NAME (Last, First): SOLUTION

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ________________________________

Instructor's Name and Section: (Circle Your Section)

Sections: **West Lafayette Campus (PWL)**

J. Osorio Pinzon, Section 033, MWF 7:30AM-8:20AM
J. Jones, Section 005, MWF 9:30AM-10:20AM
S. C. Boregowda, Section 008, MWF 10:30AM-11:20AM
J. Jones, Section 003, MWF 11:30AM-12:20PM
L. Krest, Section 009, MWF 12:30PM-1:20PM
F. Semperlotti, Section 001, MWF 1:30PM-2:20PM
A. Ramkumar, Section 010, MWF 2:30PM-3:20PM
T. Ballance, Section 032, MWF 4:30PM-5:20PM
M. Murphy, Section 007, TR 9:00AM-10:15AM
M. Murphy, Section 002, TR 10:30AM-11:45AM
J. Jones, Section Y01, Distance Learning

Indianapolis Campus (PIN)

N. Saqib, Section 031, MWF 9:30AM-10:20AM
A. McDonald, Section 029, MWF 1:30PM-2:20PM
D. Wagner, Section 030, TTh 12:00PM-1:15PM

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 25 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

- The allowable exam time for Final Exam is 120 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the backside of your exam paper.

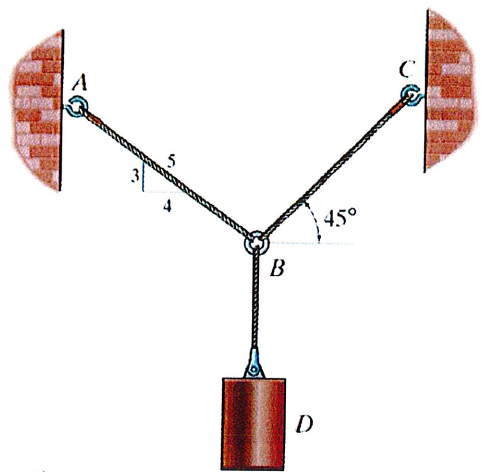
If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

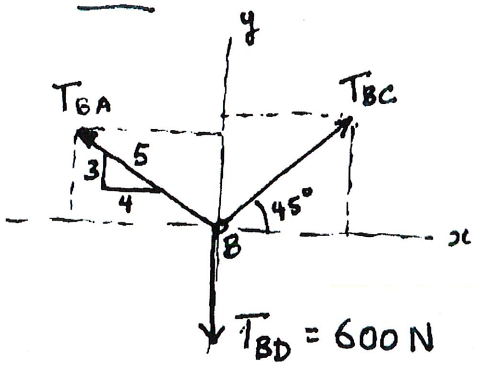
PROBLEM 1 (25 points)

PROBLEM 1. (25 points) To receive full credit, please show your work including a FBD.

1A. GIVEN: A 600-N cylinder being held in static equilibrium by two cables (BC and BA). Determine the magnitudes of the tensions in cable BC and BA to support the 600 N cylinder. **(6 pts)** – (2pts for FBD and 4 points for calculations)



FBD:



$$\sum F_x = 0 = T_{BC}\cos 45^\circ - \left(\frac{4}{5}\right) T_{BA} \longrightarrow ①$$

$$\sum F_y = 0 = T_{BC}\sin 45^\circ + \left(\frac{3}{5}\right) T_{BA} - 600\text{ N} \longrightarrow ②$$

Eqn ① $\Rightarrow$ $\boxed{T_{BA} = 0.8839\, T_{BC}}$

Substituting this into Eqn ②

$$T_{BC}(0.707) + \left(\frac{3}{5}\right)(0.8839)\, T_{BC} = 600\text{ N}$$

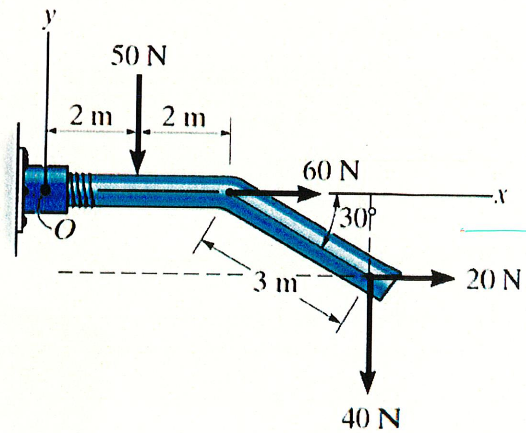
$$\Rightarrow T_{BC}\left[0.707 + 0.530\right] = 600\text{ N}$$

$$\Rightarrow \boxed{T_{BC} = 485.04\text{ N}}$$ Ans

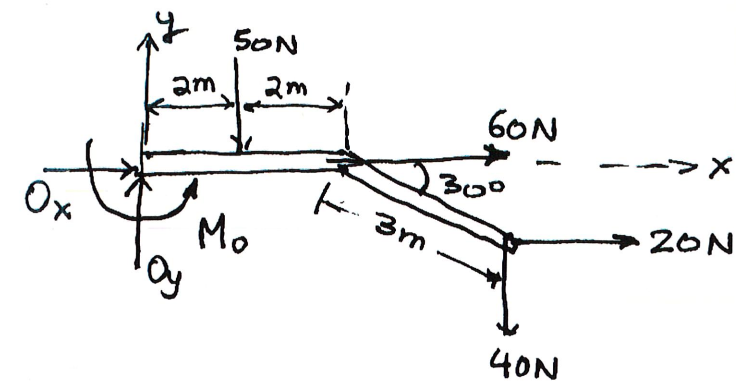
$$\Rightarrow T_{BA} = 0.8839(485.04) = \boxed{428.73\text{ N}}$$ Ans

