ME 270 – Spring 2024 Exam 1 **NAME (Last, First):** SOLUTION **(Please Print)**

PUID #: ____________________ **(Please Print)**

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ____________________________________

Instructor's Name and Section: (Circle Your Section)

Sections: J. Jones, Section 001, MWF 9:30AM-10:20AM
T. Han, Section 002, MWF 1:30PM-2:20PM
J. Jones, Section 003, MWF 11:30AM-12:20AM
J. Jones, Section 005, Distance Learning

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: ____________________________________

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, please request additional paper from your instructor.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

- The allowable exam time for Exam 1 is 90 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen or dark lead pencil** for the exam.
- Do not write on the back side of your exam paper.

If the solution does not follow a logical thought process, it will be assumed in error.

When submitting your exam on Gradescope, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of the cover page. Also, be sure to identify the page numbers for each problem before final submission on Gradescope. Do not include the cover page or the equation sheet with any of the problems.

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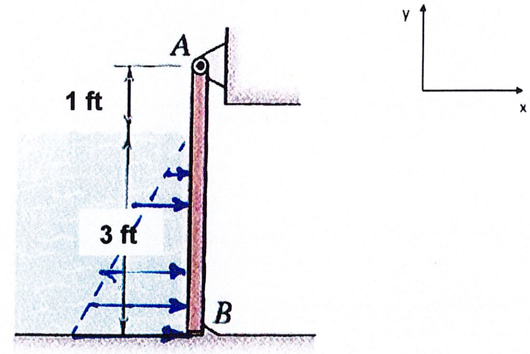
PROBLEM 1 (20 points)

1A. Gate AB hold back a small reservoir filled with 3ft of water as shown. Gate AB is held in static equilibrium by a pin joint at A and a lip shown at B that prevents the gate from opening. Determine the magnitude of the equivalent hydrostatic force (F_{eq}) due to the water acting in the gate and the magnitude of the horizontal reaction force the lip at B (F_B) exerts on Gate AB. Assume Gate AB is 2 ft wide (into the page) and that ρg = 62.5 lbs/ft^3. Remember your solution should include a Free Body Diagram and a clearly written static equilibrium equation. **(5 pts)**

$$P_B = (\rho g)\, h_B = \left(62.5 \frac{lbs}{ft^3}\right)(3ft)$$

$$P_B = 187.5\ lbs/ft^2$$



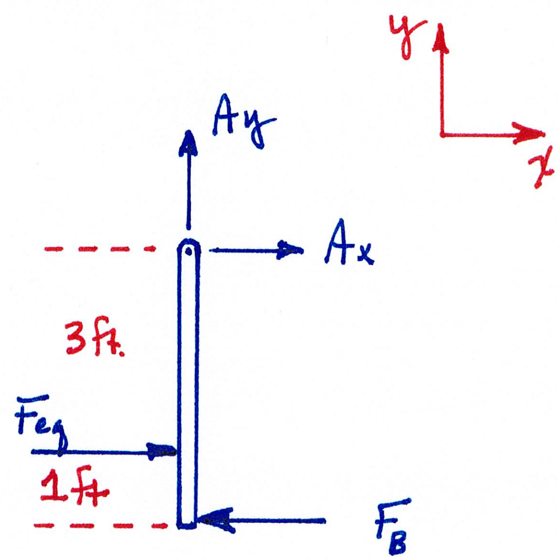
$$F_{eq} = \frac{1}{2}(3ft)(187.5\ lbs/ft^2)(2ft)$$

