ME 270 – Spring 2024 Final Exam **NAME (Last, First):** SOLUTION

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ______________________________

Instructor's Name and Section: (Circle Your Section)

Sections: J. Jones, Section 001, MWF 9:30AM-10:20AM
T. Han, Section 002, MWF 1:30PM-2:20PM
J. Jones, Section 003, MWF 11:30AM-12:20AM
J. Jones, Section 005, Distance Learning

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: ______________________________

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 25 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

- The allowable exam time for Final Exam is 120 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.
- **Do not unstaple your exam**.

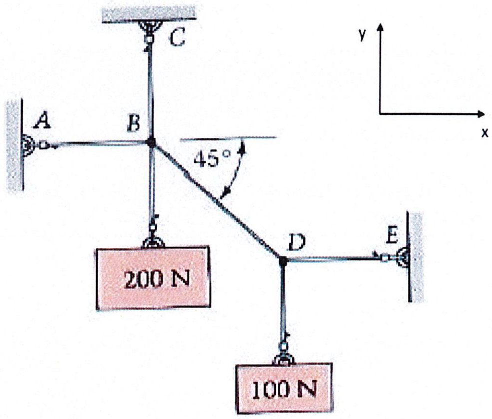
If the solution does not follow a logical thought process, it will be assumed in error.

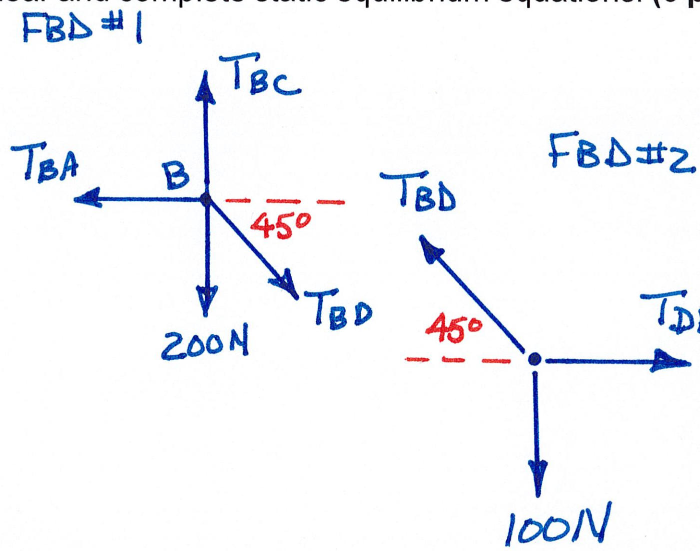
When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of every page that you wish to have graded. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

ME 270 – Spring 2024 Final Exam **NAME (Last, First):**______________________________

PROBLEM 1. (25 points) To receive full credit, please show your work including FBDs.

1A. GIVEN: The cable system shown is holding a 200N load and a 100N load as shown and is in static equilibrium. Determine the magnitudes of the tensions in cable DE, DB, BA, and BC. Be sure to sketch your Free-Body Diagrams and show clear and complete static equilibrium equations. **(6 pts)**





FBDs (2 pts)

FBD #1

$\Sigma F_x = 0 = T_{DE} - T_{BD}\cos 45°$

$\Sigma F_y = 0 = T_{BD}\sin 45° - 100$

Solving gives, $T_{BD} = 141.4\ N$ $\quad T_{DE} = 100N$

FBD #2 $\quad$ 141.4N $\qquad$ Note: $T_{BD} = T_{DB}$

$\Sigma F_x = 0 = T_{BD}\cos 45° - T_{BA}$

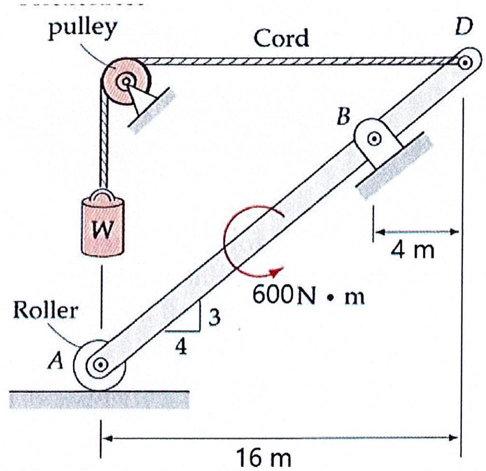
141.4N

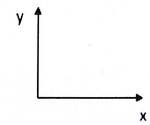
$\Sigma F_y = 0 = T_{BC} - 200 - T_{BD}\sin 45°$

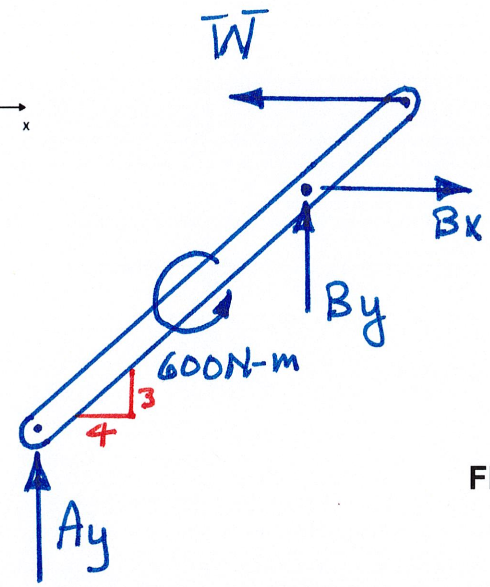
Solving gives, $T_{BA} = 100\ N$ $\quad T_{BC} = 300\ N$