T_{BC} = 485.04 N		**(2 pts)**
T_{BA} = 428.73 N		**(2 pts)**

1B. Given: A cantilever beam is loaded with four forces as shown below and held in static equilibrium by a fixed support at O. Determine the reactions at fixed support O. **(6 pts)** (2 pts for the FBD and 4 pts for calculations)



FBD:



$$\sum M_o = M_o - 50N(2m) + 60N(0) + 20N(3\sin 30^\circ) - 40N(4m + 3\cos 30^\circ) = 0$$

$$M_o = +50N(2m) - 20N(3\cdot\sin 30^\circ) + 40N(4m + 3\cos 30^\circ)$$

$$\boxed{M_o = +334 \text{ N.m}}$$

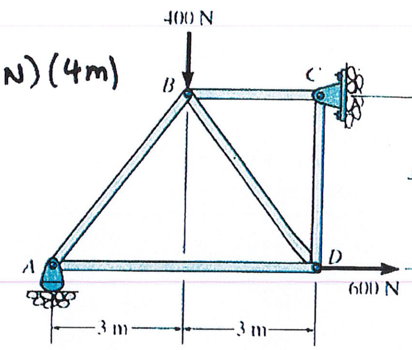
$$\sum F_x = 0 = O_x + 60N + 20N \Rightarrow \boxed{O_x = -80\text{ N}}$$

$$\sum F_y = 0 = O_y - 50N - 40N \Rightarrow \boxed{O_y = 90\text{ N}}$$

$\bar{O}$ = (−80) $\hat{\imath}$ + (90 $\hat{\jmath}$) N	**(2 pts)**
$\bar{M}_O$ = + 334 $\hat{k}$ N·m	**(2 pts)**

1C. Given: Truss ABCD is loaded with a single 400N load as shown and is held in static equilibrium by a rocker support at A and a pin support at C. Determine the reaction force at the rocker support A (1 pt). Then using the method of joints, find the magnitude and sense (tension or compression) of the force in members AB and AD. Circle the correct answer below. **(7 pts)** (2 pts for the FBDs and 5 pts for calculations)

Overall FBD:

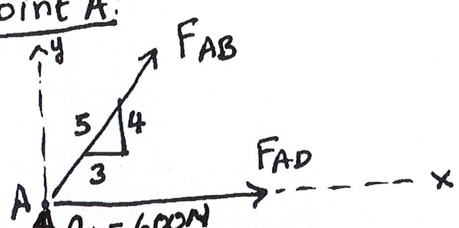


$$\sum M_C = 0 = -A_y(6m) + (400N)(3m) + (600N)(4m)$$

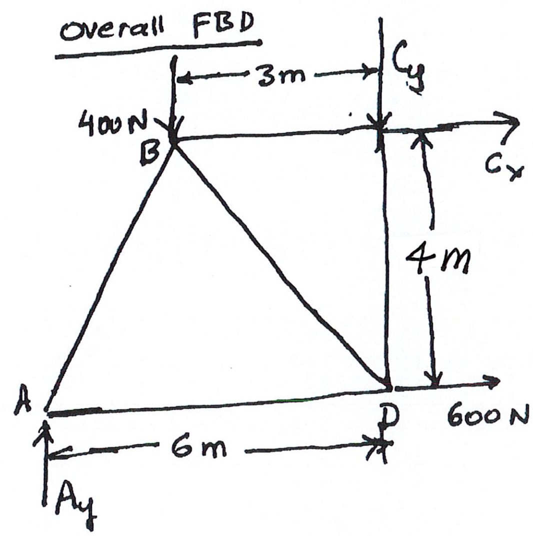
$$\Rightarrow A_y = \frac{1200\,N\cdot m + 2400\,N\cdot m}{6\,m}$$

$$\Rightarrow \boxed{A_y = 600\,N}$$

Joint A:



Overall FBD



$$\sum F_x = F_{AD} + \left(\frac{3}{5}\right)F_{AB} = 0$$

$$\sum F_y = 0 = \underset{600N}{A_y} + \left(\frac{4}{5}\right)F_{AB}$$

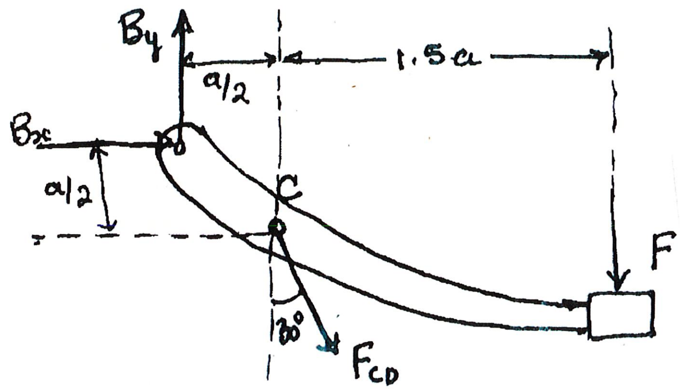
$$\Rightarrow F_{AB} = -600\,N\left(\frac{5}{4}\right)$$