$$\boxed{F_{eq} = 562.5\ lbs.}$$

$$\underline{\underline{\Sigma M_A = 0}} = F_{eq}(3ft) - F_B(4ft)$$

$$\therefore F_B = (562.5\ lbs)(3ft)/(4ft)$$

$$\boxed{F_B = 422\ lbs.}$$



F_{eq} = 563 lbs. **(2pts)** F_B = 422 lbs. **(3 pts)**

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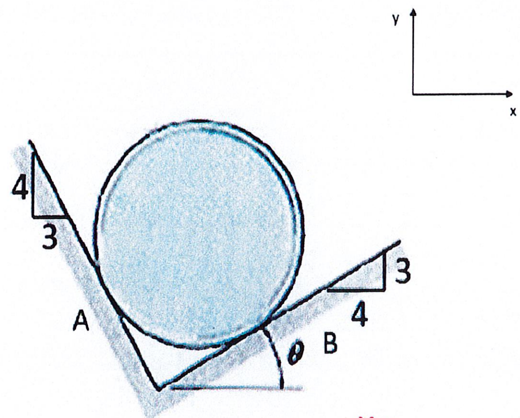
1B. The Cylinder shown weighs 10 lbs and is held in static equilibrium by two frictionless inclined surfaces (A and B) as shown. Determine the normal forces acting on the cylinder from surface A and surface B. Qualitatively, if the angle (θ) of the two inclined surfaces were increased slightly (keeping the angle between the surfaces the same), what impact would this have on the magnitude of the normal force acting at Surface B? Remember your solution should include a Free Body Diagram and clearly written static equilibrium equations. **(5 pts)**



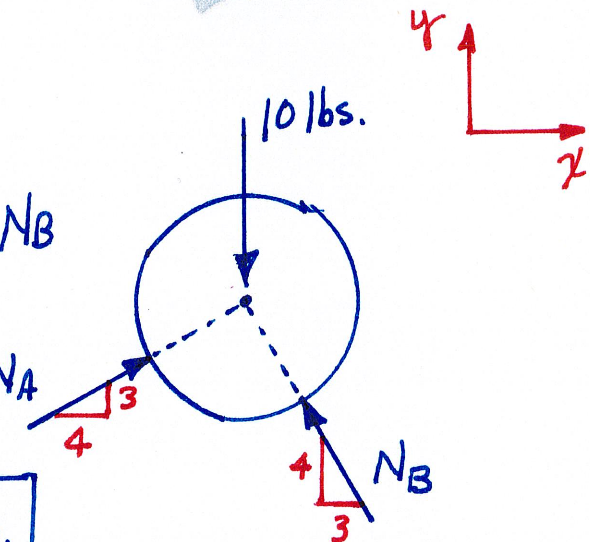
$$\underline{\underline{\Sigma F_x = 0}} = \frac{4}{5} N_A - \frac{3}{5} N_B$$

$$\underline{\underline{\Sigma F_y = 0}} = -10 + \frac{3}{5} N_A + \frac{4}{5} N_B$$

$$N_A = \left(\frac{5}{4}\right)\left(\frac{3}{5} N_B\right) = \frac{3}{4} N_B$$

$$10 = \frac{3}{5}\left(\frac{3}{4} N_B\right) + \frac{4}{5} N_B$$

$$10 = \frac{9}{20} N_B + \frac{16}{20} N_B = \frac{25}{20} N_B$$



$$\boxed{\therefore\ N_B = 8 \text{ lbs.}}$$

$$\boxed{N_A = \frac{3}{4} N_B = \frac{3}{4}(8 \text{ lbs}) = 6 \text{ lbs.}}$$

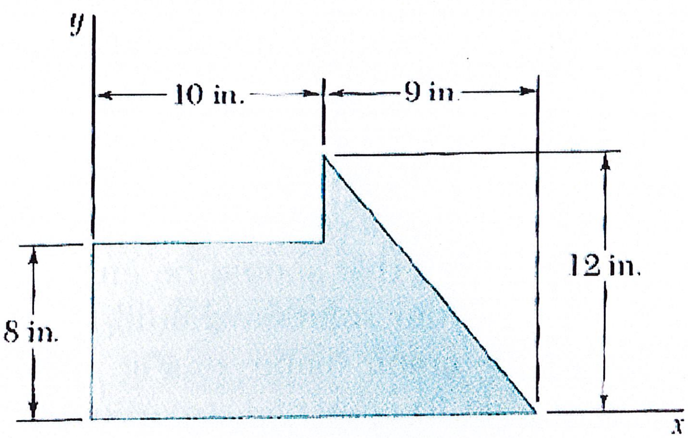
N_A = 6.0 lbs. (2 pts)	N_B = 8.0 lbs. (2 pts)
N_B: Increase Remain the Same Decrease (Circle One) (1 pt)	

