ME 270 – Spring 2026 Exam 1 **NAME (Last, First):** SOLUTION **(Please Print)**

PUID #: ____________________ **(Please Print)**

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ________________________________

Instructor's Name and Section: (Circle Your Section)

Sections: **West Lafayette Campus (PWL)**
J. Jones, Section 001, MWF 9:30AM-10:20AM
J. Jones, Section 003, MWF 11:30AM-12:20PM
A. Ramkumar, Section 005, MWF 2:30PM-3:20PM
M. Murphy, Section 013, TR 9:00AM-10:15AM
J. Jones, Section 004, Distance Learning
Indianapolis Campus (PIN)
D. Wagner, Section 015, MWF 3:30-4:20PM

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

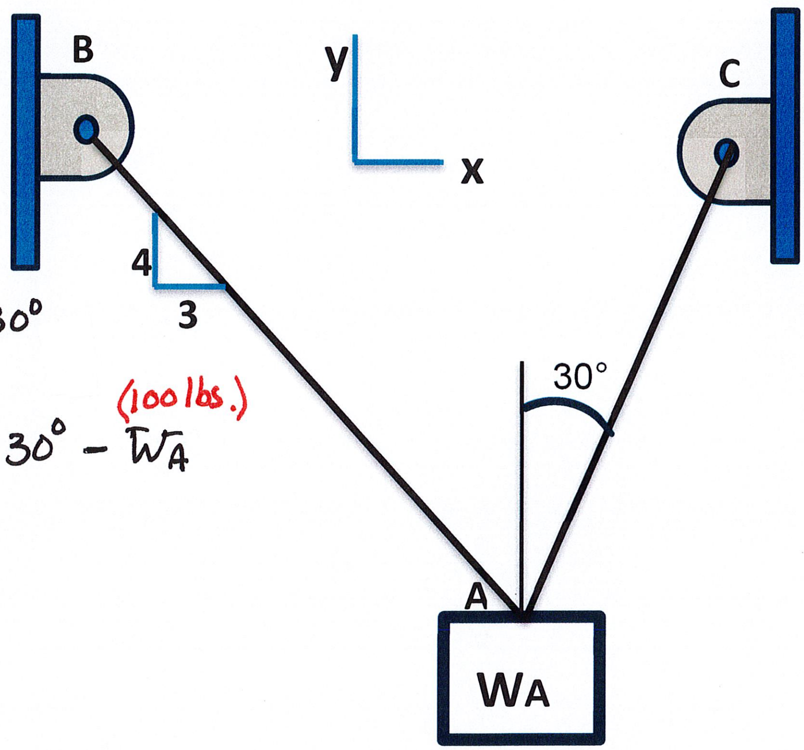
- The allowable exam time for Exam 1 is 90 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the backside of your exam paper.

If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

PROBLEM 1 (20 points)

1A. Block A (**W_A**) weighs 100lbs and is held in static equilibrium by two cables (**T_{AB}** and **T_{AC}**). Using the figure below, determine magnitudes of the tensions in cables **T_{AB}** and **T_{AC}** to maintain static equilibrium. Make sure you include your Free Body Diagram (on area provided) and write clear equations of static equilibrium. **(5 pts)**



$$\underline{\Sigma F_x = 0} = -\frac{3}{5} T_{AB} + T_{AC} \sin 30^\circ$$

(100 lbs.)

