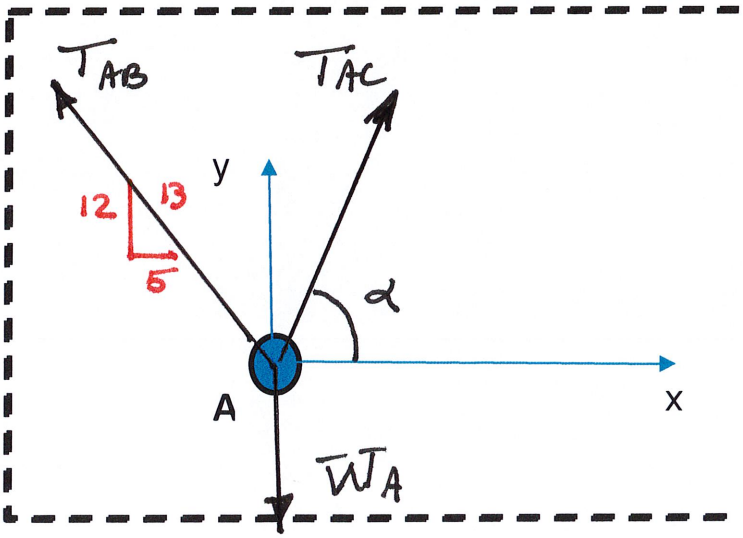
(65 lb)

$$\tan\alpha = \frac{40}{25}$$

$$\therefore \alpha = 58^\circ$$

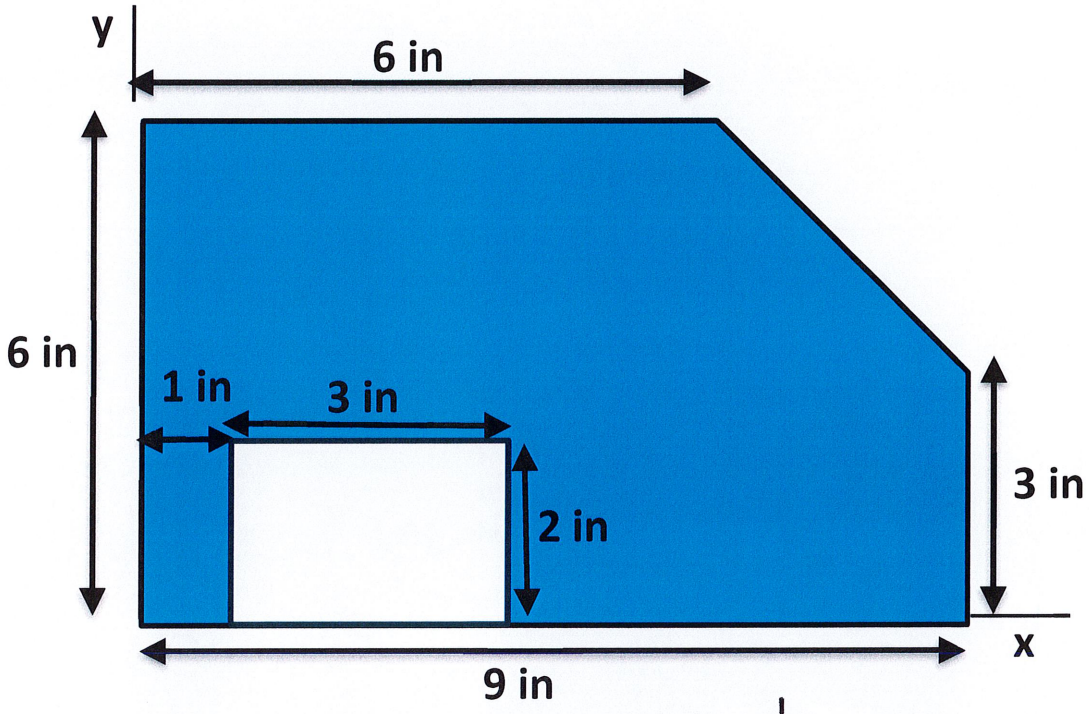
$$T_{AC} = \frac{25}{\cos 58^\circ} = \boxed{47.2 \text{ lb.}}$$

Free-body Diagram (1 pt)