1C. Using the method of composite parts, **find** the area and the x-centroid (x_C) of the shaded area in the figure below with respect to the coordinate axes provided. Please show your work to receive credit. Qualitatively, would you expect y_C be larger, the same, or smaller that x_C? (No calculations are required). **(5 pts)**

$A = (8\text{ in})(10\text{ in}) + \frac{1}{2}(9\text{ in})(12\text{ in})$

$A = 80\text{ in}^2 + 54\text{ in}^2$

$A = 134\text{ in}^2$



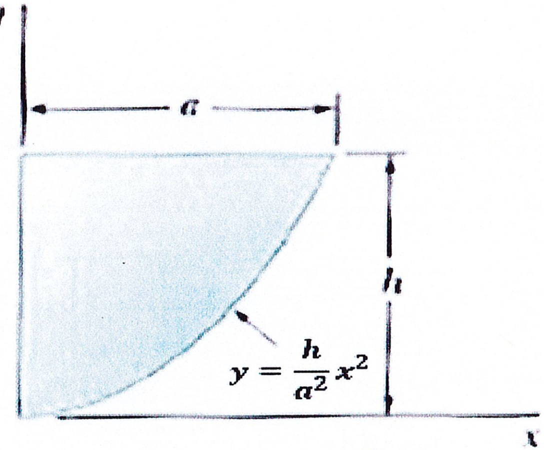
$A(x_c) = A_1(x_{c_1}) + A_2(x_{c_2})$

$134(x_c) = (80\text{ in}^2)(5\text{ in}) + (54\text{ in}^2)(13\text{ in})$

$\therefore x_c = 8.22\text{ in}$

A = 134 in² (1 pt)		x_C = 8.22 in (3 pts)		
y_C:	$y_C > x_C$	$y_C = x_C$	$y_C < x_C$	(Circle One) (1 pt)

1D. Using the method of integration, determine the area and the y-centroid (y_C) of the shaded area with respect to the coordinate axes provided. Assuming the constants "a" and "h" are equal (a=h), qualitatively would you expect for x_C to be larger, smaller or equal to y_C? (No calculations are required). **(5pts)**



$$A = \int_{x_1=0}^{x_2=a}\int_{y_1=\frac{h}{a^2}x^2}^{y_2=h} dy\,dx$$

$$A = \int_0^a [y]_{\frac{h}{a^2}x^2}^{h}\,dx = \int_0^a \left(h - \frac{h}{a^2}x^2\right)dx = hx - \frac{h}{a^2}\frac{x^3}{3}\Big]_0^a$$

$$A = ha - \frac{h}{a^2}\left(\frac{a^3}{3}\right) = h\left(a - \frac{a}{3}\right) = \boxed{\frac{2}{3}ha}$$

$$A\,y_c = \int_{x_1=0}^{x_2=a}\int_{y_1=\frac{h}{a^2}x^2}^{y_2=h} y\,dy\,dx = \int_0^a \left[\frac{y^2}{2}\right]_{\frac{h}{a^2}x^2}^{h} dx = \int_0^a \left[\frac{h^2}{2} - \frac{h^2x^4}{2a^4}\right] dx$$

$$A\,y_c = \left[\frac{h^2}{2}x - \frac{h^2}{2a^4}\left(\frac{x^5}{5}\right)\right]_0^a = \frac{h^2a}{2} - \frac{h^2a}{10} = \frac{5h^2a}{10} - \frac{h^2a}{10}$$

$$\left(\frac{2}{3}ha\right)y_c = \frac{4h^2a}{10} \qquad \therefore\ y_c = \left(\frac{4h^2a}{10}\right)\left(\frac{3}{2ha}\right) = \frac{12h}{20} = \boxed{\frac{3h}{5}}$$