$$\underline{\Sigma F_y = 0} = \frac{4}{5} T_{AB} + T_{AC} \cos 30^\circ - W_A$$

$$T_{AC} = \frac{\frac{3}{5}(T_{AB})}{\sin 30^\circ} = 1.2\, T_{AB}$$

$$\frac{4}{5} T_{AB} + (1.2 T_{AB}) \cos 30^\circ - 100 = 0$$

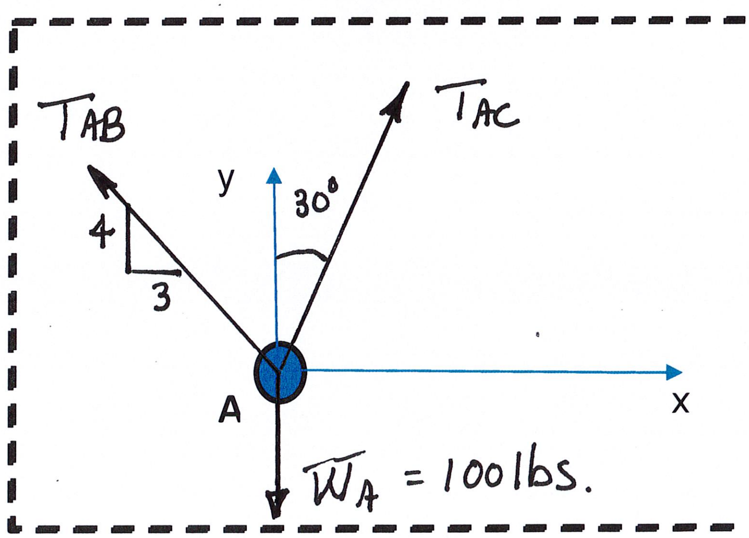
$$1.839\, T_{AB} = 100$$

$$\boxed{T_{AB} = 54.4 \text{ lbs.}}$$

$$T_{AC} = 1.2(T_{AB}) = 1.2\,(54.4)$$

$$\boxed{T_{AC} = 65.3 \text{ lbs.}}$$

Free-body Diagram (1 point)

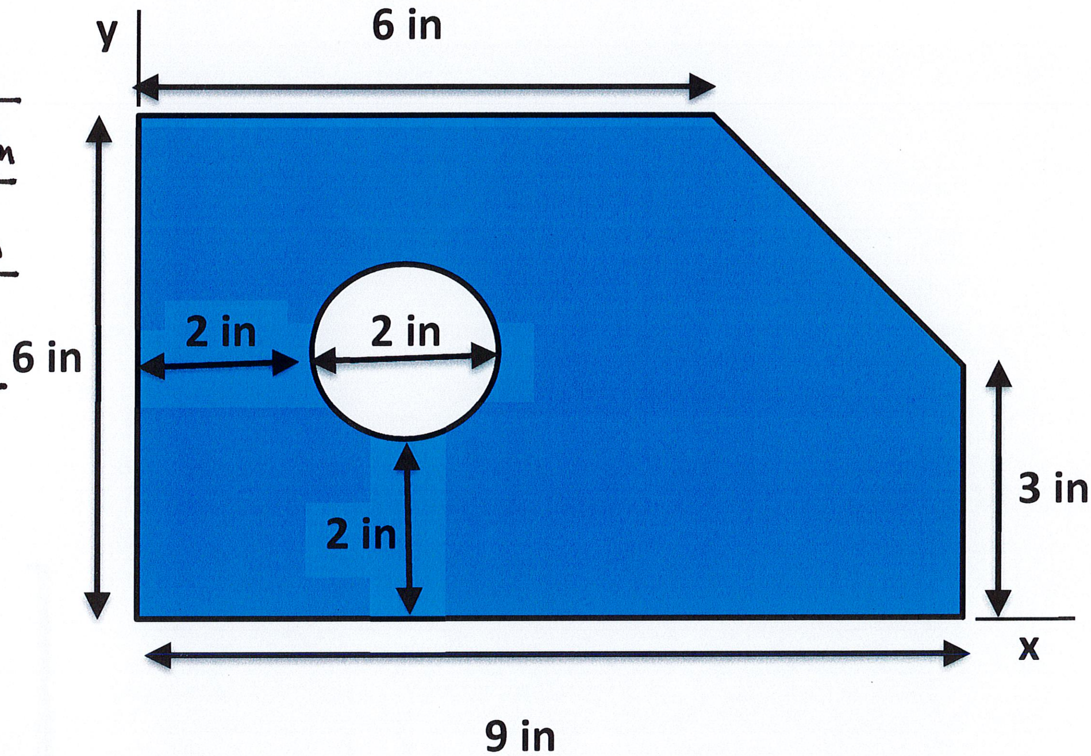


T_{AB} = 54.4 lbs (2 pts) T_{AC} = 65.3 lbs (2 pts)