T_{AC} = 47.2 lb (2 pts) α = 58 degrees (2 pts)

1B. Using the method of composite parts, find the area (A) and the x-centroid (x_C) of the shaded area in the figure below with respect to the coordinate axes provided. Please show your work to receive credit. If the rectangular cutout were filled in with material, qualitatively what impact would this have on x_C and y_C? (No calculations are required for determining this qualitative impact). **(5 pts)**



Shape	Area	x_c
Rectangle	$54\ in^2$	$4.5\ in$
Rectangle ~~Square~~	$-6\ in^2$	$2.5\ in$
Triangle	$-4.5\ in^2$	$8\ in$
	$A_{TOT} = 43.5\ in^2$	

$$A x_c = A_1 x_{c_1} + A_2 x_{c_2} + A_3 x_{c_3}$$

$$43.5(x_c) = 54(4.5) + (-6)(2.5) + (-4.5)(8)$$

$$\therefore x_c = \frac{192}{43.5} = \boxed{4.41\ in}$$

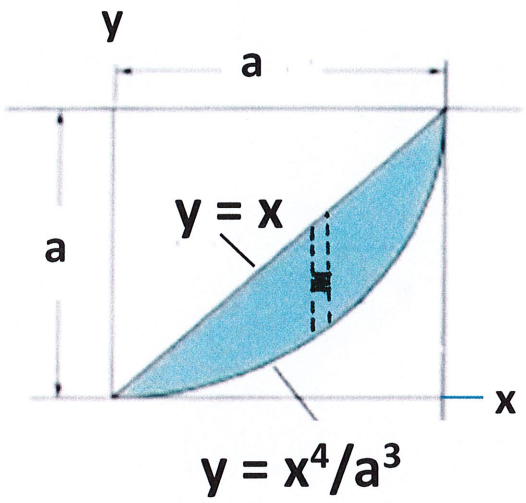
A = 43.5 in^2 (1 pt) x_C = 4.41 in (2 pts)

x_C = Increase Stay the Same **(Decrease)** (Circle One) (1 pt)

y_C = Increase Stay the Same **(Decrease)** (Circle One) (1 pt)

1C. Using the method of integration, determine the area (A) and the x-centroid (x_C) of the shaded area with respect to the coordinate axes provided as a function of the constant "a". Please show your work to receive credit. Qualitatively, would you expect y_C to be larger, smaller or equal to x_C? (No calculations are required for determining this qualitative impact). **(5 pts)**

$$A = \int_{x_1=0}^{x_2=a} \int_{y_1=x^4/a^3}^{y_2=x} dy\,dx$$



$$A = \int_0^a [y]_{\frac{x^4}{a^3}}^{x} dx = \int_0^a \left(x - \frac{x^4}{a^3}\right) dx$$

$$A = \frac{x^2}{2} - \frac{x^5}{5a^3}\Big]_0^a = \frac{a^2}{2} - \frac{a^5}{5a^3} = \boxed{\frac{3a^2}{10}}$$

$$A\,x_c = \int_{x_1=0}^{x_2=a} \int_{y_1=\frac{x^4}{a^3}}^{y_2=x} (x)\,dy\,dx = \int_0^a x\,[y]_{\frac{x^4}{a^3}}^{x}\,dx$$

$$A x_c = \int_0^a (x)\left(x - \frac{x^4}{a^3}\right) dx = \int_0^a \left(x^2 - \frac{x^5}{a^3}\right) dx$$

$$\left(\frac{3a^2}{10}\right) x_c = \left[\frac{x^3}{3} - \frac{x^6}{6a^3}\right]_0^a = \frac{a^3}{3} - \frac{a^6}{6a^3} = \frac{a^3}{3} - \frac{a^3}{6} = \frac{a^3}{6}$$

$$\frac{3a^2}{10} x_c = \frac{a^3}{6} \Rightarrow x_c = \frac{a^3}{6}\left(\frac{10}{3a^2}\right) = \boxed{\frac{5a}{9} = 0.556\,a}$$

A = $3a^2/10$	(2 pts)	x_C = $0.556\,a$	(2 pts)	
y_C Trend:	$y_C > x_C$	($y_C < x_C$)	$y_C = x_C$	(Circle One) (1 pt)

