ME 270 – Summer [redacted] Exam 1 NAME (Last, First): SOLUTION

(Please PRINT) PUID #: ______________

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ________________________________

Instructor's Name and Section: (Circle Your Section)

Sections: J. Jones MWF 9:50AM-10:50AM (Section 001)
J. Jones Asynchronous Online Section (Section Y01)

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: ________________________________

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

- The allowable exam time for Exam 1 is 90 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.

If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of every page that you wish to have graded. After uploading the exam, be sure to assign each exam page to the appropriate problem in Gradescope before submitting the exam. Do not assign the cover page, any unused blank pages, or the equation sheet to any problems.

ME 270 – Summer[redacted]Exam 1 **NAME (Last, First):** ______________________

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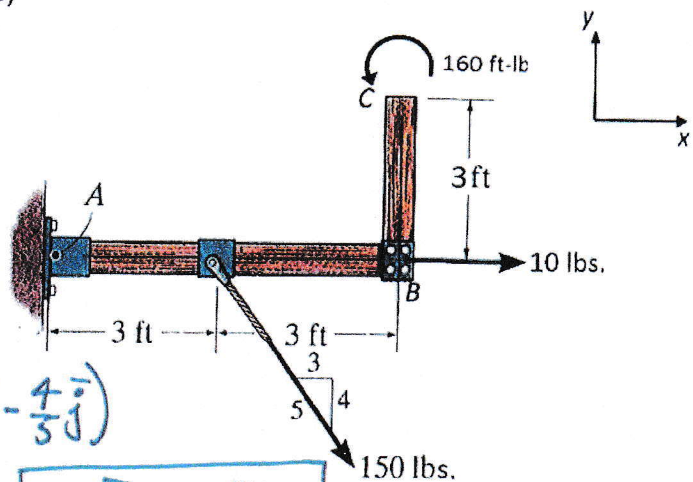
PROBLEM 1 (20 points)

1A. A L-shaped bar ABC subject to two forces and a moment as shown in the figure. Determine an equivalent force-couple system acting at A where $\vec{F}_{eq}$ is the equivalent force and $(\vec{M}_A)_{eq}$ is the equivalent moment written in vector notation. (5 pts)



$$\bar{F}_{eq} = \Sigma \bar{F} = (10\bar{i}) + 150\left(\tfrac{3}{5}\bar{i} - \tfrac{4}{5}\bar{j}\right)$$

$$= 10\bar{i} + (90\bar{i} - 120\bar{j}) = \boxed{100\bar{i} - 120\bar{j} \text{ lbs}}$$

$$(\bar{M}_A)_{eq} = \Sigma \bar{M}_A = +160\bar{k} - \tfrac{4}{5}(150)(3)\bar{k}$$

$$= 160\bar{k} - 360\bar{k} = \boxed{-200\bar{k} \text{ ft-lbs.}}$$

$\vec{F}_{eq} = (\underline{+100})\,\bar{i} + (\underline{-120})\,\bar{j}$ **lbs (2 pts)** $(\vec{M}_A)_{eq} = (\underline{-200})\,\bar{k}$ **ft-lbs (3 pts)**

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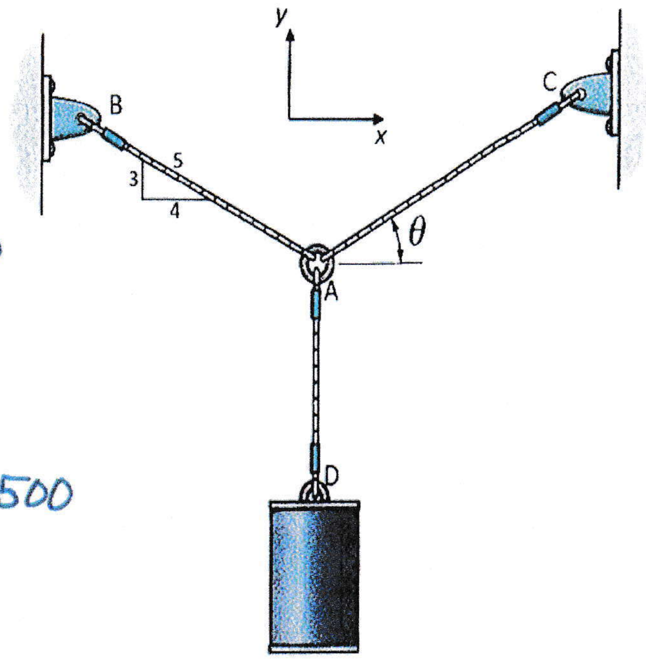
1B. A 500-lb cylinder D is held in static equilibrium by cables AB and AC. If the magnitude of the tension in cable AB = 300 lbs, determine the magnitude of the tension in cable AC and its angle (θ) to maintain static equilibrium. (5 pts)