1B. Using the method of composite parts, find the area (A) and the x-centroid (x_C) of the shaded area in the figure below with respect to the coordinate axes provided. The circular cutout has a 2-in diameter and is centered in the middle of the 6in-by-6in square part of the trapezoid. Please show your work to receive credit. If the circular hole was filled in, qualitatively what impact would this have on x_C and y_C. (No calculations are required for determining this qualitative impact). **(5 pts)**

Shape	A	$\bar{x}$
Rect.	$54 in^2$	$4.5 in$
Tri.	$-4.5 in^2$	$8 in$
Cir.	$-\pi in^2$	$3 in$

$\boxed{A_{TOT} = 46.36 in^2}$



$$A_{TOT}\, x_c = A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3$$

$$(46.36 in^2)\, x_c = (54 in^2)(4.5 in) + (-4.5 in^2)(8 in) + (-\pi)(3 in)$$

$$\therefore x_c = \frac{197.58\ in^3}{46.36\ in^2} = \boxed{4.26\ in}$$

A = 46.36 in^2 (1 pt) x_C = 4.26 in (2 pts)

x_C = Increase Stay the Same Decrease (Circle One) (1 pt)

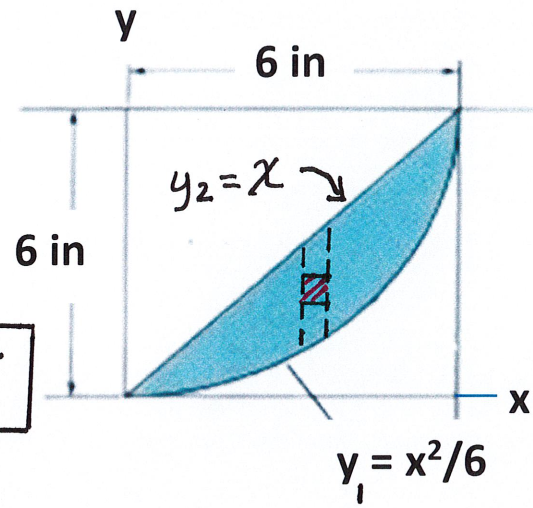
y_C = Increase Stay the Same Decrease (Circle One) (1 pt)

1C. Using the method of integration, determine the area (A) and the x-centroid (x_C) of the shaded area with respect to the coordinate axes provided as a function of the constant "a". Please show your work to receive credit. Qualitatively, would you expect for y_C to be larger, smaller or equal to x_C? (No calculations are required for determining this qualitative impact). **(5 pts)**

$$A = \int_{x_1=0}^{x_2=6} \int_{y_1=\frac{x^2}{6}}^{y_2=x} dy\, dx$$

$$A = \int_0^6 [y]_{\frac{x^2}{6}}^{x} dx = \int_0^6 \left(x - \frac{x^2}{6}\right) dx$$

$$A = \left[\frac{x^2}{2} - \frac{x^3}{18}\right]_0^6 = 18 - 12 = \boxed{6 \text{ in}^2}$$



$$A\, x_c = \int_{x_1=0}^{x_2=6} \int_{y_1=\frac{x^2}{6}}^{y_2=x} x\, dy\, dx = \int_0^6 x\, [y]_{\frac{x^2}{6}}^{x} dx$$

$$(6 \text{ in}^2)\, x_c = \int_0^6 x\left(x - \frac{x^2}{6}\right) dx = \int_0^6 \left(x^2 - \frac{x^3}{6}\right) dx$$