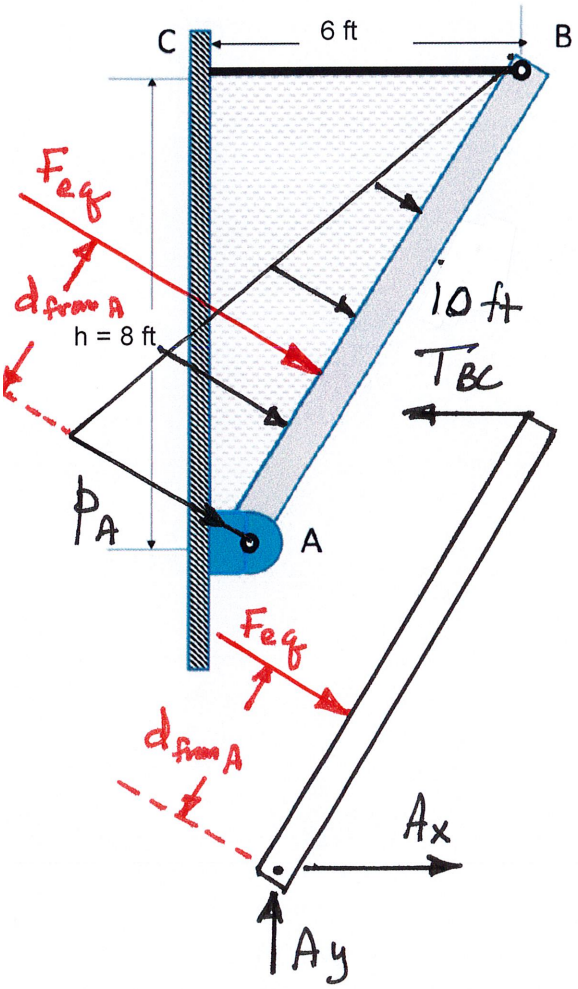
1D. A cable is attached at point B that is 8 feet above a pin joint that holds a 1-foot wide gate shut to prevent spillage of the water to a height of h as shown. The specific weight of the water is $\rho g = 62.5$ lb/ft^3. What is the tension in cable **BC** when the area is filled with water (region ABC)? **(5 pts)**

Please Determine:

i. The equivalent hydrostatic force (F_{eq}) applied to the 1-foot wide gate and its location along gate **AB.**

ii. The force carried in cable **BC.** Assume gate AB has negligible weight.

$$p_A = \rho g h_A = \left(62.5 \frac{lb}{ft^3}\right)(8 ft) = 500 \frac{lb}{ft^2}$$



$$F_{eq} = \frac{1}{2}(10 ft)\left(500 \frac{lb}{ft^2}\right)(1 ft) = \boxed{2500 lb.}$$

$$d_{from\ A} = \frac{1}{3}(10 ft.) = \boxed{3.33 ft.}$$

$$\Sigma M_A = 0 = -F_{eq}\,(d_{from\ A}) + T_{BC}\,(8 ft)$$

(2500 lb) (3.33 ft)

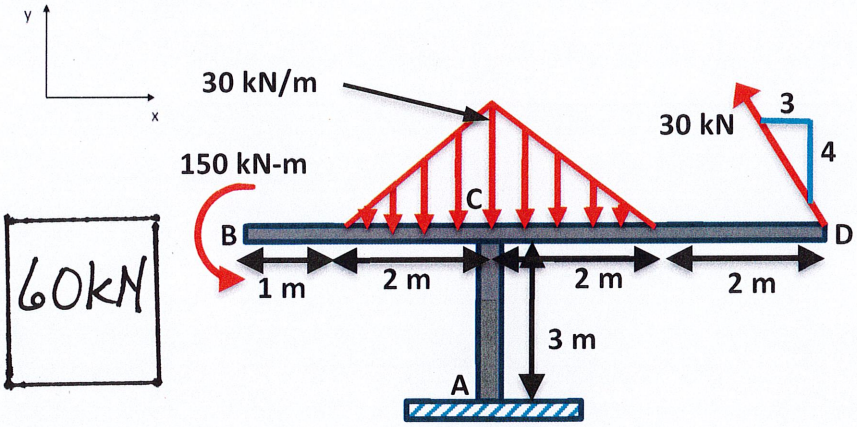
$$\therefore T_{BC} = 1041 lb.$$

F_{eq} = 2500 lb		(2 pts)
Distance of F_{eq} from A along **AB** = 3.33 ft		(1 pt)
Force carried in the cable **BC** = 1041 lb		(2 pts)