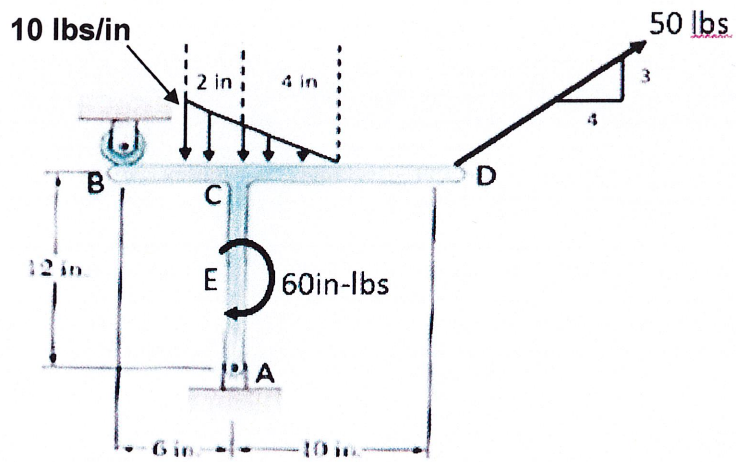
A = 2/3 ha units² (1 pt) yc = 3/5 h units (3 pts)

xc: xc > yc xc = yc xc < yc (Circle One) (1 pt)

PROBLEM 2. (20 points)

Given: Frame ABCDE is loaded with a point force at D, a distributed load as shown, and a couple at E and is held in static equilibrium by a pin support at A and a roller support at B.

Find: a) Determine the equivalent force (F_{eq}) for the distributed load and its distance from C ($\bar{x}_{eq}$). (3 pts)