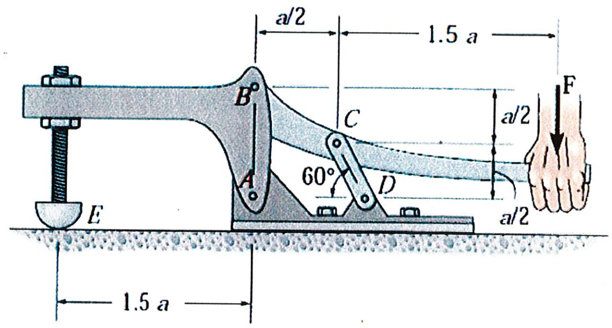
$$\boxed{F_{AB} = -750N = 750\,N\,(C)}$$

$$F_{AD} = -\frac{3}{5}F_{AB} = -\left(\frac{3}{5}\right)(-750) = +450N = \boxed{450N\,(T)}$$

A_y = 600 N		(5 pts)
a) $F_{AB} = 750$ N (Compression)	a) $F_{AD} = 450$ N (Compression)	
b) $F_{AB} = 500$ N (Compression)	b) $F_{AD} = 500$ N (Compression)	
c) $F_{AB} = 750$ N (Tension)	c) $F_{AD} = 450$ N (Tension)	
d) None of the above	d) None of the above	

1D. Given: The toggle clamp shown below is subjected to a force F = 160 N at the handle and is in static equilibrium. Determine the magnitude of the force in CD (which is a 2-force member) and circle if it is in tension or compression. Then determine the force at C on handle (BCF) in vector form. **(6 pts)** (2 pts for FBD and 4 pts for calculations).





CD → Two-force member

$$\sum M_B = 0 = -F(2a) - F_{CD}\cos 30°\left(\frac{a}{2}\right) + F_{CD}\sin 30°\left(\frac{a}{2}\right)$$

$$F_{CD} = -10.93\,F$$

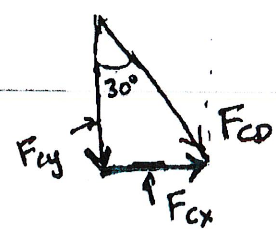
$$= -10.93(160N) = \boxed{-1748.8N}$$

$$= \boxed{1748.8\,N\,(C)}$$

$$\bar{F}_{CD} = (F_{CD}\sin 30°)\hat{i} - (F_{CD}\cos 30°)\hat{j}$$

$$= (-1748.8\times\sin 30°)\hat{i} - (-1748.8\times\cos 30°)\hat{j}$$

$$= (-874.4\,\hat{i} + 1514.5\,\hat{j})N$$



F_{CD} = 1748.8 N Tension Compression (Circle One)	(2 pts)
$(\bar{F}_C)_{on\ BCF}$ = (−874.4) $\hat{i}$ + (1514.5) $\hat{j}$ N	(2 pts)

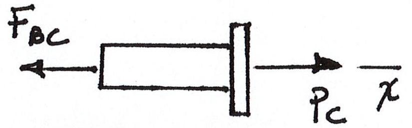
PROBLEM 2 (25 points)

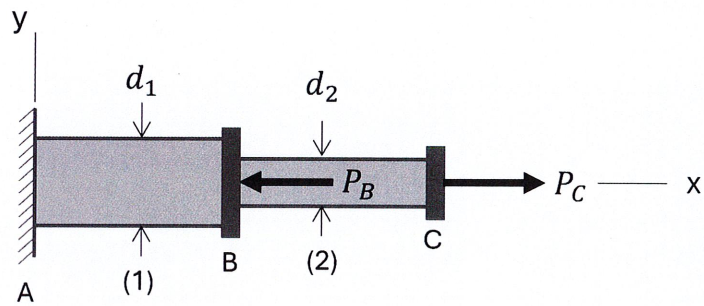
2A. Consider the stepped rod ABC with solid circular cross sections (1) and (2) of diameter $d_1 = 2\ cm$, and $d_2 = 1\ cm$, respectively. Axial loads $P_B = 50\ kN$ and $P_C = 20\ kN$ act on couplers B and C respectively. **(6 pts)**

Find:

a) Determine the magnitudes of the internal axial stresses in sections (1) and (2) of the rod. **(4 pts)**

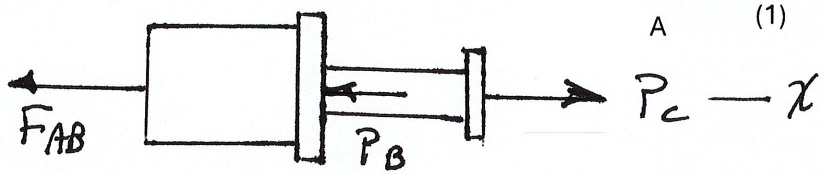
b) Assuming a factor of safety of 2.0 for the rod, what would the yield strength of the rod need to be to achieve this factor of safety? **(2 pts)**