PROBLEM 2. (20 points)

Given: The T-shaped bar shown is loaded with a 30 kN point force at D, a triangular distributed load (with a max load intensity of 30 kN/m at C), and a 150 kN-m couple at B as shown and is held in static equilibrium by a fixed support at A. The weight of the T-bar can be assumed to be negligible.

Find: a) Determine the magnitude of the equivalent force (F_{eq}) for the distributed load and its distance with respect to (wrt) end B ($\bar{x}_{eq}$)$_{\text{wrt B}}$. **(3 pts)**

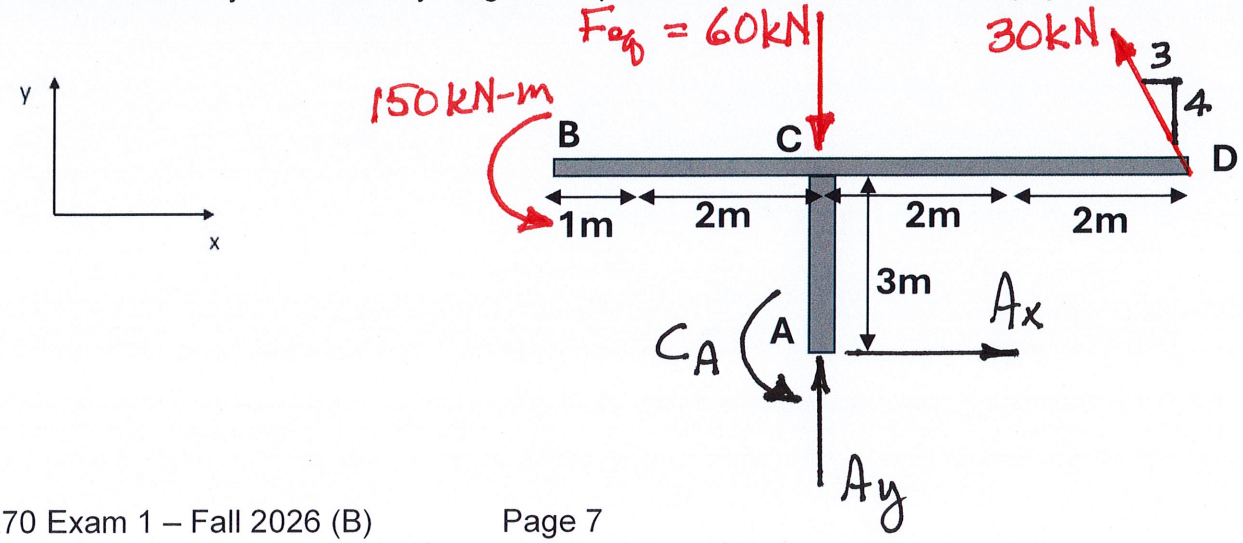


$F_{eq} = \frac{1}{2}(4m)(30\ \frac{kN}{m}) = \boxed{60kN}$

$\boxed{(\bar{x}_{eq})_{\text{wrt B}} = 3m}$

F_{eq}= 60 kN (2 pts)	$(\bar{x}_{eq})_{\text{wrt B}}$ = 3 m (1 pts)

b) On the artwork provided, complete the free body diagram for the T-shaped bar. Use the F_{eq} determined above in your free body diagram in place of the distributed load. **(2 pts)**



c) Clearly write the equilibrium equations and solve for the reactions at the fixed support at A. Express your solution in vector form. **(12 pts)**

$$\Sigma M_A = 0 = C_A + 150\,\text{kN-m} + (30\text{kN})\left(\tfrac{3}{5}\right)(3\text{m}) + (30\text{kN})\left(\tfrac{4}{5}\right)(4\text{m})$$

