ME 270 – Fall 2025 Exam 1 NAME (Last, First): SOLUTION (Please Print)

PUID #: ____________________ (Please Print)

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ____________________________________

Instructor's Name and Section: (Circle Your Section)

Sections: **West Lafayette Campus (PWL)**

J. Osorio Pinzon, Section 033, MWF 7:30AM-8:20AM
J. Jones, Section 005, MWF 9:30AM-10:20AM
S. C. Boregowda, Section 008, MWF 10:30AM-11:20AM
J. Jones, Section 003, MWF 11:30AM-12:20PM
L. Krest, Section 009, MWF 12:30PM-1:20PM
F. Semperlotti, Section 001, MWF 1:30PM-2:20PM
A. Ramkumar, Section 010, MWF 2:30PM-3:20PM
T. Ballance, Section 032, MWF 4:30PM-5:20PM
M. Murphy, Section 007, TR 9:00AM-10:15AM
M. Murphy, Section 002, TR 10:30AM-11:45AM
J. Jones, Section Y01, Distance Learning

Indianapolis Campus (PIN)

N. Saqib, Section 031, MWF 9:30AM-10:20AM
A. McDonald, Section 029, MWF 1:30PM-2:20PM
D. Wagner, Section 030, TTh 12:00PM-1:15PM

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

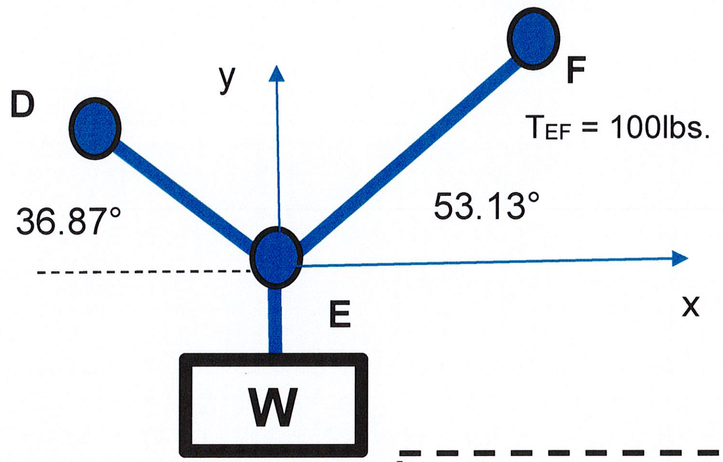
- The allowable exam time for Exam 1 is 90 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.

If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

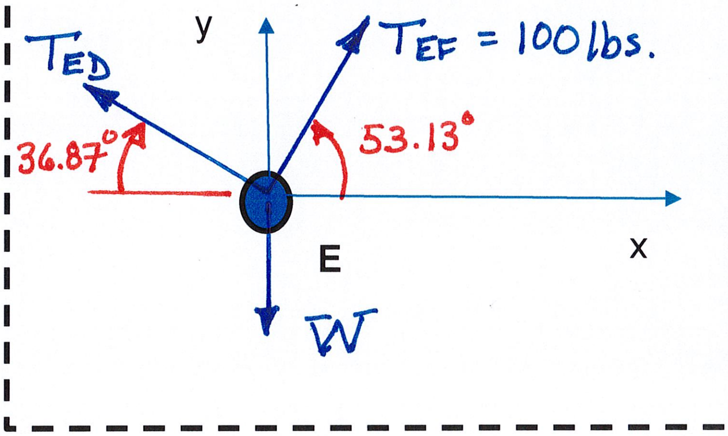
PROBLEM 1 (20 points)