$\Sigma F_x = 0 = -F_{BC} + P_c$

$\therefore F_{BC} = P_c = 20kN$ (tension)



$\Sigma F_x = 0 = -F_{AB} - P_B + P_c \Rightarrow F_{AB} = P_c - P_B = 20 - 50 = -30kN$ (compression)

$$\sigma_{x,1} = \frac{F_{AB}}{A_1} = \frac{-30kN}{\pi(0.01)^2} = \frac{-30{,}000\ N}{0.000314159\ m^2} = -95.5\ MPa$$

$$\sigma_{x,2} = \frac{F_{BC}}{A_2} = \frac{20kN}{\pi(0.005)^2} = \frac{20{,}000\ N}{0.00007854\ m^2} = 254.6\ MPa$$

$$\frac{\sigma_{yield,rod}}{\sigma_{x,2}} = F.S. = 2 \Rightarrow \sigma_{yield,rod} = 2(254.6) = 509.3\ MPa$$

$\sigma_{x,1}$ = -95.5	**MPa**	**(2 pts)**
$\sigma_{x,2}$ = 254.6	**MPa**	**(2 pts)**
$\sigma_{yield,rod}$ = 509.3	**MPa**	**(2 pts)**

2B. An L-shaped rigid bar ABC is supported by a double shear cylindrical pin at A, and a cable at B. An applied force of $P = 200\ lb$ acts at point C. The pin at support A has a diameter of 1 inch. Determine the magnitude of the average shear stress τ in the double shear pin at A. **(6 pts)**

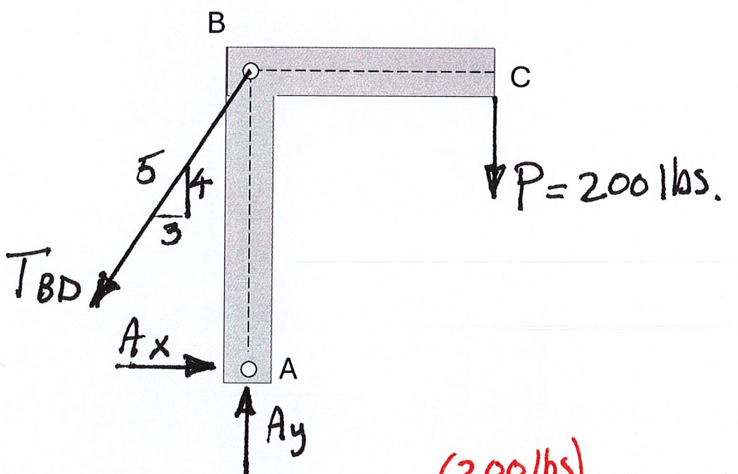
Find:

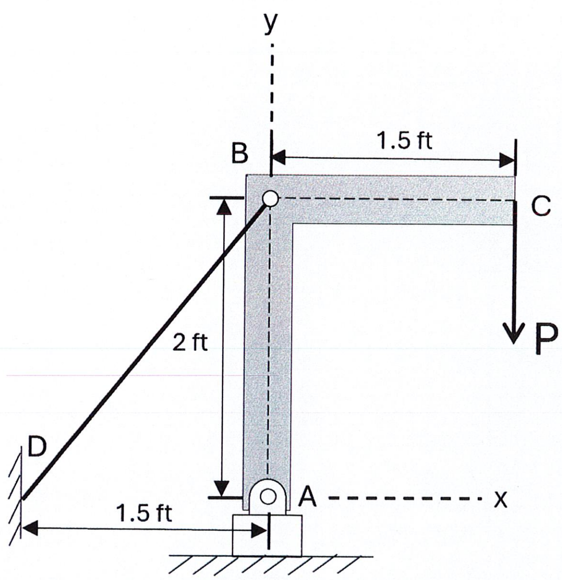
a) FBD of bar ABC. **(1 pt)**

b) Determine the support reaction of the Pin at A. Express your answer in Vector Form. **(2 pts)**

c) Determine the magnitude of the average shear stress τ in the double shear pin at A. **(3 pts)**

FBD (1 pt):





$$\sum M_A = 0 = -\overset{(200\,lbs)}{P}(1.5) + T_{BD}\left(\tfrac{3}{5}\right)(2) \Rightarrow T_{BD} = 250\,lbs.$$

$$\sum F_x = 0 = A_x - \overset{(250\,lbs)}{T_{BD}}\left(\tfrac{3}{5}\right) \Rightarrow A_x = 150\,lbs.$$

$$\sum F_y = 0 = A_y - \overset{(250\,lbs)}{T_{BD}}\left(\tfrac{4}{5}\right) - \overset{(200\,lbs)}{P} \Rightarrow A_y = 400\,lbs.$$

$$V_A = \left[A_x^2 + A_y^2\right]^{1/2} = \left[150^2 + 400^2\right]^{1/2} \Rightarrow V_A = 427.2\,lbs.$$

$$\tau = \frac{V_A}{2A} = \frac{427.4\ lbs}{2\left[\pi(0.5)^2\right] in^2} = 272.1\ psi$$