T_{DE} = 100 N	T_{DB} = 141.4 N	**(2 pts)**
T_{BA} = 100N N	T_{BC} = 300 N	**(2 pts)**

1B. Given: Rigid bar ABC has negligible weight and is loaded with cylinder W weighing 200N and a 600N-m couple as shown and is in static equilibrium. Assume the pulley is an ideal frictionless pulley. Determine the reaction at the roller support A and the pin reaction B in vector form. **HINT**: For support B, show your reactions in the horizontal and vertical directions. **(6 pts)**







FBD (1 pt)

$$\Sigma M_B = 0 = -A_y(12) + 600 + \overset{200N}{W}(3)$$

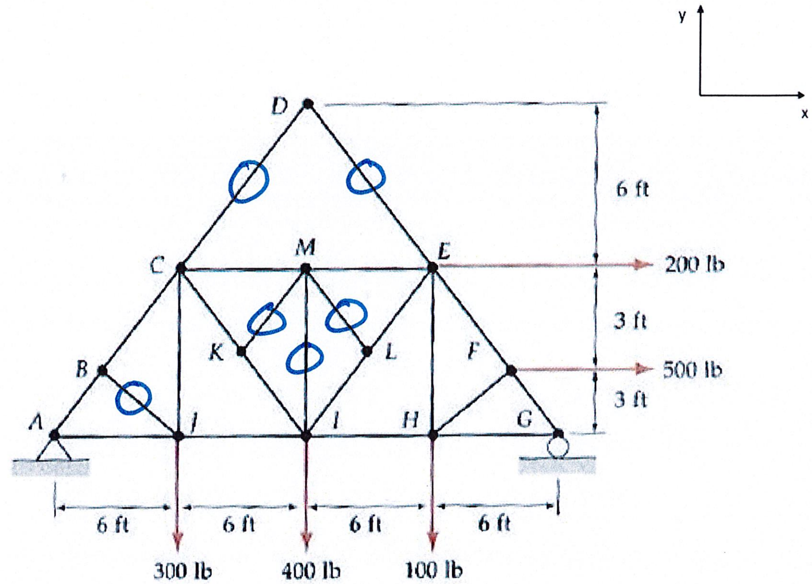
$$\therefore A_y = 100N$$

$$\Sigma F_x = 0 = -\overset{200N}{W} + B_x \Rightarrow B_x = +200N$$

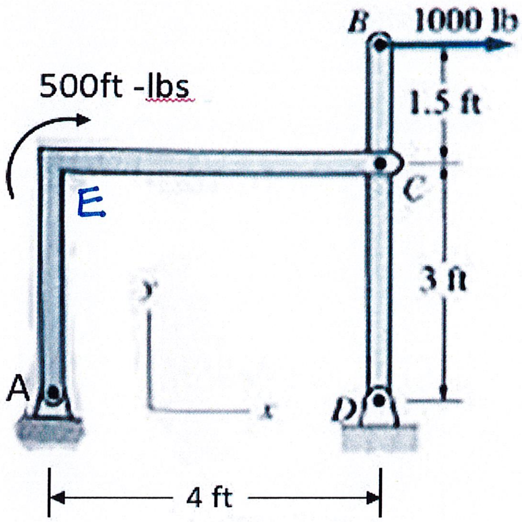
$$\Sigma F_y = 0 = \overset{100N}{A_y} + B_y \Rightarrow B_y = -100N$$

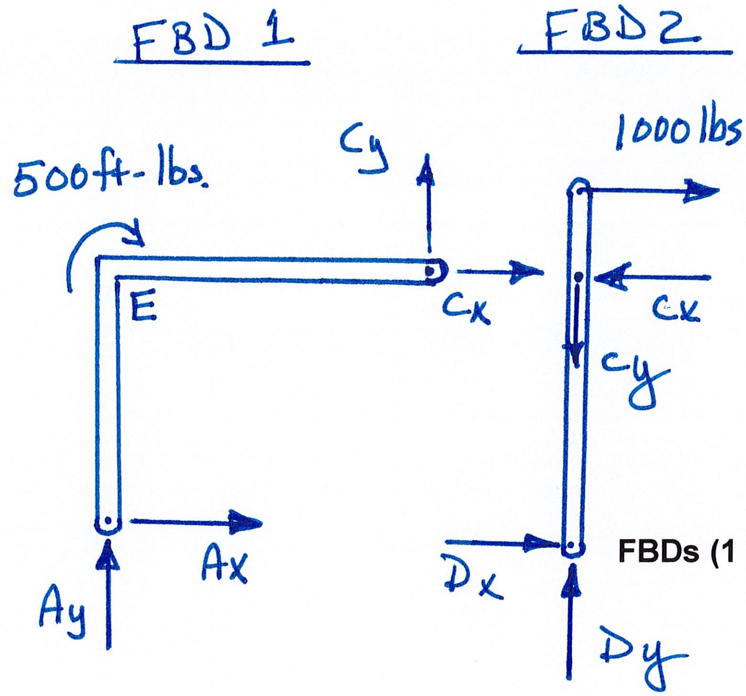
$\bar{A}$ = __+200__ $\hat{j}$ N	**(3 pts)**
$\bar{B}$ = (__+200__) $\hat{\imath}$ + (__−100__ $\hat{j}$) N	**(2 pts)**