$$\therefore C_A = 300\,\text{kN-m}$$

$$\Sigma F_x = 0 = A_x - 30\text{kN}\left(\tfrac{3}{5}\right)$$

$$\therefore A_x = 18\,\text{kN}$$

$$\Sigma F_y = 0 = A_y - 60\,\text{kN} + 30\text{kN}\left(\tfrac{4}{5}\right)$$

$$\therefore A_y = 36\,\text{kN}$$

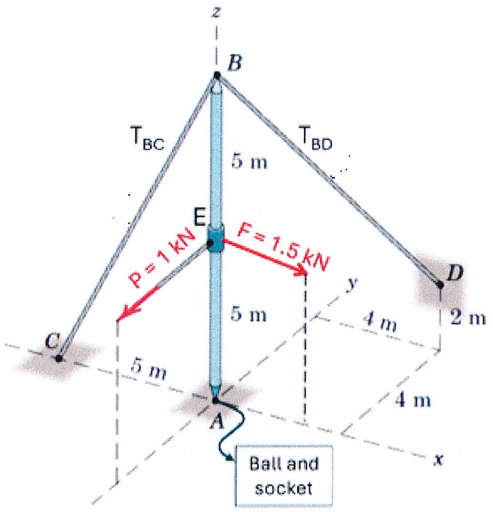
$\vec{C}_A =$ **−300** $\hat{k}$ **kN·m (6pts)** $\vec{F}_A =$ (**18** $\hat{\imath}$ + **36** $\hat{\jmath}$) **kN (6 pts)**

d) If the 30 kN point load at D were removed from the T-bar, would the **magnitudes** of the reactions at A increase, decrease, or remain the same. Circle the appropriate response. (No work needs to be shown)? **(3 pts)**

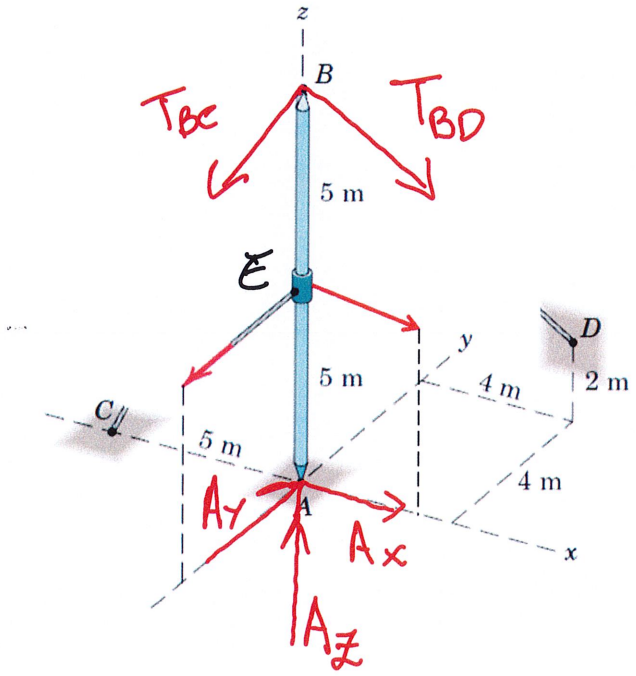
M_A would:	**increase**	**remain the same**	**(decrease)**	**(circle one)**	**(1 pt)**
A_x would:	**increase**	**remain the same**	**(decrease)**	**(circle one)**	**(1 pt)**
A_y would:	**(increase)**	**remain the same**	**decrease**	**(circle one)**	**(1 pt)**

PROBLEM 3. (20 points)

GIVEN: A pole of negligible mass supports a force P = 1 kN (aligned along the y axis) and a force F = 1.5 kN (aligned along the x axis). The pole is held in static equilibrium by two cables (BC and BD) attached to the floor and the wall at C and D, respectively. There is a ball-and-socket constraint at point A.