$$\underline{\Sigma F_x = 0} = -\frac{4}{5}(300) + T_{AC}\cos\theta$$

$$\therefore T_{AC}\cos\theta = 240$$

$$\underline{\Sigma F_y = 0} = \frac{3}{5}(300) + T_{AC}\sin\theta - 500$$

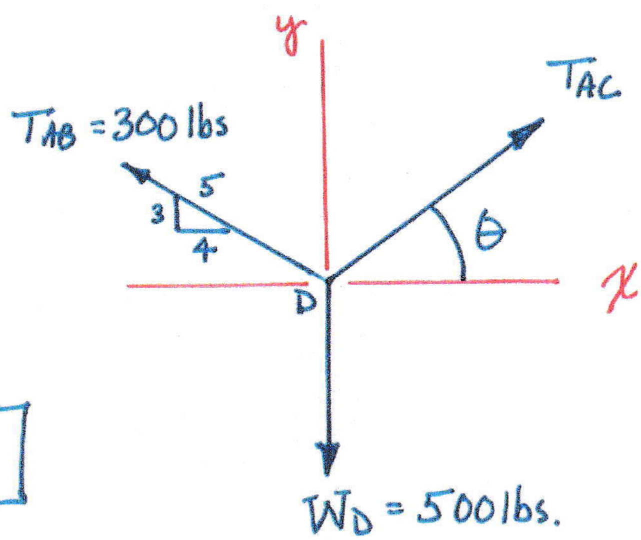
$$\therefore T_{AC}\sin\theta = 500 - 180 = 320$$

$$\frac{T_{AC}\sin\theta}{T_{AC}\cos\theta} = \tan\theta = \frac{320}{240} = \frac{4}{3}$$

$$\tan\theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\boxed{\theta = 53.13^\circ}$$

$$T_{AC} = \frac{240}{\cos 53.13^\circ} = \boxed{400\text{ lbs}}$$



T_{AC} =

T_{AC} = 400 lbs (3 pts)	θ = 53.13 Degrees (2 pts)

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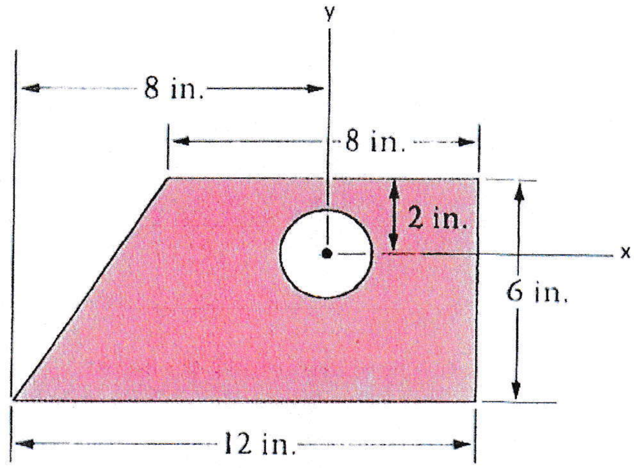
1C. The circular cutout shown has an area of 12 in². Using the method of composite parts, determine the y-centroid ($\mathbf{y_c}$) **about the x-y axis provided**. If the circular cutout was filled in, would the y-centroid shift up, remain the same or shift down? (5 pts)



	Area (in²)	yc (in)
Triangle	12	-2
Square	48	-1
Circle	-12	0
	48 in²	

$$A_{TOT}(y_c) = A_1 y_{c_1} + A_2 y_{c_2} + A_3 y_{c_3}$$

$$52(y_c) = 12(-2) + 48(-1) + 12(0)$$

$$y_c = \frac{-72}{48} = -1.5$$

$A =$ 48 in^2 (2 pts) $(y_c) =$ -1.5 in (2 pts)

$y_c =$ Shift Up Remain the Same Shift Down (Circle One) (1 pt)