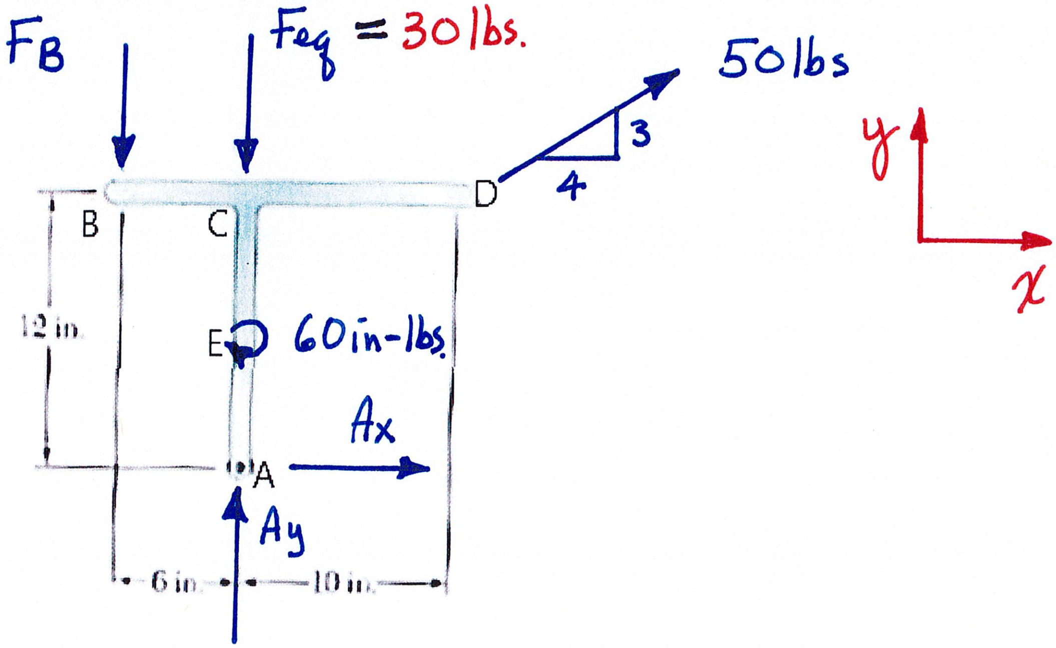


$F_{eq} = \frac{1}{2}(10\,lb/in)(6\,in) = 30\,lbs.$

$(x_{eq})_{from\ C} = 0\,in$

F_{eq} = ____30____ lbs. (2 pts) $(\bar{x}_{eq})_{from\ C}$ = ____0____ in (1 pt)

b) On the artwork provided below, complete the free body diagram for frame ABCDE. Use the F_{eq} determined above in your free body diagram. (2 points)



c) Clearly write the equilibrium equations and solve for the reactions at supports in vector form. (12 pts)

$$\Sigma M_A = 0 = -60 + F_B(6in) - \tfrac{4}{5}(50lbs)(12in) + \tfrac{3}{5}(50lbs)(10in)$$

$$\boxed{F_B = 40\ lbs.}$$

$$\Sigma F_x = 0 = A_x + \tfrac{4}{5}(50\ lbs) \Rightarrow \boxed{A_x = -40\ lbs.}$$

40lbs. 30lbs.

$$\Sigma F_y = 0 = A_y - F_B - F_{eq} + \tfrac{3}{5}(50\ lbs)$$

$$\Rightarrow \boxed{A_y = +40\ lbs.}$$

$\bar{F}_B$ = - 40 $\hat{\jmath}$ *lbs.* (6 pts)	$\bar{F}_A$ = - 40 $\hat{\imath}$+ + 40 $\hat{\jmath}$ lbs. (6 pts)

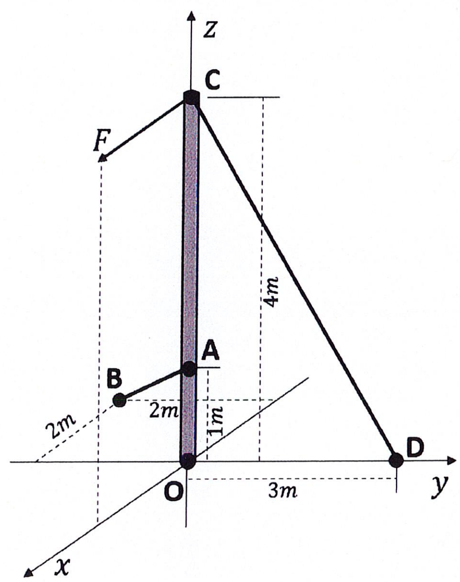
d) If the distributed load were removed from the frame, what qualitative effect would this have on the magnitudes (i.e., neglect any sign changes) of the reactions (no work need be shown)? (3 pts)

B_y would:	increase	remain the same	decrease (circle one)	(1 pt)
A_x would:	increase	remain the same	decrease (circle one)	(1 pt)
A_y would:	increase	remain the same	decrease (circle one)	(1 pt)

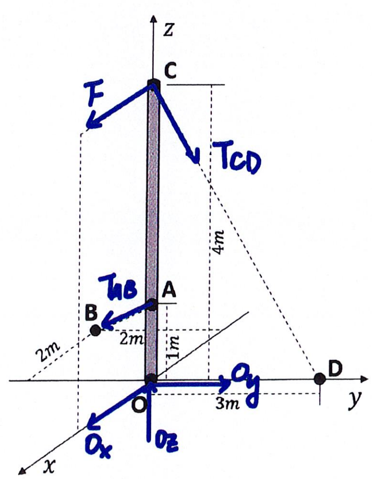
PROBLEM 3. (20 points)