$\vec{A}$	=	150 $\hat{\imath}$ + 400 $\hat{\jmath}$ lb **(2 pts)**
τ	=	272.1 psi **(3 pts)**

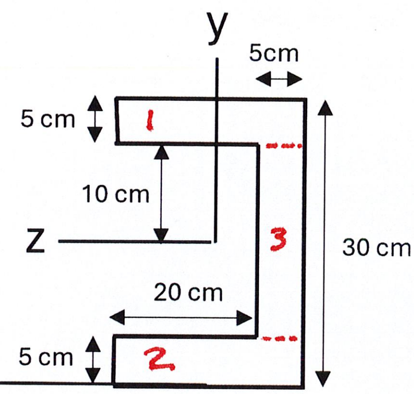
2C. Consider the C-beam with a cross section as shown in the figure below. **(6 pts)**

Find:

a) Determine the second area moment of the U-beam about the z-axis, I_z **(4 pts).**

b) Qualitatively, with no work needed, would $I_{Z'}$ of the U-beam be greater, smaller, or the same compared to I_z? **(2 pts)**

	Area	d
Area 1	125 cm²	12.5 cm
Area 2	125 cm²	−12.5 cm
Area 3	100 cm²	0 cm



$$I_z = I_1 + A_1 d_1^2 + I_2 + A_2 d_2^2 + I_3 + A_3 d_3^2$$

$$= \frac{1}{12}(25\text{cm})(5\text{cm})^3 + (125\text{cm}^2)(12.5\text{cm})^2$$

$$+ \frac{1}{12}(25\text{cm})(5\text{cm})^3 + (125\text{cm}^2)(12.5\text{cm})^2$$

$$+ \frac{1}{12}(5\text{cm})(20\text{cm})^3 + (100\text{cm}^2)(0\text{cm})^2 \rightarrow 0$$

$$I_z = 260.42\text{cm}^4 + 19{,}531\text{cm}^4$$

$$+ 260.42\text{cm}^4 + 19{,}531\text{cm}^4$$

$$+ 3333.33\text{cm}^4 + 0$$

$$\therefore I_z = 19{,}791.4 + 19{,}791.4 + 3333. = 42{,}916.$$

I_z = 42,916 cm⁴				**(4 pts)**
$I_{Z'}$ **will be:**	**Larger**	**Smaller**	**Equal to** I_z **(Circle the correct answer.)**	**(2 pts)**

2D. Given a shaded region enclosed by a parabola $y = x^3$ and a line $y = 8\ in.$ as shown in the figure below. Assume I_x is the second area moment about the x-axis shown, I_y is the second area moment of inertia about the y-axis shown, and I_{xc} to be the second moment of area about the centroid of the shaded area.. **(7 pts)**

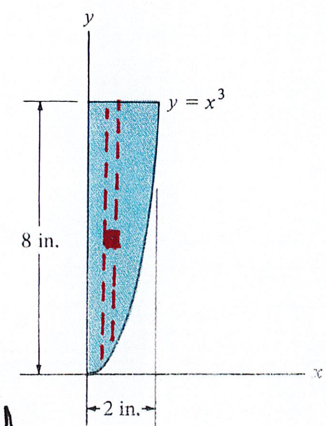
Find:

a) Determine the second area moment of inertia about x-axis I_x. **(3 pts)**

b) Based on the shaded area, would you expect I_x to be greater than, equal to or smaller than I_y? **(2 pts)**

c) Based on the shaded area, would you expect the second moment of area about the centroid of the shaded area (I_{xc}) to be qualitatively greater than, equal to, or less than I_x about the x-axis shown? **(2 pts)**

$$I_x = \int_A y^2\, dy\, dx$$



$$I_x = \int_{x_1=0}^{x_2=2} \int_{y_1=x^3}^{y_2=8} y^2\, dy\, dx = \int_0^2 \left.\frac{y^3}{3}\right]_{x^3}^{8} dx$$

$$I_x = \int_0^2 \left[\frac{8^3}{3} - \frac{x^9}{3}\right] dx = \int_0^2 \left[170.67 - \frac{x^9}{3}\right] dx$$

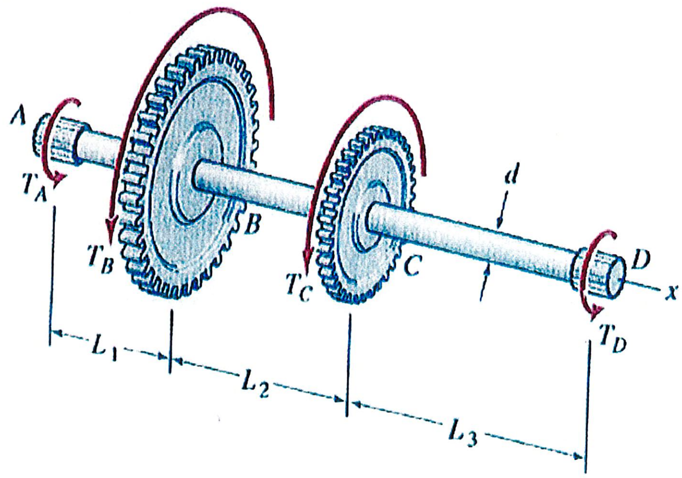
$$I_x = 170.67x - \left.\frac{x^{10}}{30}\right]_0^2 = 170.67(2) - \frac{(2)^{10}}{30} = 341.33 - 34.13$$

$$I_x = 307.2\ in^4$$

$I_x =$ 307.2			in^4	**(3 pts)**
$I_x > I_y$	$I_x = I_y$	$I_x < I_y$	**(Circle One)**	**(2 pts)**
$I_{xc} > I_x$	$I_{xc} = I_x$	$I_{xc} < I_x$	**(Circle One)**	**(2 pts)**