1C. Given: The truss is loaded as shown and is held in static equilibrium by a pin support at A and a roller support at G. Place a zero on all members that can be identified as zero-force members. (No work need be shown). **(6 pts)**



1D. Given: Frame ABCDE is loaded with one force and one couple as shown and is held in static equilibrium by pin supports at joints A and D. Determine the force at joint C on member AEC. Also, determine the force at joint C on member BCD. Express these forces in vector form. Be sure to sketch your Free-Body Diagrams and show complete and clear equilibrium equations. (**HINT**: You don't need to solve for the reactions at A and D to determine the forces at joint C.) **(7 pts)**





FBDs (1 pt)

FBD 2

$\Sigma M_D = 0 = C_x(3) - 1000(4.5) \Rightarrow C_x = +1500\text{ lbs.}$

FBD 1

1500 lbs.

$\Sigma M_A = 0 = -500 - C_x(3) + C_y(4)$

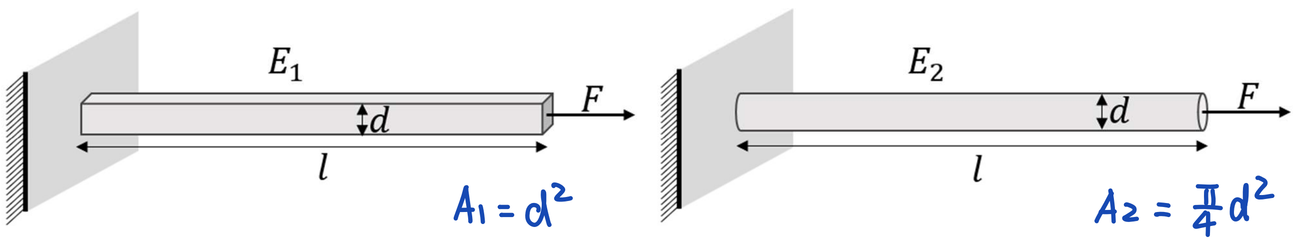
$\Rightarrow C_y = +1250\text{ lbs.}$

$(\bar{F}_C)_{\text{on AEC}}$ = (+1500) $\hat{\imath}$ + (+1250) $\hat{\jmath}$ N	**(4 pts)**
$(\bar{F}_C)_{\text{on BCD}}$ = (−1500) $\hat{\imath}$ + (−1250) $\hat{\jmath}$ N	**(2 pts)**

Problem 2. (25 points) To receive full credit, please show your work including FBDs.

2A. Given: Two systems are given as the figure shown below. In each system, a bar has an axial load of F applied on the right end and fixed to a wall on the left end with Young's Modulus of E_1 and E_2 respectively. The cross section of the bar in *System 1* is a square with each edge length of d. The cross section of the bar in *System 2* is a circle with a diameter of d. Both bars have the same length l.

Please answer the following question by circling the correct choice in each given scenario (**5 pts**)

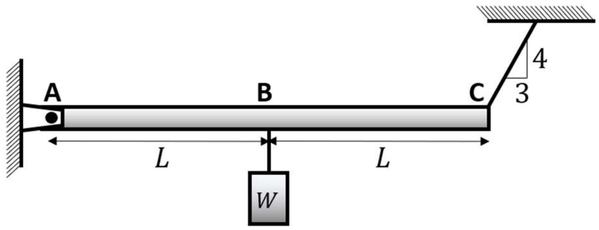


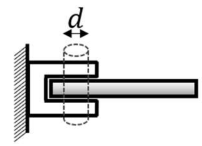
System 1 System 2

If $E_1 = E_2$, which of the following is true regarding the normal stress due to the axial load?
A. $\sigma_1 > \sigma_2$ B. $\sigma_1 = \sigma_2$ C. $\sigma_1 < \sigma_2$ D. Not enough information is given to compare σ_1 and σ_2. $\sigma = \frac{F}{A}$ $A_1 > A_2 \Rightarrow \sigma_1 < \sigma_2$
If $E_1 > E_2$, which of the following is true regarding the normal stress due to the axial load?
A. $\sigma_1 > \sigma_2$ B. $\sigma_1 = \sigma_2$ C. $\sigma_1 < \sigma_2$ D. Not enough information is given to compare σ_1 and σ_2. E doesn't affect σ.
If $E_1 = E_2$, which of the following is true regarding the normal strain due to the axial load?
A. $\epsilon_1 > \epsilon_2$ B. $\epsilon_1 = \epsilon_2$ C. $\epsilon_1 < \epsilon_2$ D. Not enough information is given to compare ϵ_1 and ϵ_2. $\varepsilon = \frac{\sigma}{E}$ $\sigma_1 < \sigma_2 \Rightarrow \varepsilon_1 < \varepsilon_2$
If $E_1 > E_2$, which of the following is true regarding the normal strain due to the axial load?
A. $\epsilon_1 > \epsilon_2$ B. $\epsilon_1 = \epsilon_2$ C. $\epsilon_1 < \epsilon_2$ D. Not enough information is given to compare ϵ_1 and ϵ_2. $\sigma_1 < \sigma_2$ $E_1 > E_2$ $\rightarrow \frac{\sigma_1}{E_1} < \frac{\sigma_2}{E_2}$
If $E_1 < E_2$, which of the following is true regarding the normal strain due to the axial load?
A. $\epsilon_1 > \epsilon_2$ B. $\epsilon_1 = \epsilon_2$ C. $\epsilon_1 < \epsilon_2$ D. Not enough information is given to compare ϵ_1 and ϵ_2. $\sigma_1 < \sigma_2$ $E_1 < E_2$. not enough info

2B. Given: As shown in the figure below, bar ABC with total length of $2L$ is pinned to a wall at end A and attached to a cable at end C. A weight of W is attached to the bar at the middle B. Pin A is a double-shear pin with diameter d. **(9 pts)**

Consider that $W = 800lbs$, $d = 2in$ and $L = 2ft$.