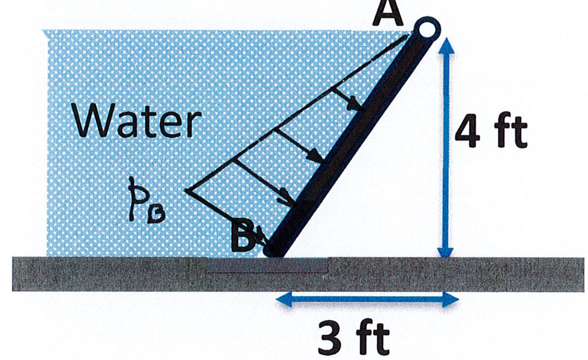
$$(6 \text{ in}^2)\, x_c = \left[\frac{x^3}{3} - \frac{x^4}{24}\right]_0^6 = 72 - 54 = 18 \text{ in}^3$$

$$\therefore x_c = \frac{18 \text{ in}^3}{6 \text{ in}^2} = \boxed{3 \text{ in}}$$

A = 6 in² (2 pts)			x_C = 3 in (2 pts)	
y_C **Trend:**	$y_C > x_C$	$y_C < x_C$	$y_C = x_C$	(Circle One) (1 pt)

1D. Gate AB is holding back a 4-ft deep reservoir of water and is held in static equilibrium by a pin at A and a vertical normal force at B. The specific weight of the water is ρg = 62.5 lbs/ft^3 and Gate AB is 10-ft wide (into the page). Determine:

i. the hydrostatic pressure at the bottom of the reservoir (p_B),
ii. the magnitude of the equivalent force acting on Gate AB (F_{eq}), and
iii. the magnitude of the normal force at B. Include a FBD of Gate AB.

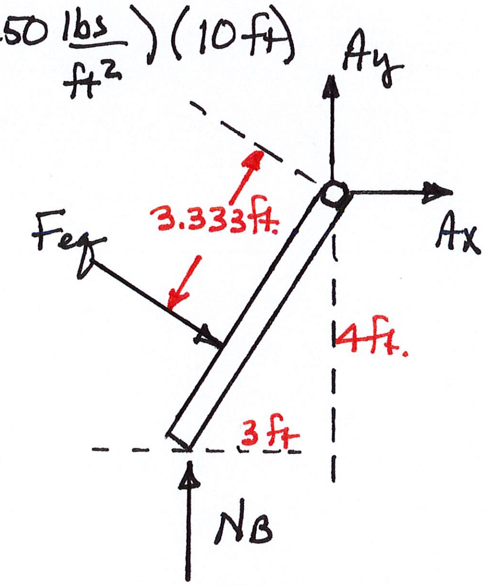


i) $p_B = \rho g h_B = \left(62.5 \frac{lbs}{ft^3}\right)(4 ft)$

$$\boxed{p_B = 250 \ \frac{lbs}{ft^2}}$$

ii) $F_{eq} = \frac{1}{2}(L_{AB})(p_B)(w) = \frac{1}{2}(5 ft)\left(250 \frac{lbs}{ft^2}\right)(10 ft)$