PROBLEM 3. (25 points)

Given: The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. The gears are subjected to the torques: T_A = +300 N.m; T_C = -500 N.m; T_C = -200 N.m; and T_D = +400 N.m.



Find:

a) Determine the polar moment of inertia for this solid shaft?

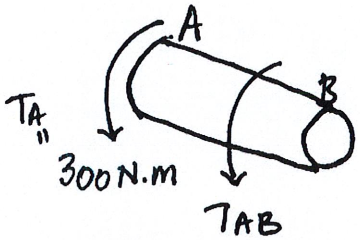
$$J = \frac{\pi}{2} r_o^4$$

$$= \frac{\pi}{2}(0.015)^4$$

$$= \boxed{7.95 \times 10^{-8} \text{ m}^4}$$

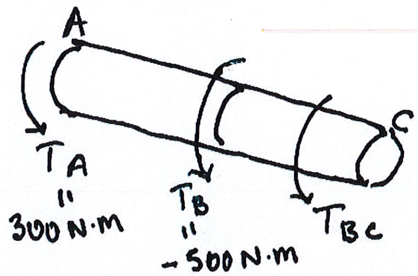
J = 7.95 x 10^{-8} m^4	**(4 points)**

b) Determine the absolute maximum torque on the shaft and on which segment this absolute maximum torque occurs. In that segment, determine the absolute maximum shear stress.



$$\sum M_x = 0 =$$

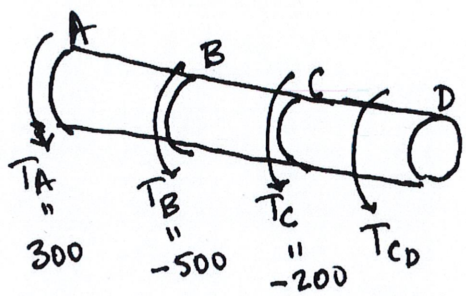
$$\Rightarrow T_{AB} = -T_A = \boxed{-300\ N\cdot m}$$



$$\sum M_x = 0 = T_A + T_B + T_{BC}$$

$$T_{BC} = -T_A - T_B = -300\ N\cdot m - (-500\ N\cdot m)$$

$$\boxed{T_{BC} = +200\ N\cdot m}.$$



$$\sum M_x = 0 = T_A + T_B + T_C + T_{CD}$$

$$T_{CD} = -T_A - T_B - T_C$$

$$= -300 - (-500) - (-200) = -300 + 500 + 200$$

$$\boxed{T_{CD} = 400\ N\cdot m}\ \checkmark$$

$$|T_{max}| = |T_{CD}| = 400\ N\cdot m$$

$$|\tau_{max}| = \frac{|T_{CD}|\, r}{J} = \frac{(400\ N\cdot m)(0.015 m)}{7.95\times 10^{-8}\ m^4}\left|\frac{1\ MPa}{10^6\ N/m^2}\right|$$

$$= \boxed{75.47\ MPa}$$

$\lvert T_{max}\rvert$ = 400 N.m N.m;	AB	BC	CD		(circle one)	(6 pts)
$\lvert \tau_{max}\rvert$ = 75.47 MPa						(6 pts)

c) If this solid shaft is to be replaced by a new solid circular shaft made of the same material to withstand the maximum allowable stress of $\tau_{allowable}$ = 20 MPa, determine the minimum diameter (d) of this new replaced solid shaft to the nearest 10 mm.

$$\frac{J}{r} = \frac{T_{max}}{\tau_{allowable}}$$

$$\frac{(\pi/2)\, r^{\cancel{4}\,3}}{\cancel{r}} = \frac{T_{max}}{\tau_{allow}}$$

$$r = \left(\frac{2\,T_{max}}{\pi\,\tau_{allow}}\right)^{1/3}$$

$$= \left(\frac{2 \times 400\ N.m}{\pi \times 20 \times 10^{6}\ N/m^{2}}\right)^{1/3}$$