1A. Block A with an unknown weight **W** is held in static equilibrium by two cables (T_{EF} and T_{ED}). The tension in cable EF is T_{EF} = 100lbs. Using the figure below, determine magnitudes of the weight **W** and of the tension in cable **ED** required to maintain static equilibrium. Make sure you include your Free Body Diagram (on area provided) and write clear equations of static equilibrium. **(5 pts)**



$$\underline{\underline{\Sigma F_x = 0}} = -T_{ED}\cos 36.87 + T_{EF}\cos 53.13^\circ$$
(100lbs)

$$\underline{\underline{\Sigma F_y = 0}} = T_{ED}\sin 36.87 + T_{EF}\sin 53.13^\circ - W$$
(100lbs)



$$\therefore\ T_{ED} = (100\,lbs)\cos 53.13^\circ / \cos 36.87^\circ$$

$$T_{ED} = (100\,lbs)(0.6)/(0.8) = \boxed{75\ lbs.}$$

(75lbs). (100 lbs)

$$\therefore\ W = T_{ED}\sin 36.87 + T_{EF}\sin 53.13 = 75(0.6) + (100)(0.8)$$

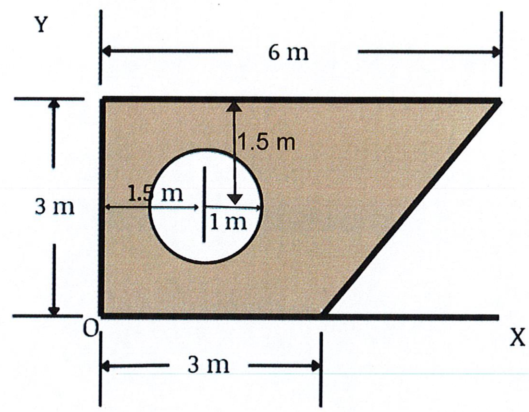
$$\boxed{W = 125\,lbs.}$$

$W =$ 125 lbs.	**(2 pts)**
$\lvert\vec{T}_{ED}\rvert =$ 75 lbs.	**(2 pts)**

1B. Using the method of composite parts, find the area (A) and the x-centroid (x_C) of the shaded area in the figure below with respect to the coordinate axes provided. The circular cutout has a 1-m radius and is centered in the middle of the 3m-by-3m square part of the trapezoid. Please show your work to receive credit. If the circular hole was filled in, qualitatively what impact would this have on x_C and y_C. (No calculations are required for determining this qualitative impact). **(5 pts)**

Shape	Area (m^2)	x_c (m)
Square	$9\ m^2$	1.5 m
Triangle	$4.5\ m^2$	4 m
Circle	$-\pi\ m^2$	1.5 m

Area Tot = $10.36\ m^2$



$$A_{TOT}(x_c) = A_1\, x_{c_1} + A_2\, x_{c_2} + A_3\, x_{c_3}$$

$$(10.36\,m^2)(x_c) = (9\,m^2)(1.5\,m) + (4.5\,m)(4\,m) + (-3.1416\,m^2)(1.5\,m)$$

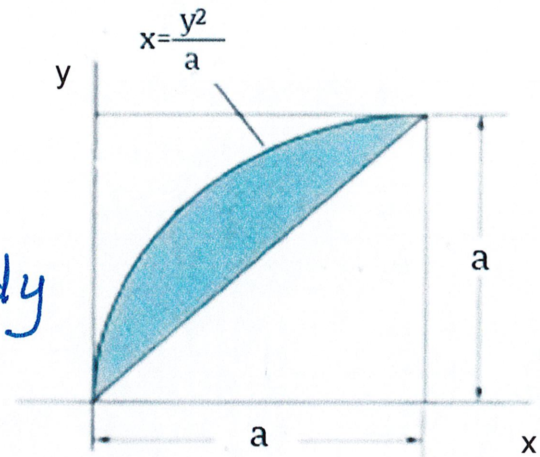
$$x_c = 2.59\ m$$

A = 10.36 m^2 (1 pt) x_C = 2.59 m (2 pts)

x_C = Increase Stay the Same Decrease (Circle One) (1 pt)

y_C = Increase Stay the Same Decrease (Circle One) (1 pt)