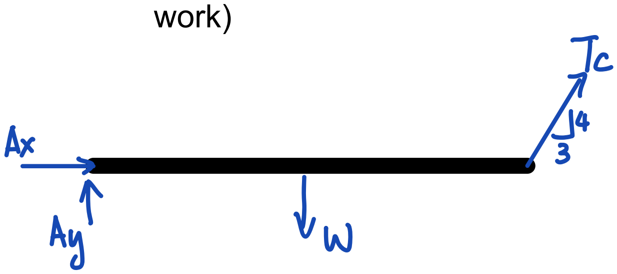




Top View of **Pin A**

Find:

a) The resultant force at A. (3 pts)
b) Shear stress on pin A. (2 pts)
c) Whether there is normal stress due to Axial Load and Bending on the bar. (No need to show your work)
d) Whether there is shear stress due to Pure Shear and Torsion on the bar. (No need to show your work)



$$\Sigma M_A = -L \cdot W + 2L \cdot \tfrac{4}{5} T_c = 0$$

$$\hookrightarrow T_c = \tfrac{5}{8} W$$

$$\Sigma F_x = A_x + \tfrac{3}{5} T_c = 0 \rightarrow A_x = -\tfrac{3}{8} W$$

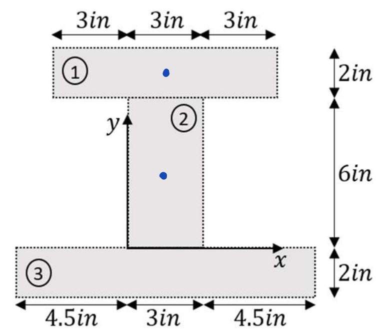
$$\Sigma F_y = A_y - W + \tfrac{4}{5} T_c = 0 \rightarrow A_y = \tfrac{1}{2} W$$

$$|\vec{F_A}| = \sqrt{(\tfrac{3}{8}W)^2 + (\tfrac{1}{2}W)^2} = \tfrac{5}{8} W = 500\,lb$$

$$\tau = \frac{|\vec{F_A}|}{2 \cdot \pi \cdot (\frac{2in}{2})^2} = 79.58\ psi$$

$\|F_A\|$ = 500 **lbs**	**(1 pts)**
$\|\tau_A\|$ = 79.58 **psi**	**(1 pts)**
Is there normal stress due to Axial Load? Yes No (Circle one)	**(1 pts)**
Is there normal stress due to Bending? Yes No (Circle one)	**(1 pts)**
Is there shear stress due to Pure Shear? Yes No (Circle one)	**(1 pts)**
Is there shear stress due to Torsion? Yes No (Circle one)	**(1 pts)**

2C. Given: Consider the following beam's cross-sectional area with the given coordinate system. **(6 pts)**



Find:

a) Fill out the following table based on the labeled section. (1 pts)
b) Centroid of the full cross-sectional area. (3 pts)
c) Use method of section to find I_x of the cross-sectional area. (2 pts)

Section Number	Area $[in^2]$	$\bar{y}$ $[in]$ (y centroid of the section)	I_x $[in^4]$ (based on the neutral axis of the section)
1	18	7	6
2	18	3	54
3	24	-1	8

$A_1 = 9in \times 2in = 18\ in^2$

$A_2 = 6in \times 3in = 18\ in^2$

$A_3 = 12in \times 2in = 24 in^2$

$$\bar{y}_{Tot} = \frac{18\times7 + 18\times3 + 24\times(-1)}{18+18+24} = 2.6\ in$$

$I_{x1} = \frac{1}{12} \cdot 9 \cdot 2^3 = 6 in^4$

$I_{x2} = \frac{1}{12} \cdot 3 \cdot 6^3 = 54 in^4$

$I_{x3} = \frac{1}{12} \cdot 12 \cdot 2^3 = 8 in^4$

$$I_{Totx} = I_{x1} + 18\times(7-2.6)^2 + I_{x2} + 18\times(3-2.6)^2 + I_{x3} + 24\times(-1-2.6)^2$$

$$= 730.4\ in^4$$

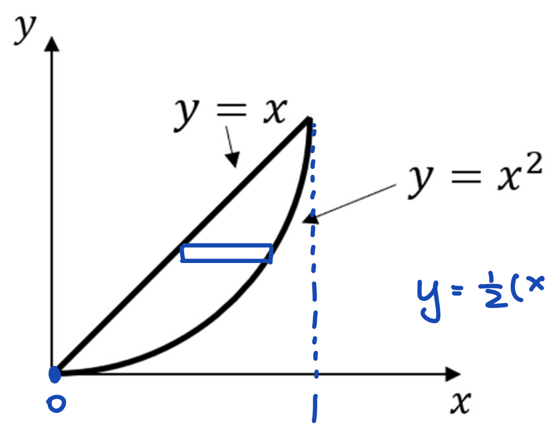
$\bar{y}_{total} =$ ____2.6____ in		**(1 pts)**
$I_x =$ ____730.4____ in^4		**(1 pts)**