$$r = 0.02335\ m = 23.35\ mm$$

$$d = 2 \times 23.35\ mm = 46.70\ mm$$

$$\simeq 50\ mm$$

$d =$ 50 mm	mm (round to the nearest 10mm)	**(4 pts)**

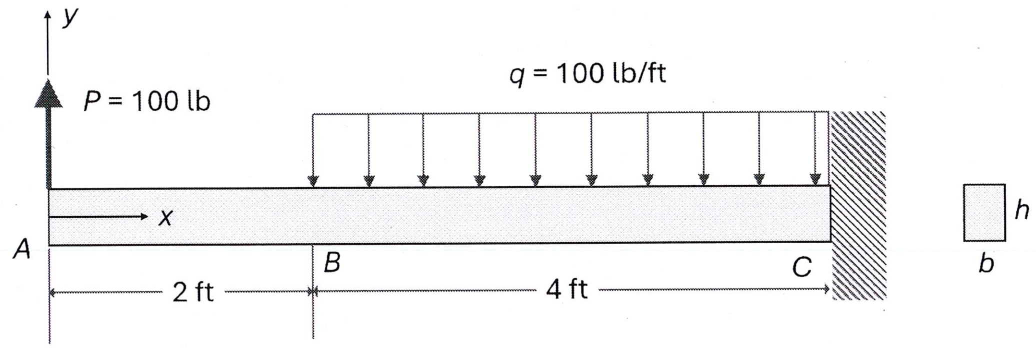
d) If a solid shaft is replaced by a hollow circular shaft, will the maximum shear stress qualitatively increase, remains the same, or decrease given the applied torque and the outer diameter are the same for both shafts? No work needs to be shown

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) \downarrow \qquad \tau_{max} = \frac{T\, r_o}{J} \uparrow$$

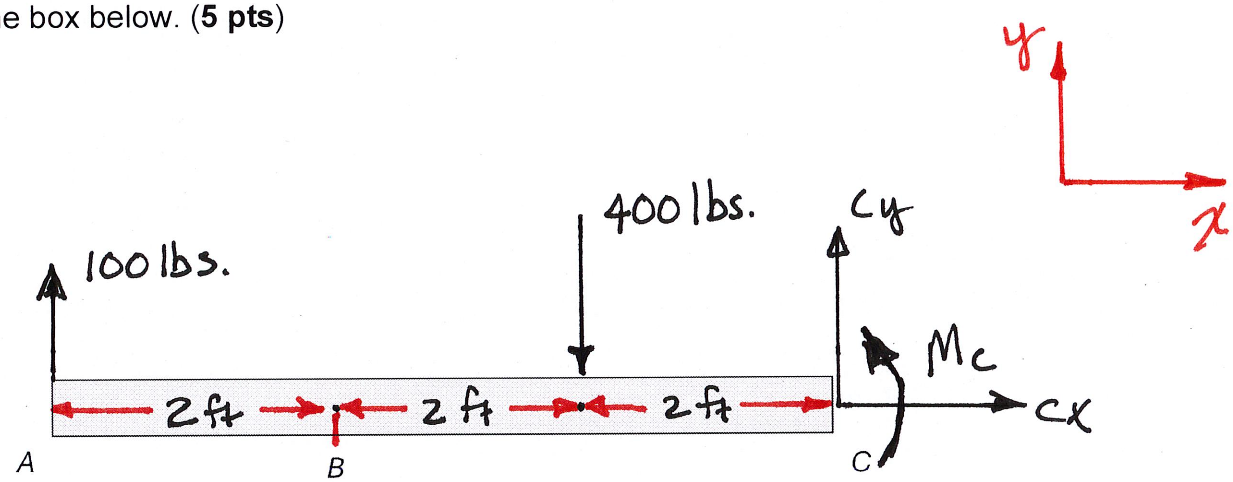
τ_{max} = **Increase**	**Remains the Same**	**Decrease**		**(Circle One)**	**(3 pts)**

PROBLEM 4. (25 points)

GIVEN: A cantilever beam supports a uniformly distributed load and a concentrated load as shown below. The beam has a rectangular cross section with b = 3.0 in and h = 4.0 in.



FIND: a) Draw the overall free body diagram on the artwork provided below (**2 pts**). Use the single equivalent force for the uniform distributed load. Determine the reactions at the fixed end C and write them in the box below. (**5 pts**)



$$\underline{\Sigma M_c = 0} = M_c - 100(6) + 400(2)$$

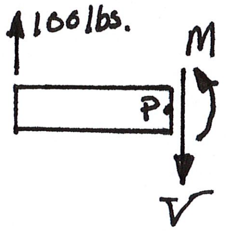
$$\therefore \quad M_c = -200 \text{ ft-lbs.}$$

$$\underline{\Sigma F_x = 0} = C_x \quad \Rightarrow \quad C_x = 0 \text{ lbs.}$$

$$\underline{\Sigma F_y = 0} = C_y + 100 - 400 \quad \Rightarrow \quad C_y = +300 \text{ lbs.}$$

$\vec{C}$ = (0)$\hat{\imath}$ + (300)$\hat{\jmath}$ lbs (**3 pts**) $\quad$ $\vec{M}_C$ = (−200)$\hat{k}$ ft-lbs (**2 pts**)

b) Find the expressions describing the shear force $V(x)$ and the bending moment $M(x)$ as a function of the spatial coordinate x. Write your results for the segements AB and BC in the box below. (**8 pts**) Seg. AB