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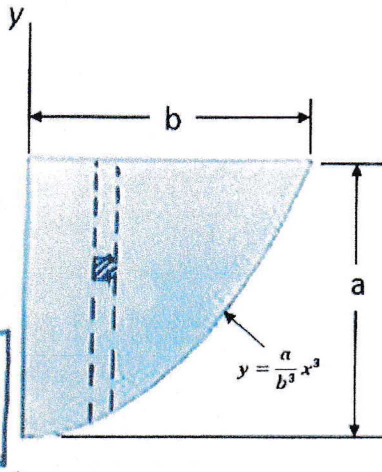
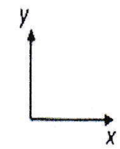
1D. Using the method of integration, find the area (A) and the x-centroid (x_c) of the shaded area about the x-y axis shown as a function of constants a and b. Also, by observation (i.e., no calculations are required), if constants a and b are equal, will x_c be larger, the same as, or smaller than y_c? (5 pts)





$$A = \int_{x_1=0}^{x_2=b}\int_{y_1=\frac{a}{b^3}x^3}^{y_2=a} dy\,dx$$

$$A = \int_0^b [y]_{\frac{a}{b^3}x^3}^{a}\,dx = \int_0^b \left(a - \frac{a}{b^3}x^3\right)dx$$

$$A = ax - \frac{a}{b^3}\frac{x^4}{4}\Big]_0^b = ab - \frac{a}{b^3}\frac{b^4}{4} = \boxed{\frac{3ab}{4}}$$

$$A(x_c) = \int_{x_1=0}^{x_2=b}\int_{y_1=\frac{a}{b^3}x^3}^{y_2=a} x\,dy\,dx = \int_0^b x[y]_{\frac{a}{b^3}x^3}^{a}\,dx = \int_0^b x\left(a - \frac{a}{b^3}x^3\right)dx$$

$$A(x_c) = \int_0^b \left(ax - \frac{a}{b^3}x^4\right)dx = \left[\frac{ax^2}{2} - \frac{a}{b^3}\frac{x^5}{5}\right]_0^b = \frac{ab^2}{2} - \frac{ab^2}{5}$$

$$\therefore \left(\frac{3ab}{4}\right)x_c = \frac{3ab^2}{10} \Rightarrow x_c = \left(\frac{3ab^2}{10}\right)\left(\frac{4}{3ab}\right)$$

$$\boxed{x_c = \frac{2}{5}b}$$

A = $3ab/4$	units² (2 pts)		x_c = $(2/5)b$	units (2 pts)	
(x_c):	$x_c > y_c$	$x_c = y_c$	$x_c < y_c$	(Circle One)	(1 pts)

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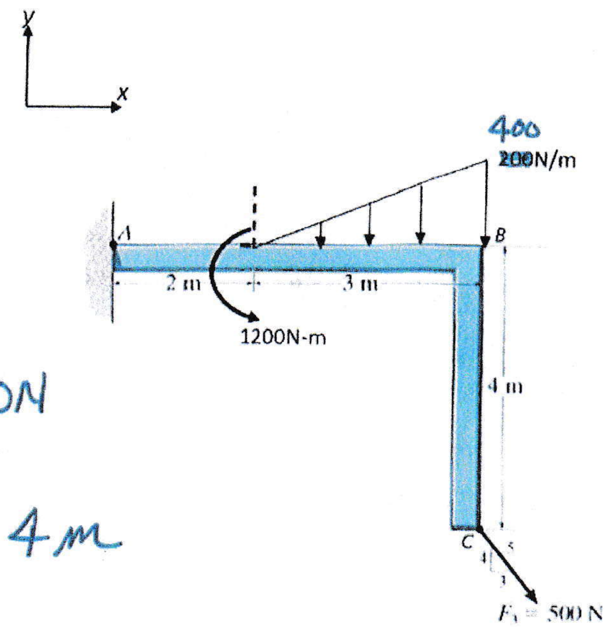
PROBLEM 2. (20 points)

Given: Angled bar ABC is loaded with a point force, a couple, and a distributed load as shown. The triangular line load has a weight/unit length of 400 N/m at B. The bar is held in static equilibrium by a fixed support at A.