2D. Given: Consider the shaded area shown. **(5 pts)**

Find: (a) Using the method of integration determine the moment of inertia of the area enclosed by function $y = x$ and $y = x^2$ about the given x-axis. Length units are in meters. (4 pts)

(b) Intuitively, do you expect the moment of inertia about the y axis I_y to be larger, the same or smaller than I_x. No work need be shown. (1 pt)

(a) $x = x^2 \rightarrow x = 0$ or $x = 1$,



$y = \frac{1}{2}(x + x^2)$

$I_x = \int_A y^2 dA \qquad dA = dy \cdot (x_R - x_L)$

$= dy \cdot (y^{1/2} - y)$

$I_x = \int_0^1 y^2 \cdot (y^{1/2} - y)\, dy$

$= \int_0^1 y^{5/2} - y^3 \, dy$

$= \left[\frac{2}{7} y^{7/2} - \frac{1}{4} y^4 \right]_{y=0}^{y=1}$

$= \frac{2}{7} - \frac{1}{4} = \frac{8-7}{28} = \frac{1}{28}$

Alternatively

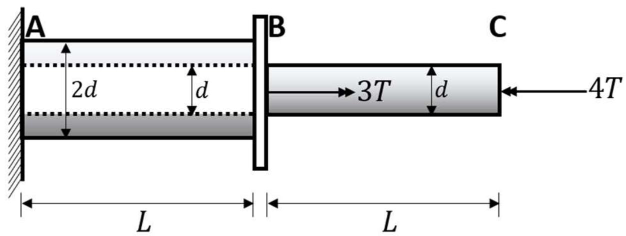
$I_x = \int_0^1 \int_{x^2}^{x} y^2 \, dy\, dx$

$= \frac{1}{28}$

$I_x =$ ______ $\frac{1}{28}$ ______ m⁴		**(1 pts)**
$I_x > I_y$ $\quad$ $I_x = I_y$ $\quad$ $I_x < I_y$	**(Circle One)**	**(1 pts)**

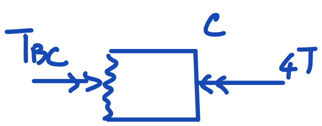
PROBLEM 3. (25 points)

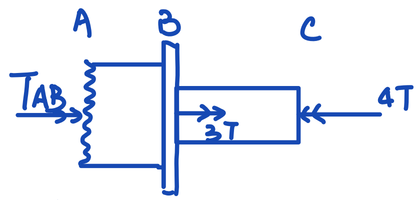
Given: Consider the following shaft under torsion. Section AB has an outer diameter of $2d$ and a circular hollow with diameter of d. Section BC is a solid cylinder with diameter of d. Use the following parameters: $L = 8m$, $d = 2m$. Feel free to include π in your expression.



Find:

a) Assume T is known and is given in kN, determine the magnitude of torque carried by Section AB and BC. (2 pts)





$T_{BC} = 4T$

$T_{AB} + 3T - 4T = 0 \rightarrow T_{AB} = T$

$\lvert T_{AB} \rvert$ = ____T____ kN·m	**(1 pt)**
$\lvert T_{BC} \rvert$ = ____4T____ kN·m	**(1 pt)**

b) Determine the polar moment of inertia for Section AB and BC. (4pts)

$J_{AB} = \frac{\pi}{2}\left[\left(\frac{2d}{2}\right)^4 - \left(\frac{d}{2}\right)^4\right]$

$= \frac{\pi}{2}\left[(2m)^4 - (1m)^4\right]$

$= \frac{15}{2}\pi\ m^4$

$J_{BC} = \frac{\pi}{2} \cdot \left(\frac{d}{2}\right)^4$

$= \frac{\pi}{2} \cdot (1m)^4$

$= \frac{\pi}{2}\ m^4$

J_{AB} = ____$\frac{15}{2}\pi$____ m⁴	**(1 pt)**
J_{BC} = ____$\frac{\pi}{2}$____ m⁴	**(1 pt)**

c) Assume T is known and is given in kNm determine the maximum and minimum magnitude and location(s) of shear stress on Section AB. (8 pts)

$$\tau_{AB\,max} = \frac{T\cdot\frac{1}{2}(2d)}{\frac{15}{2}\pi}\cdot\left[\frac{kN\cdot m\cdot m}{m^4}\right] = \frac{T\cdot 2}{\frac{15}{2}\pi}\ kPa = \frac{4}{15}\frac{T}{\pi}\ kPa$$

$$\tau_{AB\,min} = \frac{T\cdot\frac{1}{2}(d)}{\frac{15}{2}\pi} = \frac{2}{15}\frac{T}{\pi}\ kPa$$