a) Complete the free body diagram of the pole using the artwork below by adding all the necessary FBD components. **(2 pts)**



b) Write expressions for tension vectors $\mathbf{T_{BD}}$ and $\mathbf{T_{BC}}$ acting on the pole using their magnitudes (at this stage, magnitude of $\mathbf{T_{BD}}$ and $\mathbf{T_{BC}}$ are unknown and will be determined at part (c)) and known unit vectors. Please express your result in decimal form. **(4 pts)**

$$\vec{T}_{BD} = T_{BD}\,\hat{u}_{BD} = T_{BD}\,\frac{\vec{r}_{BD}}{|\vec{r}_{BD}|} = T_{BD}\,\frac{4\hat{i}+4\hat{j}-8\hat{k}}{\sqrt{(4)^2+(4)^2+(8)^2}} =$$

$$= T_{BD}\,\frac{4\hat{i}+4\hat{j}-8\hat{k}}{\sqrt{96}} = T_{BD}\,(0.408\,\hat{i}+0.408\,\hat{j}-0.816\,\hat{k})$$

$$T_{BC} = T_{BC}\,\hat{u}_{BC} = T_{BC}\,\frac{\vec{r}_{BC}}{|\vec{r}_{BC}|} = T_{BC}\,\frac{-5\hat{i}-10\hat{k}}{\sqrt{5^2+10^2}} =$$

$$= T_{BC}\,\frac{-5\hat{i}-10\hat{k}}{\sqrt{125}} = T_{BC}\,(-0.447\,\hat{i}-0.894\,\hat{k})$$

$\vec{T}_{BD} = |T_{BD}| * [(\underline{0.408})\,\hat{i} + (\underline{0.408})\,\hat{j} + (\underline{-0.816})\hat{k}]$ [redacted]. (2 pts)

$\vec{T}_{BC} = |T_{BC}| * [(\underline{-0.447})\,\hat{i} + (\underline{0})\,\hat{j} + (\underline{-0.894})\,\hat{k}]$ [redacted]. (2 pts)

c) Determine the magnitudes of the tension in cables T_{BD} and T_{BC}. **(8 pts)**

The moment wrt A:

$$\vec{M}_A = (\vec{r}_{AE} \times \vec{P}) + (\vec{r}_{AE} \times \vec{F}) + (\vec{r}_{AB} \times \vec{T}_{BC}) + (\vec{r}_{AB} \times \vec{T}_{BD})$$

$$\begin{cases} \vec{r}_{AE} = 5\hat{k} \; ; \; \vec{r}_{AB} = 10\hat{k} \\ \vec{P} = -1\hat{j} \qquad \vec{F} = 1.5\hat{i} \end{cases}$$

Substitute:

$$\vec{M}_A = (+5\,\hat{i}) + (7.5\hat{j}) + T_{BC}(-4.47\hat{j}) + T_{BD}(4.08\hat{j} - 4.08\hat{i})$$

Project to get scalar eqns:

$\Sigma M_x = 0$ $\quad 5 - 4.08\,T_{BD} = 0$ $\quad \boxed{T_{BD} = 1.225\ kN}$

$\Sigma M_y = 0$ $\quad 7.5 + 4.08\,T_{BD} - 4.47\,T_{BC} = 0$

$\boxed{T_{BC} = 2.795\ kN}$

$\lvert\vec{T}_{BD}\rvert =$	1.225 $kN.$	(4 pts)
$\lvert\vec{T}_{BC}\rvert =$	2.795 $kN.$	(4 pts)

d) Determine the reactions at point **A** and express them as a vector. **(6 pts)**

$$\sum \vec{F} = \vec{0} \Rightarrow \vec{A} + \vec{P} + \vec{F} + \vec{T}_{BD} + \vec{T}_{BC} = \vec{0}$$

$$x: \sum F_x = 0 \quad A_x + F + 0.408\,T_{BD} - 0.447\,T_{BC} = 0$$

$$\hookrightarrow \boxed{A_x = -0.750 \text{ kN}}$$

$$y: \sum F_y = 0 \quad A_y - P + 0.408\,T_{BD} = 0$$

$$\hookrightarrow \boxed{A_y = 0.5 \text{ kN}}$$

$$z: \sum F_z = 0 \quad A_z - 0.816\,T_{BD} - 0.894\,T_{BC} = 0$$

$$\hookrightarrow \boxed{A_z = 3.498 \text{ kN}}$$

$\vec{A} =$ [(−0.750) $\hat{\imath}$ + (0.5) $\hat{\jmath}$ + (3.498) $\hat{k}$] kN. **(6 pts)**