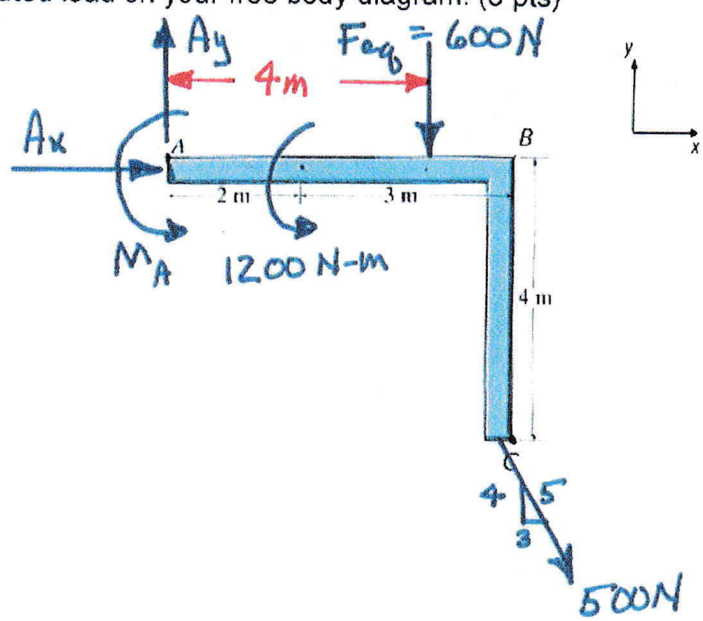
Find: a) Determine the equivalent force (F_{eq}) for the distributed load and its distance from A (x_{eq}). (3 pts)

$$F_{eq} = \frac{1}{2}(3\,m)(400\,N/m) = 600N$$

$$(\bar{x}_{eq})_{from\,A} = 2m + \frac{2}{3}(3m) = 4m$$

F_{eq} = 600 N (2 pts) $(x_{eq})_{from\ A}$ = 4 m (1 pt)

b) On the artwork provided, complete the free body diagram for the L-shaped bar. Use F_{eq} determined above in place of the distributed load on your free body diagram. (3 pts)



c) Clearly write the equilibrium equations and solve for the reactions at the fixed support A in vector form. (12 pts)

$$\underline{\Sigma M_A = 0} = M_A + 1200 - 600(4) + \tfrac{3}{5}(500)(4) - \tfrac{4}{5}(500)(5)$$

$$\therefore M_A = -1200 + 2400 - 1200 + 2000$$

$$\boxed{\bar{M}_A = +2000 \text{kN-m}}$$

$$\underline{\Sigma F_x = 0} = A_x + \tfrac{3}{5}(500)$$

$$\boxed{\therefore A_x = -300\text{N}}$$

$$\underline{\Sigma F_y = 0} = A_y - 600 - \tfrac{4}{5}(500)$$

$$\boxed{\therefore A_y = +1000\text{N}}$$

$\bar{A}$ = -300 $\bar{i}$ + +1000 $\bar{j}$ N (6 pts) $\bar{M}_A$ = +2000 $\bar{k}$ N-m (6 pts)

d) If the distributed load were removed from angled bar ABC, what qualitative effect would this have on the <u>magnitude</u> at the reaction moment at A. (No work need be shown.) (2 pts)

M_A would: increase remain the same decrease (circle one) (2 pt)

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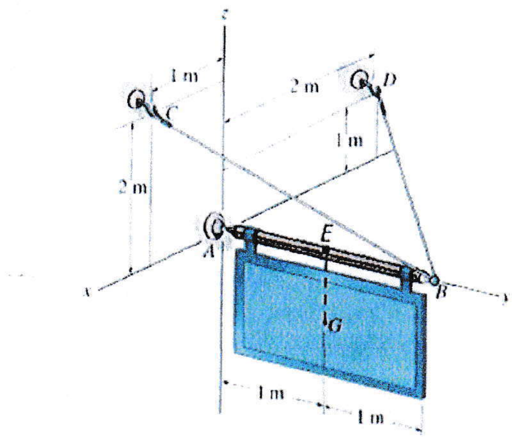
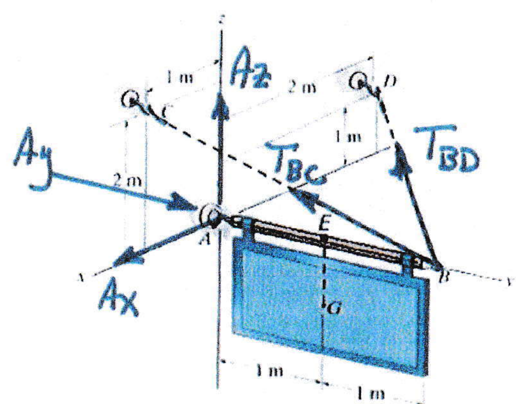
PROBLEM 3. (20 points)

GIVEN: A pole of negligible weight is held in static equilibrium by two cables, BC and BD, and a ball-and-socket support at the ground A. A 1000-N sign with a center of mass G is supported by the pole. (**<u>Hint</u>**: Point E is on the line of action of the center of mass G of the sign).