$$\boxed{F_{eq} = 6250 \text{ lbs.}}$$



iii) $\Sigma M_A = 0 = F_{eq}(3.333) - N_B(3)$

$$\therefore N_B = \frac{(6250 \text{ lbs})(3.333 ft)}{3 ft.}$$

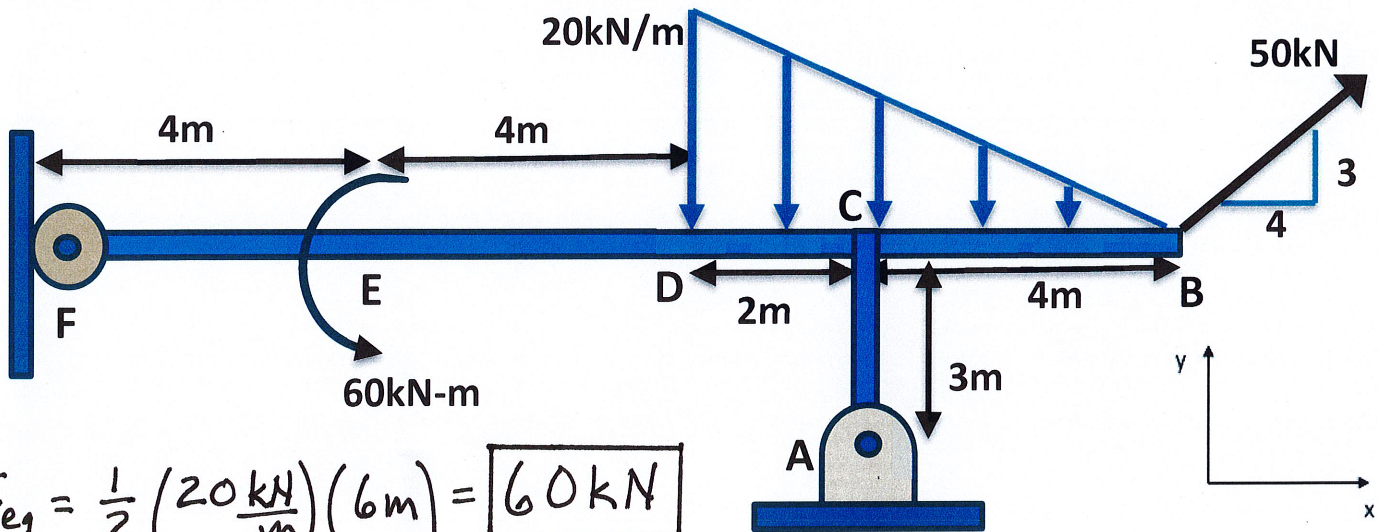
$$\boxed{N_B = 6944 \text{ lbs.}}$$

p_B = 250 lbs/ft^2 (1 pt) F_{eq} = 6250 lbs (2 pts) N_B = 6944 lbs (2 pts)

PROBLEM 2. (20 points)

Given: The extended T-bar shown is loaded with a 50kN point force at B, a distributed load between B and D (with a max load per foot of 20kN/m at end D), and an 60kN-m couple at E and is held in static equilibrium by a pin support at A and a roller support at F.

Find: a) Determine the magnitude of the equivalent force (F_{eq}) for the distributed load and its distance measured from B ($(\bar{x}_{eq})_{\text{from B}}$). **(3 pts)**