$\lvert\tau_{AB}\rvert_{max}$ = $\frac{4}{15}\frac{T}{\pi}$ **kPa**	**(1 pt)**
$\lvert\tau_{AB}\rvert_{max}$ location(s): Outside layer of the AB section.	**(2 pts)**
$\lvert\tau_{AB}\rvert_{min}$ = $\frac{2}{15}\frac{T}{\pi}$ **kPa**	**(1 pt)**
$\lvert\tau_{AB}\rvert_{min}$ location(s): Inside layer (hollow) of the AB section.	**(2 pts)**

d) Assume T is known and is given in kNm determine the maximum and minimum magnitude and location(s) of shear stress on Section BC. (8 pts)

$$\tau_{BC\,max} = \frac{4T\cdot\frac{1}{2}d}{\frac{1}{2}\pi} = 8\frac{T}{\pi}\ kPa$$

$$\tau_{BC\,min} = 0$$

$\lvert\tau_{BC}\rvert_{max}$ = $8\frac{T}{\pi}$ **kPa**	**(1 pt)**
$\lvert\tau_{BC}\rvert_{max}$ location(s): Outside layer of BC section	**(2 pts)**
$\lvert\tau_{BC}\rvert_{min}$ = 0 **kPa**	**(1 pt)**
$\lvert\tau_{BC}\rvert_{min}$ location(s): Center line (neutral axis) of BC section.	**(2 pts)**

e) If the material of both Sections fails at τ_{fail} given in kPa, what is the maximum allowable value for T when the design requires a Factor of Safety of 2. (3 pts)
(Hint: Your expression for T should include π and τ_{fail} and be in units of kN.

$$\tau_{BC max} = 8\frac{T}{\pi} = \frac{\tau_{fail}}{FoS}$$

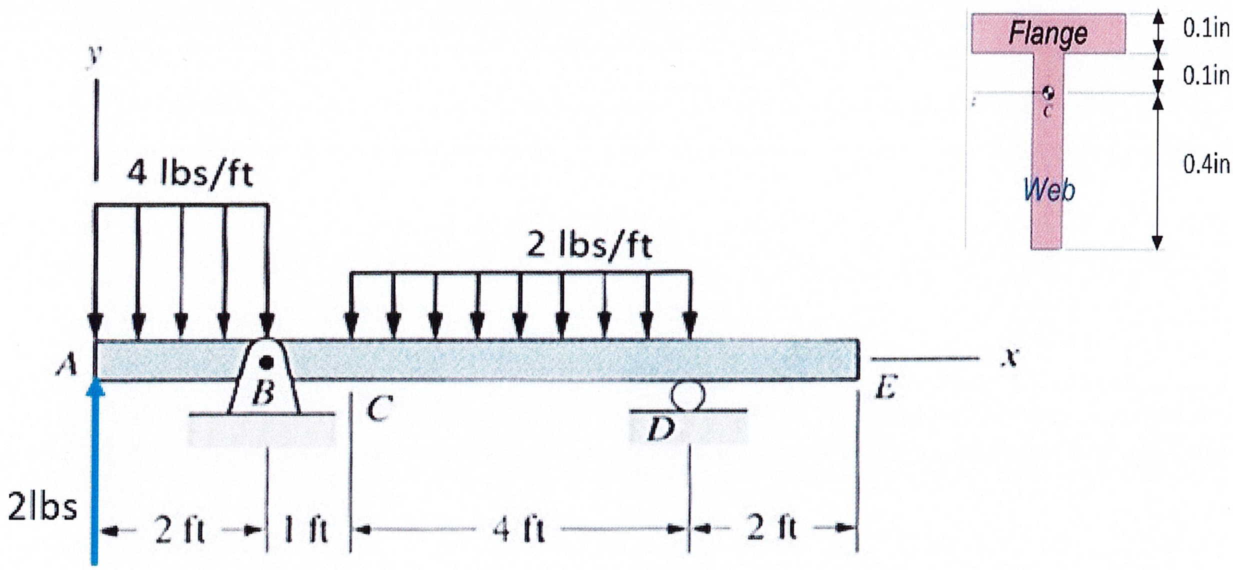
$$8\frac{T}{\pi} = \frac{\tau_{fail}}{2}$$

$$T = \frac{1}{16}\frac{\tau_{fail}}{\pi}$$

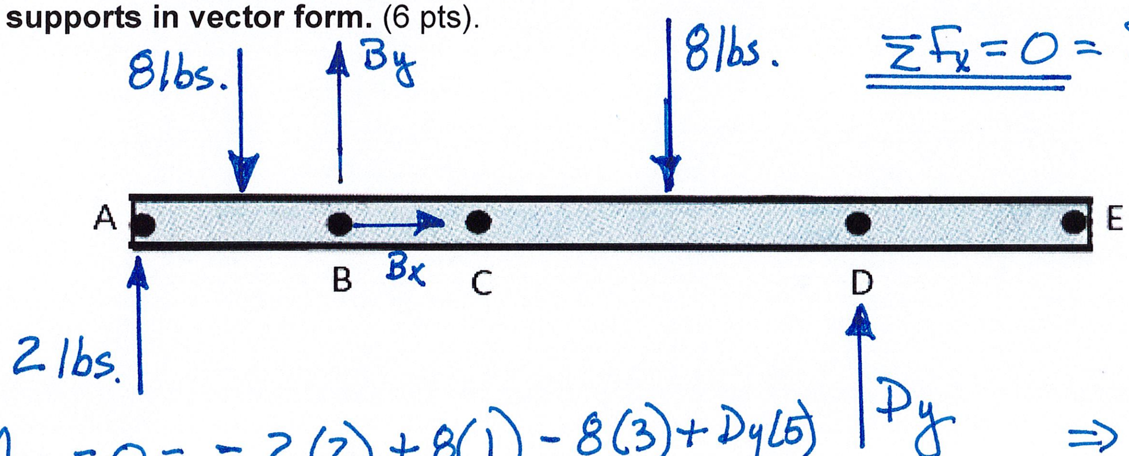
$\lvert \boldsymbol{T} \rvert_{allowed}$ = $\frac{1}{16}\frac{\tau_{fail}}{\pi}$ **kN·m**	**(1 pt)**