a) Complete the free body diagram of the pole using the artwork below. (2 pts)



b) Write expressions for tension vectors BC and BD acting on the pole using their unknown magnitudes and known unit vectors. (5 pts)

$\bar{T}_{BC} = (1,0,2) - (0,2,0) / \sqrt{1^2+2^2+2^2} = 1/3\,\bar{i} - 2/3\,\bar{j} + 2/3\,\bar{k}$

$T_{BD} = (-2,0,1) - (0,2,0) / \sqrt{2^2+2^2+1^2} = -2/3\,\bar{i} - \frac{2}{3}\bar{j} + 1/3\,\bar{k}$

$\bar{T}_{BC} = \lvert\vec{T}_{BC}\rvert * [(\ +1/3\ )\hat{\imath} + (\ -2/3\ )\hat{\jmath} + (\ +2/3\ )\hat{k}]$		(2 pts)
$\bar{T}_{BD} = \lvert\vec{T}_{BD}\rvert * [(\ -2/3\ )\hat{\imath} + (\ -2/3\ )\hat{\jmath} + (\ +1/3\ )\hat{k}]$		(2 pts)
Applied Load= 1000 N * [(0) $\hat{\imath}$ + (0) $\hat{\jmath}$ + (−1) $\hat{k}$]		(1 pt)

c) Determine the magnitude of the tension in each of the cables BC and BD. (8 pts)

$\Sigma \bar{M}_A = \bar{0} = (\bar{r}_{AB} \times \bar{T}_{BC}) + (\bar{r}_{AB} \times \bar{T}_{BD}) + (\bar{r}_{AE} \times -1000\bar{k})$

$\bar{0} = \left[(2\bar{j}) \times \left(\frac{1}{3}\bar{i} - \frac{2}{3}\bar{j} + \frac{2}{3}\bar{k}\right) T_{BC}\right] + \left[(2\bar{j}) \times \left(-\frac{2}{3}\bar{i} - \frac{2}{3}\bar{j} + \frac{1}{3}\bar{k}\right) T_{BD}\right]$
$+ \left[(4\bar{j}) \times -1000\bar{k}\right]$

$\bar{0} = \left(-\frac{2}{3}T_{BC}\bar{k} + \frac{4}{3}T_{BC}\bar{i}\right) + \left(\frac{4}{3}T_{BD}\bar{k} + \frac{2}{3}T_{BD}\bar{i}\right) + (-1000\bar{i})$

$\underline{M_x = 0} = \frac{4}{3}T_{BC} + \frac{2}{3}T_{BD} - 1000$

$M_z = 0 = -\frac{2}{3}T_{BC} + \frac{4}{3}T_{BD} \Rightarrow T_{BC} = 2T_{BD}$

$\therefore \frac{8}{3}T_{BD} + \frac{2}{3}T_{BD} = 1000 \Rightarrow \boxed{T_{BD} = 300N}$

$T_{BC} = 2T_{BD} = 2(300) = \boxed{600N}$

$\lvert\vec{T}_{BC}\rvert = 600$	$[N]$	(4 pts)
$\lvert\vec{T}_{BD}\rvert = 300$	$[N]$	(4 pts)

d) At point A determine the reaction force in the z-direction (A_z). (3 pts)

$$\Sigma F_z = 0 = \frac{2}{3}(600) + \frac{1}{3}(300) + O_z - 1000$$

$$\boxed{\therefore A_z = +500N}$$

A_z = 500 N	(3 pts)

e) If cables BC and BD were limited to a load of no more than 800N, what is the heaviest sign that could be supported without exceeding this tension limit? (2 pts)

$$\frac{T_{Largest}}{W_{curr.}} = \frac{T_{Limit}}{W_{MAX}} \Rightarrow \frac{600N}{1000N} = \frac{800N}{W_{MAX}}$$

$$\therefore W_{MAX} = \frac{(800)(1000)}{600} = 1333N$$

W_{max} = 1333 N	(2 pts)

ME 270 Exam 1 Equations

Distributed Loads

$$F_{eq} = \int_0^L w(x)\, dx$$

$$\bar{x}F_{eq} = \int_0^L x\, w(x)\, dx$$

Centroids

$$\bar{x} = \frac{\int x_c dA}{\int dA}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA}$$

$$\bar{x} = \frac{\sum_i x_{ci} A_i}{\sum_i A_i}$$

$$\bar{y} = \frac{\sum_i y_{ci} A_i}{\sum_i A_i}$$

In 3D, $\bar{x} = \frac{\sum_i x_{ci} V_i}{\sum_i V_i}$