1C. Using the method of integration, determine the area (A) and the x-centroid (x_C) of the shaded area with respect to the coordinate axes provided as a function of the constant "a". Please show your work to receive credit. Qualitatively, would you expect for y_C to be larger, smaller or equal to x_C? (No calculations are required for determining this qualitative impact). **(5 pts)**



$$A = \int_{y_1=0}^{y_2=a} \int_{x_1=\frac{y^2}{a}}^{x_2=y} dx\,dy$$

$$A = \int_0^a [x]_{\frac{y^2}{a}}^{y}\, dy = \int_0^a \left[y - \frac{y^2}{a}\right] dy$$

$$A = \left[\frac{y^2}{2} - \frac{y^3}{3a}\right]_0^a = \frac{a^2}{2} - \frac{a^3}{3a}$$

$$\therefore A = \frac{a^2}{2} - \frac{a^2}{3} = \frac{3a^2 - 2a^2}{6} = \boxed{\frac{a^2}{6} \text{ units}^2}$$

$$A x_c = \int_{y_1=0}^{y_2=a} \int_{x_1=\frac{y^2}{a}}^{x_2=y} x\,dx\,dy = \int_0^a \left[\frac{x^2}{2}\right]_{\frac{y^2}{a}}^{y} = \int_0^a \left[\frac{y^2}{2} - \frac{y^4}{2a}\right] dy$$

$$\left(\frac{a^2}{6}\right) x_c = \left[\frac{y^3}{6} - \frac{y^5}{10a^2}\right]_0^a = \frac{a^3}{6} - \frac{a^5}{10a^2} = \frac{5a^3 - 3a^3}{30} = \frac{2a^3}{30}$$

$$\therefore x_c = \left(\frac{2a^3}{30}\right)\left(\frac{6}{a^2}\right) = \boxed{\frac{2a}{5} \text{ units.}}$$

A = $a^2/6$ **units^2 (2 pts)**			**x_C =** $2a/5$ **units (2 pts)**	
y_C Trend:	$y_C > x_C$	$y_C < x_C$	$y_C = x_C$	**(Circle One) (1 pt)**

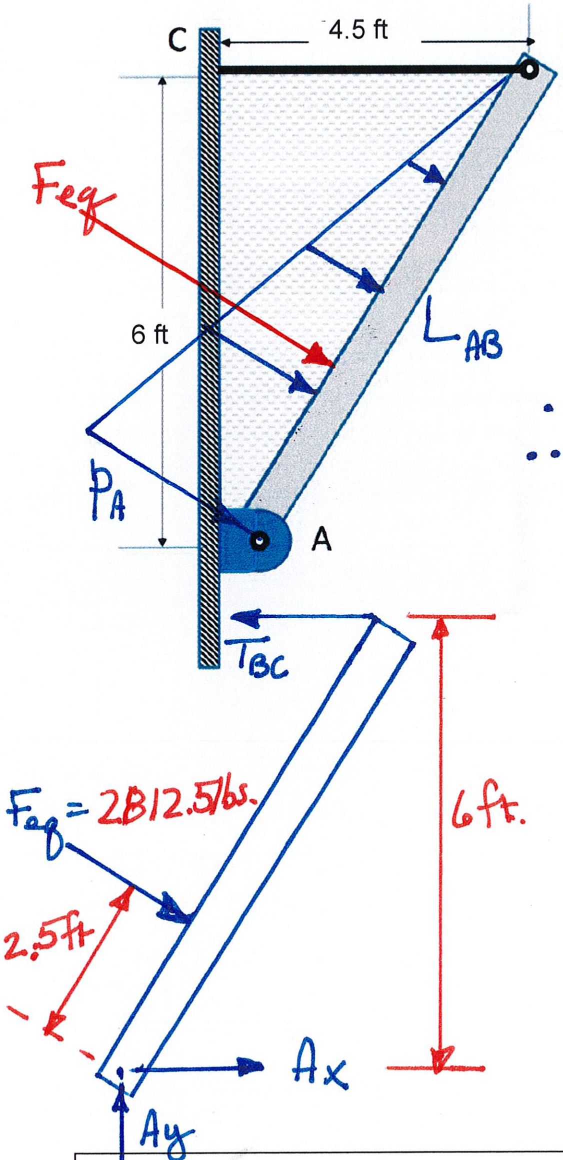
1D. A cable is attached at point B that is 6 feet above a pin joint that holds a 1-foot wide gate shut to prevent spillage of the water to a height of 6 ft as shown. The specific weight of the water is ρg = 62.5 lb/ft^3. What is the tension in cable **CB** when the area is filled with water? **(5 pts)**