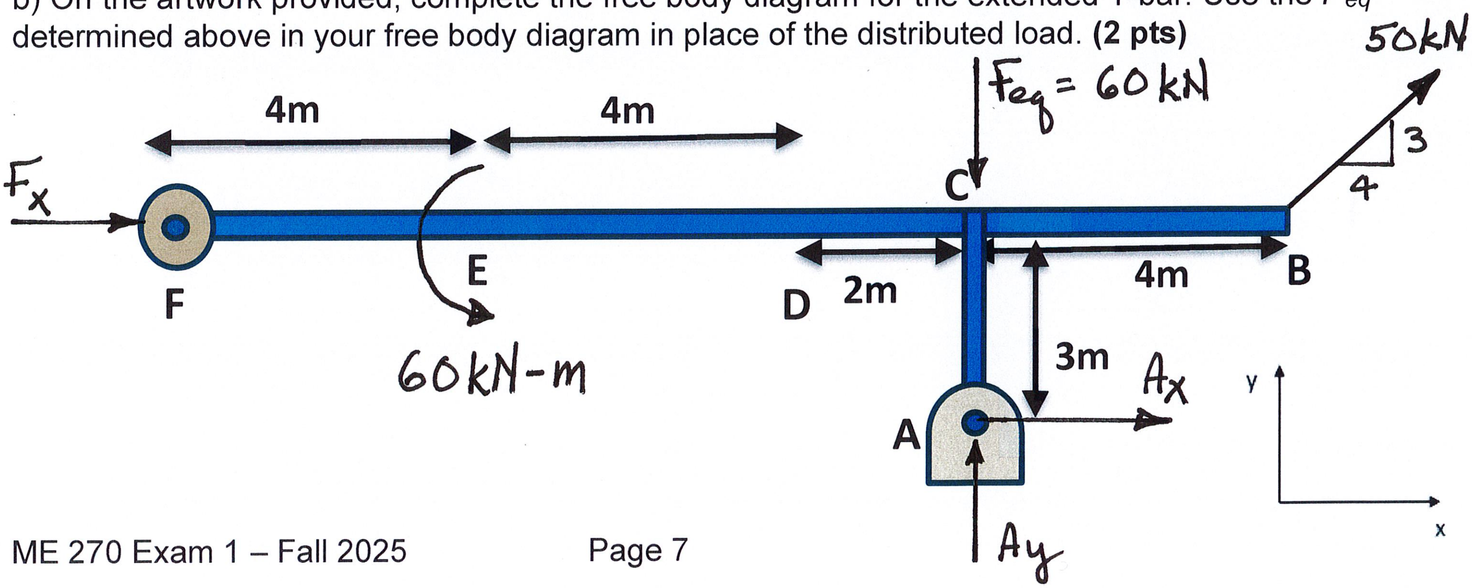


$$F_{eq} = \frac{1}{2}\left(20\frac{kN}{m}\right)(6m) = \boxed{60kN}$$

$$(\bar{x}_{eq})_{\text{from B}} = \frac{2}{3}(6m) = \boxed{4m}$$

F_{eq}= 60 kN **(2 pts)**	$(\bar{x}_{eq})_{\text{from B}}$ = 4 m **(1 pt)**

b) On the artwork provided, complete the free body diagram for the extended T-bar. Use the F_{eq} determined above in your free body diagram in place of the distributed load. **(2 pts)**



c) Clearly write the equilibrium equations and solve for the reactions at the pin support at A and the roller support at F. Express your solution in vector form. **(12 pts)**

$$\underline{\sum M_A = 0} = -F_x(3m) + 60kN\text{-}m + F_{eq}(\cancel{6m})^{0} - 50kN\left(\tfrac{4}{5}\right)(3m) + 50kN\left(\tfrac{3}{5}\right)(4m)$$

$$\boxed{\therefore F_x = +20kN}$$

$$\underline{\sum F_x = 0} = A_x + \underset{(20kN)}{F_x} + \underset{(40kN)}{50kN\left(\tfrac{4}{5}\right)}$$

$$\boxed{\therefore A_x = -60kN}$$

$$\underline{\sum F_y = 0} = A_y - \underset{(60kN)}{F_{eq}} + \underset{(30kN)}{50kN\left(\tfrac{3}{5}\right)}$$