PROBLEM 4. (25 points)

GIVEN: Beam ABCDE is loaded wit two distributed loads and one point load as shown and is held in static equilibrium by a pin support at B and a roller support at D. The cross section of the beam is a **T-shaped as shown, which is made of a "flange" on top and a vertical "web" member.** The centroid of the T-beam is also shown. Assume I = 3.0 x 10^{-3} in^4.



FIND: a) Draw the **free body diagram** (1 pt) of the beam ABCDE and **calculate the reaction forces at the supports in vector form.** (6 pts).



$\underline{\Sigma F_x = 0} = B_x$

$B_x = 0$ lbs.

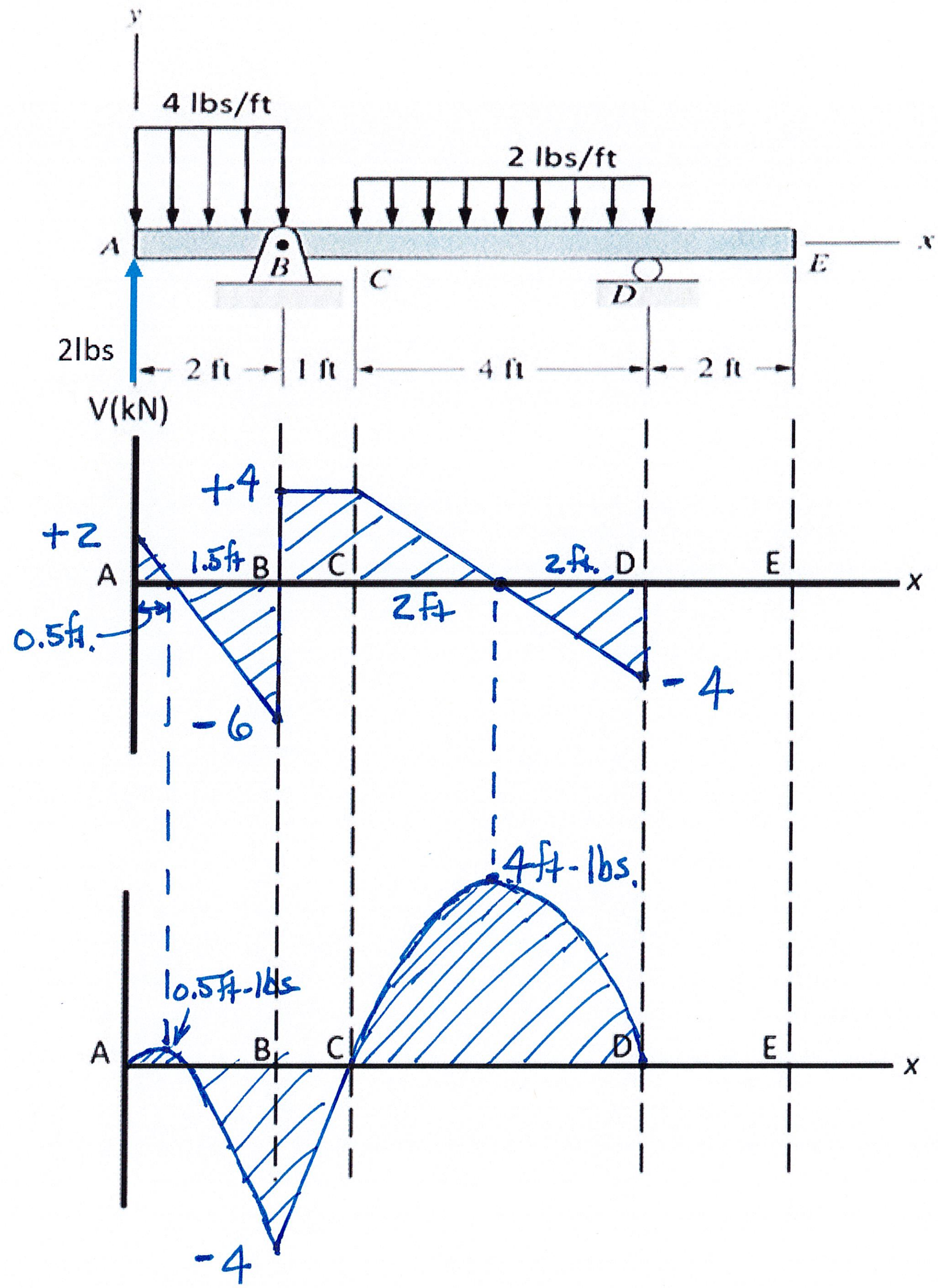
$\underline{\Sigma M_B = 0} = -2(2) + 8(1) - 8(3) + D_y(5) \Rightarrow D_y = 4$ lbs.