Please Determine:

i. The equivalent hydrostatic force (F_{eq}) applied to the 1-foot wide gate and its location along gate **AB.**

ii. The force carried in cable **AB.**

$$P_A = \rho g h_A = \left(62.5 \frac{lbs}{ft^3}\right)(6 ft) = 375 \frac{lbs}{ft^2}$$



$$L_{AB} = \left[6^2 + 4.5^2\right]^{1/2} = 7.5 ft$$

$$F_{eq} = \frac{1}{2}(L_{AB})(P_A)(w)$$

$$F_{eq} = \frac{1}{2}(7.5 ft)\left(375 \frac{lbs}{ft^2}\right)(1 ft)$$

$$\therefore F_{eq} = 1406\ lbs$$

$$\underline{\underline{\Sigma M_A = 0}} = T_{BC}(6 ft) - F_{eq}(2.5 ft)$$

$$\therefore T_{BC} = \frac{F_{eq}(2.5 ft)}{6 ft}$$

$$= \frac{(1406)(2.5 ft)}{6 ft} = 586\ lbs.$$

F_{eq} = 1406	lbs		**(2 pts)**
Distance of F_{eq} from A along **AB** = 2.5	ft		**(1 pt)**
Force carried in the cable **BC** = 586	lbs		**(2 pts)**

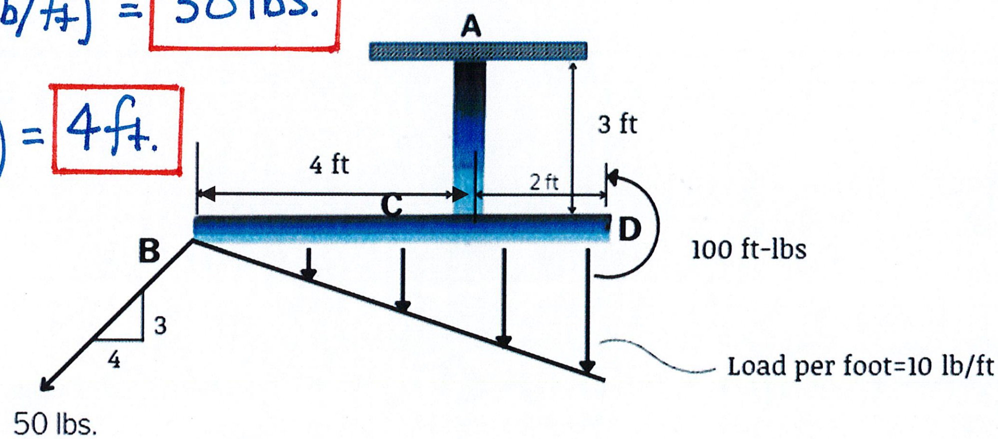
PROBLEM 2. (20 points)

Given: The inverted T-shaped bar shown is loaded with a 50lb point force at B, a distributed load from B to D (with a max load per foot of 10lbs/ft at D), and an 100ft-lb couple at D as shown and is held in static equilibrium by a fixed support at A.