$$\boxed{\therefore A_y = +30kN}$$

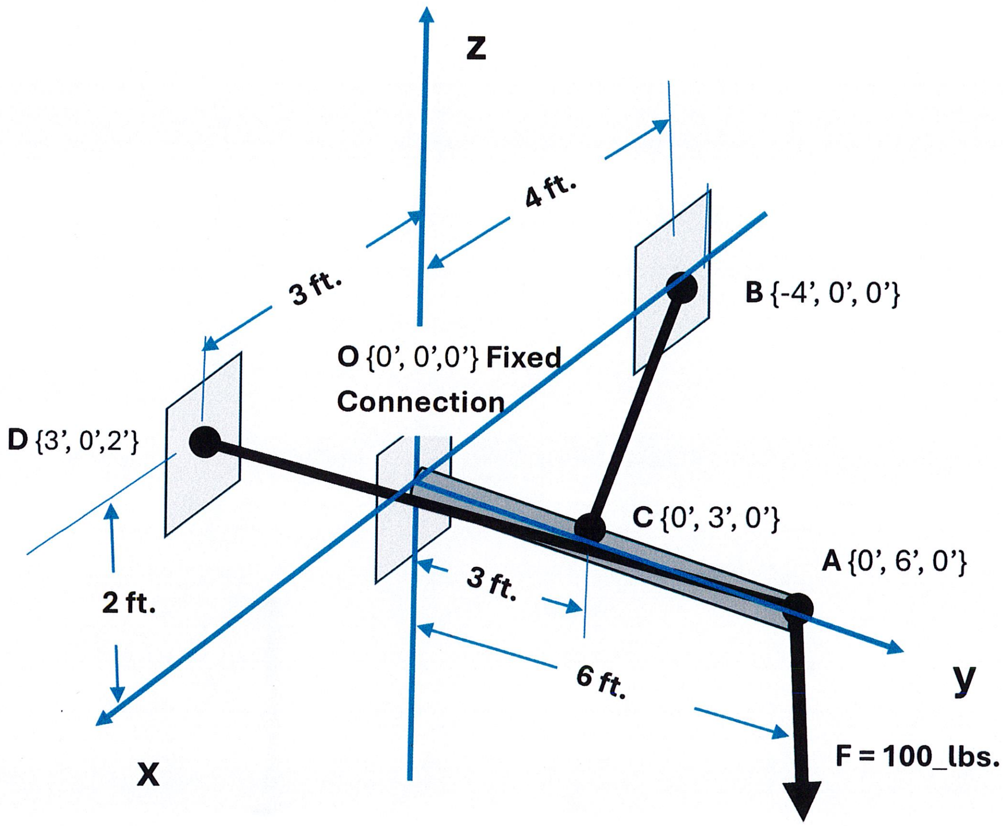
$\bar{\mathbf{F}}$ = + 20 $\hat{\imath}$ **kN (6 pts)** $\bar{\mathbf{A}}$ = − 60 $\hat{\imath}$+ + 30 $\hat{\jmath}$ **kN (6 pts)**

d) If the 50kN point load were removed from the T-bar, how would the **magnitude** of the reactions at A and F change (i.e., ignore any sign changes). Circle the correct change (no work need be shown). **(2 pts)**

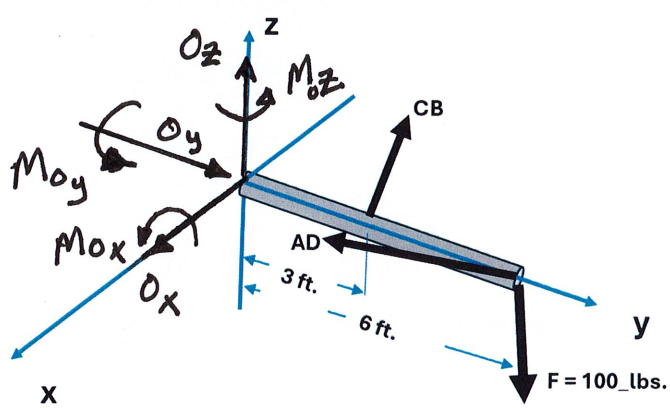
A_x :	Increase	Remain the Same	Decrease	(Circle One)	(1 pt)
A_y :	Increase	Remain the Same	Decrease	(Circle One)	(1 pt)
F_x :	Increase	Remain the Same	Decrease	(Circle One)	(1 pt)

PROBLEM 3. (20 points)

GIVEN: A pole of negligible mass is **fixed into a wall** at point **O.** The pole has a force **F** of 100-lbs applied at point **A**, the tension in cable **CB** (which lies in the horizontal x-y plane) is 350-lbs, and the tension in cable **AD** is 350-lbs.