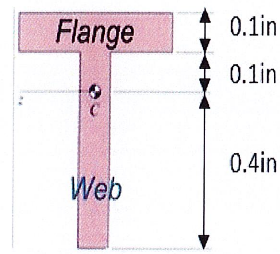
(+4 lbs)

$\underline{\Sigma F_y = 0} = 2 - 8 + B_y - 8 + D_y \Rightarrow B_y = 10$ lbs.

$\overline{F}_B$ = ___0___ $\overline{i}$ +	___+10___ $\overline{j}$ lbs		(4 pts)
$\overline{F}_D$ = ___+4___ $\overline{j}$ lbs			(2 pts)

b) Draw the **shear force and bending moment diagrams** of the beam ABCDE. You must **label** the **shear force** and **bending moment values** on the diagram at points **A, B, C, D, and E as well as any max or min values** to receive full credit (8 pts). You may use the graphical method. (**8 pts**)



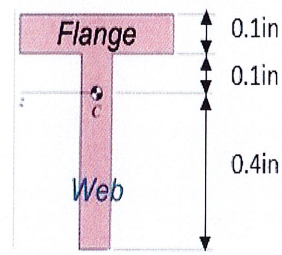


c) Identify all sections of the beam which have a point or section in **pure bending** (2 pts)?

Sections: AB BC CD DE (Circle all that have a pt or section in Pure Bending)	**(2 pts)**

d) Among the points and sections where pure bending occurs, determine the **magnitude of the overall maximum bending stress (σ_{max}) of the beam (and whether it is in tension or compression)**. Also, calculate the **magnitude of the bending stress at the bottom of the flange in the section where the maximum pure bending occurs (and determine whether it is in tension or compression)**. (**Hint:** Pay close attention to your sign convention and your units). (8 pts).

$$\sigma_{MAX} = -\frac{My}{I} = -\frac{(+4\text{ ft-lbs})(-0.4\text{ in})\left(\frac{12\text{ in}}{1\text{ ft}}\right)}{3\times10^{-3}\text{ in}^4}$$



$$\sigma_{MAX} = +6400\text{ psi} = 6400\text{ psi, Tension}$$

$$\sigma_{\text{Flange Bottom}} = -\frac{My}{I} = \frac{-(+4\text{ ft-lbs})(0.1\text{ in})\left(\frac{12\text{ in}}{1\text{ ft}}\right)}{3\times10^{-3}\text{ in}^4}$$

$$\sigma_{\text{Flange Bottom}} = -1600\text{ psi} = 1600\text{ psi, Compression}$$

σ_{max} = 6400 psi **Compression** or **Tension** (circle one)	(4 pts)
$\sigma_{\text{Bottom of Flange}}$ = 1600 psi **Compression** or **Tension** (circle one)	(4 pts)

Fall 2022 Final Exam – Equation Sheet

Normal Stress and Strain

$$\sigma_x = \frac{F_n}{A}$$

$$\sigma_x(y) = \frac{-My}{I}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{\Delta L}{L}$$

$$\varepsilon_y = \varepsilon_z = -\vartheta\varepsilon_x$$

$$\varepsilon_x(y) = \frac{-y}{\rho}$$

$$FS = \frac{\sigma_{fail}}{\sigma_{allow}}$$

Shear Stress and Strain

$$\tau = \frac{V}{A}$$

$$\tau(\rho) = \frac{T\rho}{J}$$

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\vartheta)}$$

$$\gamma = \frac{\delta_s}{L_s} = \frac{\pi}{2} - \theta$$

Second Area Moment

$$I = \int_A y^2 dA$$

$$I = \frac{1}{12}bh^3 \quad \text{Rectangle}$$

$$I = \frac{\pi}{4}r^4 \quad \text{Circle}$$

$$I_B = I_O + Ad_{OB}{}^2$$

Polar Area Moment

$$J = \frac{\pi}{2}r^4 \quad \text{Circle}$$

$$J = \frac{\pi}{2}(r_o^4 - r_i^4) \quad \text{Tube}$$

Shear Force and Bending Moment

$$V(x) = V(0) + \int_0^x p(\epsilon)d\epsilon$$

$$M(x) = M(0) + \int_0^x V(\epsilon)d\epsilon$$

Buoyancy

$$F_B = \rho g V$$

Fluid Statics

$$p = \rho g h$$

$$F_{eq} = p_{avg}(Lw)$$

Belt Friction

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

Distributed Loads

$$F_{eq} = \int_0^L w(x)dx$$

$$\bar{x}F_{eq} = \int_0^L x\, w(x)dx$$

Centroids

$$\bar{x} = \frac{\int x_c dA}{\int dA} \qquad \bar{y} = \frac{\int y_c dA}{\int dA}$$

$$\bar{x} = \frac{\sum_i x_{ci}A_i}{\sum_i A_i} \qquad \bar{y} = \frac{\sum_i y_{ci}A_i}{\sum_i A_i}$$

In 3D, $\bar{x} = \frac{\sum_i x_{ci}V_i}{\sum_i V_i}$

Centers of Mass

$$\tilde{x} = \frac{\int x_{cm}\rho dA}{\int \rho dA} \qquad \tilde{y} = \frac{\int y_{cm}\rho dA}{\int \rho dA}$$

$$\tilde{x} = \frac{\sum_i x_{cmi}\rho_i A_i}{\sum_i \rho_i A_i} \qquad \tilde{y} = \frac{\sum_i y_{cmi}\rho_i A_i}{\sum_i \rho_i A_i}$$