Find: a) Determine the equivalent force (F_{eq}) for the distributed load and its distance from B ($(\bar{x}_{eq})_{\text{from B}}$). **(3 pts)**

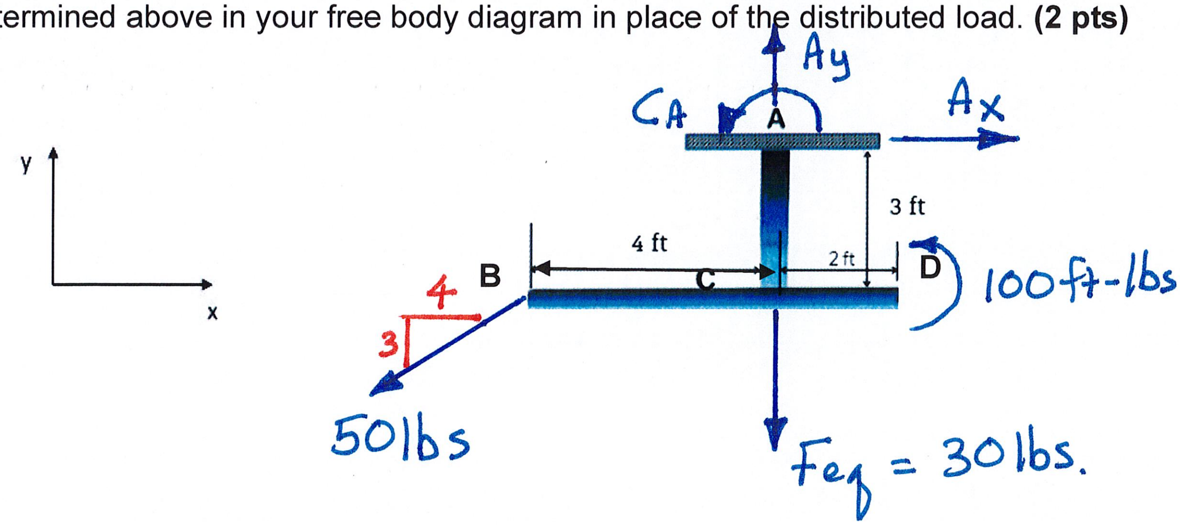
$$F_{eq} = \frac{1}{2}(6\,ft)(10\,lb/ft) = \boxed{30\,lbs.}$$

$$(x_{eq})_{\text{from B}} = (6\,ft)\left(\frac{2}{3}\right) = \boxed{4\,ft.}$$



F_{eq}= 30 lbs (2 pts)	$(\bar{x}_{eq})_{\text{from B}}$ = 4 ft (2 pts)

b) On the artwork provided, complete the free body diagram for the inverted T-shaped bar. Use the F_{eq} determined above in your free body diagram in place of the distributed load. **(2 pts)**



c) Clearly write the equilibrium equations and solve for the reactions at the fixed support at A. Express your solution in vector form. **(12 pts)**

$$\underline{\underline{\Sigma M_A = 0}} = C_A + \left(\tfrac{3}{5}\right) 50\text{ lbs}\,(4\text{ ft}) - \left(\tfrac{4}{5}\right)(50\text{ lbs})(3\text{ ft}) + 100$$

$$\boxed{\therefore\ C_A = -100\text{ ft-lbs}}$$

(Note: F_{eq} passes through pt. A so causes No moment).

$$\underline{\underline{\Sigma F_x = 0}} = A_x - \tfrac{4}{5}(50\text{ lbs})$$

$$\boxed{\therefore\ A_x = +40\text{ lbs}}$$

$$\underline{\underline{\Sigma F_y = 0}} = A_y - \tfrac{3}{5}(50\text{ lbs}) - \underset{(30\text{ lbs})}{F_{eq}}$$

$$\boxed{\therefore\ A_y = +60\text{ lbs}}$$

$\bar{C}_A$ = − 100 $\hat{k}$ ft – lbs (6 pts) $\bar{F}_A$ = + 40 $\hat{\imath}$ + + 60 $\hat{\jmath}$ lbs (6 pts)