a) Please complete the free-body diagram for the pole **OA** by denoting all of the reactions at **O** on the artwork provided below. **(2 pts).**



b) Write expressions for tension vectors T_{AD} and T_{CB} acting on the pole using their magnitudes and known unit vectors. The applied load is shown as an example. Please express your result in **decimal** form. **(4 pts)**

$$\bar{T}_{AD} = 350\left[\frac{3\bar{i} - 6\bar{j} + 2\bar{k}}{[3^2 + (-6)^2 + 2^2]^{1/2}}\right] = 350\left(\frac{3}{7}\bar{i} - \frac{6}{7}\bar{j} + \frac{2}{7}\bar{k}\right)$$

$$\bar{T}_{AD} = [150\bar{i} - 300\bar{j} + 100\bar{k}] \text{ lbs}$$

$$\bar{T}_{CB} = 350\left[\frac{-4\bar{i} - 3\bar{j}}{[(-4)^2 + (-3)^2]^{1/2}}\right] = 350\left(-\frac{4}{5}\bar{i} - \frac{3}{5}\bar{j}\right)$$

$$\bar{T}_{CB} = [-280\bar{i} - 210\bar{j}] \text{ lbs.}$$

$\bar{T}_{AD} = 350 * [(\underline{0.429})\hat{\imath} + (\underline{-0.857})\hat{\jmath} + (\underline{0.286})\hat{k}]$ lbs.	**(2 pts)**
$\bar{T}_{CB} = 350 * [(\underline{-0.8})\hat{\imath} + (\underline{-0.6})\hat{\jmath} + (\underline{0})\hat{k}]$ lbs.	**(2 pts)**
Applied Load=100 * $[(\underline{0})\hat{\imath} + (\underline{0})\hat{\jmath} + (\underline{-1})\hat{k}]$ lbs.	**(example)**

c) Determine the <u>components</u> of the reaction Moment at point **O.** Express the moment as a vector **(6 pts)**. To earn credit ALL work must be shown.

$$\bar{M}_o = \bar{0} = (\bar{r}_{OC} \times \bar{T}_{CB}) + (\bar{r}_{OA} \times \bar{T}_{AD}) + (\bar{r}_{OA} \times \bar{F})$$

$$\bar{0} = [3\bar{j} \times (-280\bar{i} - 210\bar{j})] + [6\bar{j} \times (150\bar{i} - 300\bar{j} + 100\bar{k})] + [6\bar{j} \times (-100\bar{k})]$$

$$\bar{0} = [+840\bar{k}] + [-900\bar{k} + 600\bar{i}] + [-600\bar{i}]$$

$$\underline{\Sigma(M_o)_x} = 0 = M_{o_x} + 600 - 600 \Rightarrow \boxed{M_{o_x} = 0 \text{ ft-lbs.}}$$

$$\underline{\Sigma(M_o)_y} = 0 = M_{o_y} \Rightarrow \boxed{M_{o_y} = 0 \text{ ft-lbs.}}$$

$$\underline{\Sigma(M_o)_z} = 0 = M_{o_z} + 840 - 900 \Rightarrow \boxed{M_{o_z} = +60 \text{ ft-lbs}}$$

$\bar{M}_O = [(\underline{0})\hat{\imath} + (\underline{0})\hat{\jmath} + (\underline{60})\hat{k}]$ ft.-lbs.	**(6 pts)**

c) At point **O,** determine the components of the force reaction at **O** and express as a vector. **(6 pts)**

$$\underline{\Sigma \bar{F} = \bar{0}} = (O_x \bar{i} + O_y \bar{j} + O_z \bar{k}) + \bar{T}_{AD} + \bar{T}_{CB} + \bar{F}$$

$$\underline{\Sigma F_x = 0} = O_x + 150 - 280 \Rightarrow \boxed{O_x = +130 \text{ lbs.}}$$

$$\underline{\Sigma F_y = 0} = O_y - 300 - 210 \Rightarrow \boxed{O_y = +510 \text{ lbs.}}$$

$$\underline{\Sigma F_z = 0} = O_z + 100 - 100 \Rightarrow \boxed{O_z = 0 \text{ lbs.}}$$

$\vec{O}$ = [(+130) $\hat{\imath}$ + (+510) $\hat{\jmath}$ + (0) $\hat{k}$] lbs. **(6 pts)**

d) If the cable **AD** is cut (removed) what would happen to the z-component of the reaction force at **O**?

Circle the correct response **(2 pts).**

O_z would: **Decrease** **Increase** **Remain the Same**