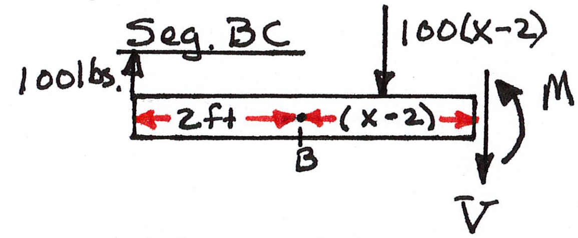


$\Sigma M_P = 0 = M - 100x$

$\therefore M(x) = 100(x)$

$\Sigma F_y = 0 = -V + 100$

$\therefore V(x) = 100 \text{ lbs.}$



$\Sigma M_P = 0 = M - 100(x) + 100(x-2)\frac{(x-2)}{2}$

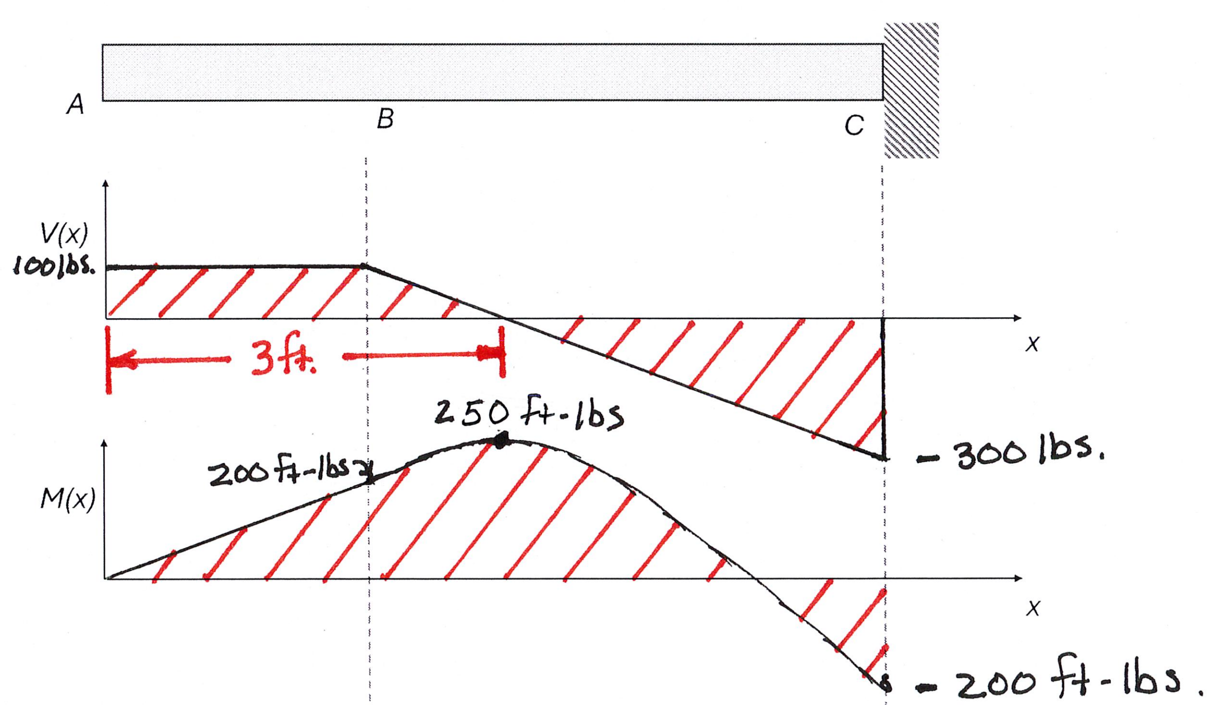
$\therefore M(x) = -50x^2 + 300(x) - 200$

$\Sigma F_y = 0 = -V + 100 - 100(x-2)$

$\therefore V(x) = -100x + 300$

Segment *AB*	$V(x) =$ 100 lbs	$M(x) =$ 100 (x) ft-lbs	(**4 pts**)
Segment *BC*	$V(x) = -100x + 300$ lbs	$M(x) = -50x^2 + 300(x) - 200$ ft-lbs	(**4 pts**)

a) Draw $V(x)$ and $M(x)$ on the diagram provided below. Please label the shear force and bending moment values on the diagram at points A, B and C, as well as any max or min values. (**4 pts**)



d) Indicate the location on the beam which has a point in pure bending by stating its location from the left end of the beam. For the point experiencing pure bending, find the value of the maximum tensile normal stress and the location on the cross section where it occurs. (**6 pts**)

Pure Bending occurs where $V=0$ and $M \neq 0$

From $V(x) = -100(x) + 300 = 0$

$x = 3$ ft from left end of beam.

$M_{MAX} = 250$ ft-lbs. (tensile will occur on Bottom)

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(3\text{in})(4\text{in})^3 = 16\text{ in}^4$$

$$\sigma_{bottom} = -\frac{My}{I} = \frac{-(250\ \cancel{ft}\text{-lbs})(-2\text{ in})\left(\frac{12\text{ in}}{1\ \cancel{ft}}\right)}{16\text{ in}^4}$$

$$\therefore \sigma_{bottom} = 375\ \frac{\text{lbs}}{\text{in}^2} = 375\text{ psi}$$

Pure bending	Located 3 ft from the left end of the beam	(**2 pts**)
Max tensile stress	σ = 375 psi	(**2 pts**)
Location of max tensile stress	Top Bottom Center (Circle one)	(**2 pts**)