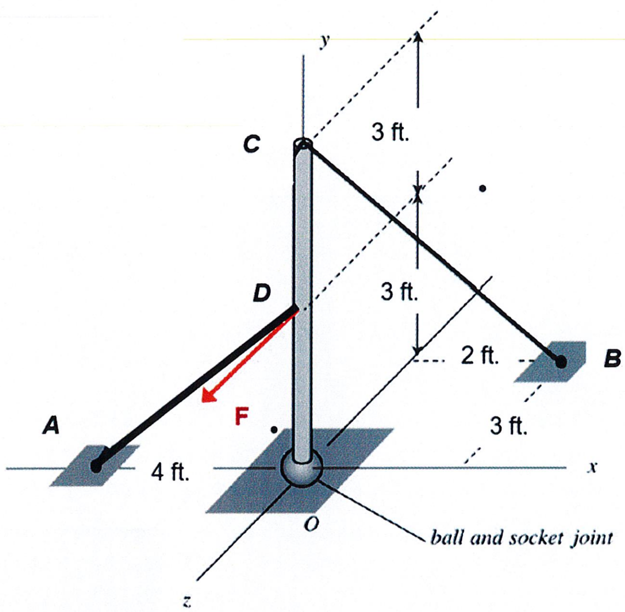
d) If the distributed load were removed from the inverted T-bar, which of the reactions at A (if any) would change in magnitude (either increase or decrease). Circle the reaction or reactions that would change from the eight options provided below. (no work need be shown)? **(2 pts)**

Circle One: None	C_A	A_x	A_y	C_A & A_x	C_A & A_y	A_x & A_y	C_A, A_x & A_y
(2 pts)							

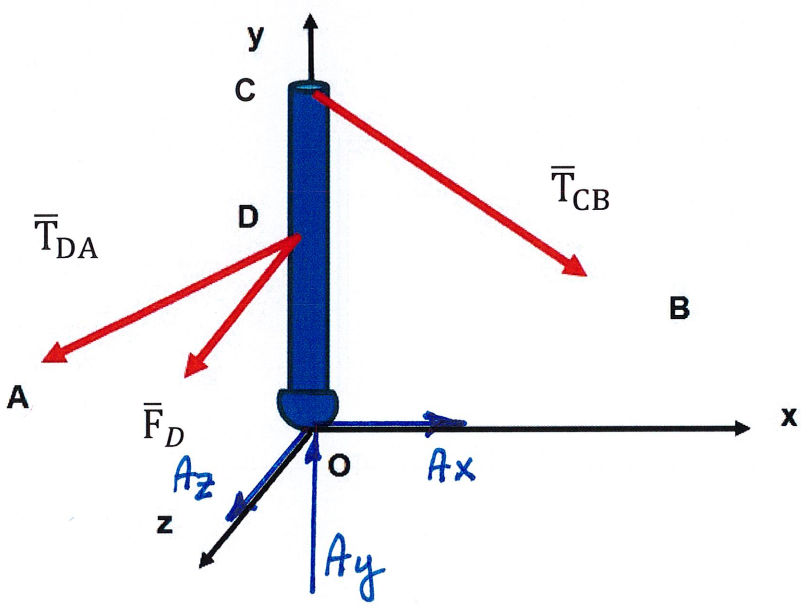
PROBLEM 3. (20 points)

GIVEN: A pole of negligible mass with an unknown-force **F** in the z-direction at point **D** is held in static equilibrium by two cables (**DA** and **CB**) attached to the floor at **A** & **B** respectively. There is a ball-and-socket at point **O.** The tension in **CB** is 70-lb.

A at (-4', 0', 0')
B at (2', 0', -3')
C at (0', 6', 0')
D at (0', 3', 0')



a) Complete the free body diagram of the pole using the artwork below by adding FBD components at **point O. ([illegible] pts)**
2



b) Write expressions for tension vectors T_{DA} and T_{CB} acting on the pole using their magnitudes (magnitude of T_{DA} is unknown) and known unit vectors. The applied load is shown as an example. Please express your result in either fraction or decimal form. **(4 pts)**

$$\bar{T}_{DA} = T_{DA}\left[\frac{-4\bar{i} - 3\bar{j}}{\sqrt{4^2+3^2}}\right] = T_{DA}\left[-\frac{4}{5}\bar{i} - \frac{3}{5}\bar{j}\right] = \boxed{T_{DA}[-0.8\bar{i} - 0.6\bar{j}]}$$

$$\bar{T}_{CB} = 70\left[\frac{2\bar{i} - 6\bar{j} - 3\bar{k}}{\sqrt{2^2+6^2+3^2}}\right] = 70\left[\frac{2}{7}\bar{i} - \frac{6}{7}\bar{j} - \frac{3}{7}\bar{k}\right] = \boxed{20\bar{i} - 60\bar{j} - 30\bar{k}\ \text{lbs.}}$$

$\bar{T}_{DA} = \lvert\vec{T}DA\rvert * [(\ -0.8\ )\hat{\imath} + (\ -0.6\ )\hat{\jmath} + (\ 0\ )\hat{k}]$ lb.	(2 pts)
$\bar{T}_{CB} = 70 * [(\ 0.286\ )\hat{\imath} + (\ -0.857\ )\hat{\jmath} + (\ -0.429\ )\hat{k}]$ lb.	(2 pts)
Applied Load= $\lvert\vec{F}\rvert * [(\ 0\ )\hat{\imath} + (\ 0\ )\hat{\jmath} + (\ 1\ )\hat{k}]$ lb.	(example)

c) Determine the magnitudes of the tension in cables T_{DA} and the load at **F**. **(8 pts)**

$$\Sigma \bar{M}_O = \bar{0} = [\bar{r}_{OD} \times \bar{T}_{DA}] + [\bar{r}_{OC} \times \bar{T}_{CB}] + [\bar{r}_{OD} \times \bar{F}]$$

$$\bar{0} = [(3\bar{j}) \times T_{DA}(-0.8\bar{i} - 0.6\bar{j})] + [(6\bar{j}) \times (20\bar{i} - 60\bar{j} - 30\bar{k})] + [(3\bar{j}) \times F\bar{k}]$$

$$\bar{0} = (2.4\bar{k})T_{DA} + (-120\bar{k} - 180\bar{i}) + 3F\bar{i}$$

$$\therefore \Sigma M_x = 0 = 3F - 180 \Rightarrow \boxed{F = 60\ \text{lbs.}}$$

$$\Sigma M_z = 0 = 2.4T_{DA} - 120 \Rightarrow \boxed{T_{DA} = 50\ \text{lbs.}}$$

$\lvert\vec{T}_{DA}\rvert =$ 50 $lb.$	(4 pts)
$\lvert\vec{F}\rvert =$ 60 $lb.$	(4 pts)

d) At point **O,** determine the reactions at **O** and express as a vector. **(6 pts)**

$$\Sigma \bar{F} = \bar{O} = (O_x\bar{i} + O_y\bar{j} + O_z\bar{k}) + \bar{T}_{DA} + \bar{T}_{CB} + \bar{F}$$

$$\bar{O} = (O_x\bar{i} + O_y\bar{j} + O_z\bar{k}) + \overset{(50\,lbs)}{T_{DA}}(-0.8\bar{i} - 0.6\bar{j}) \quad (60\,lbs)$$

$$+ (20\bar{i} - 60\bar{j} - 30\bar{k}) + F\bar{k}$$

$$\Sigma F_x = 0 = O_x - 40 + 20 \Rightarrow \boxed{O_x = +20\text{ lbs.}}$$

$$\Sigma F_y = 0 = O_y - 30 - 60 \Rightarrow \boxed{O_y = +90\text{ lbs.}}$$

$$\Sigma F_z = 0 = O_z - 30 + 60 \Rightarrow \boxed{O_z = -30\text{ lbs.}}$$

$\vec{O}$ = [(+20) $\hat{\imath}$ + (+ 90) $\hat{\jmath}$ + (− 30) $\hat{k}$] lb.	**(6 pts)**