GIVEN:

Pole **OAC** with negligible mass is supported by the ground with ball-and-socket joint. Two cables **AB** and **CD** are attached to the pole. Load **F** in x-direction is applied to the pole at **C**. The tension in cable **AB** is 300-N.



a) Complete the free body diagram of the pole using the figure below. (2 pts)



b) Write expressions for tension vectors **AB** and **CD** acting on the pole using their unknown magnitudes (magnitude of **AB** is known) and known unit vectors. (4 pts) The applied load is shown as an example, you may express as a **simplified** fraction or use the decimal representation. (4 pts)

$\vec{R}_{AB} = [-2\hat{i} - 2\hat{j} - 1\hat{k}]\,m$

$\rightarrow \vec{U}_{AB} = -\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

$R_{CD} = [3\hat{j} - 4\hat{k}]\,m$

$\rightarrow \vec{U}_{CD} = (\frac{3}{5}\hat{j} - \frac{4}{5}\hat{k})$

$\bar{T}_{AB} = 300 * [(\underline{\ -2/3\ })\hat{\imath} + (\underline{\ -2/3\ })\hat{\jmath} + (\underline{\ -1/3\ })\hat{k}]N$	(2 pts)
$\bar{T}_{CD} = \lvert\vec{T}_{CD}\rvert * [(\underline{\ 0\ })\hat{\imath} + (\underline{\ 3/5\ })\hat{\jmath} + (\underline{\ -4/5\ })\hat{k}]N$	(2 pts)
Applied Load= $\lvert\vec{F}\rvert * [(\underline{\ 1\ })\hat{\imath} + (\underline{\ 0\ })\hat{\jmath} + (\underline{\ 0\ })\hat{k}]N$	(example)

c) Determine the <u>magnitudes</u> of the tension in cables **CD** and the load at **F**. (8 pts)

$\Sigma\vec{M}_O = \vec{R}_{OA} \times \vec{T}_{AB} + \vec{R}_{OC} \times \vec{T}_{CD} + \vec{R}_{OC} \times \vec{F}$

$= (1\hat{k})m \times [-200\hat{i} - 200\hat{j} - 100\hat{k}]N$

$+ (4\hat{k})m \times [\frac{3}{5}T_{CD}\hat{j} - \frac{4}{5}T_{CD}\hat{k}]$

$+ (4\hat{k})m \times [F\hat{i}]$

$= [-200\hat{j} + 200\hat{i}] - \frac{12}{5}T_{CD}\hat{i} + 4F\hat{j}$

$\hat{i}: \quad 200 - \frac{12}{5}T_{CD} = 0$

$\hat{j}: \quad -200 + 4F = 0$

$\Rightarrow \quad T_{CD} = \frac{250}{3}N = 83.3N$

$F = 50\,N$

$\lvert\vec{T}_{CD}\rvert = \underline{\quad 83.3 \quad}\ N$	(4 pts)
$\lvert\vec{F}\rvert = \underline{\quad 50 \quad}\ N$	(4 pts)

d) At point **O**, determine the reactions at **O** and express as a vector. (6 points)

$\Sigma F_x = O_x - 200N + 50N = 0$

$\Sigma F_y = O_y - 200N + 50N = 0$

$\Sigma F_y = O_z - 100N - 66.67N = 0$

$\Rightarrow O_x = 150N$

$O_y = 150N$

$O_z = 166.67N$

$\vec{O} = [(\underline{\ 150\ })\,\hat{\imath} + (\underline{\ 150\ })\,\hat{\jmath} + (\underline{\ 166.67\ })\,\hat{k}]$ N. (6 pts)

ME 270 Exam 1 Equations

Distributed Loads

$$F_{eq} = \int_0^L w(x)\,dx$$

$$\bar{x}F_{eq} = \int_0^L x\,w(x)\,dx$$

Centroids

$$\bar{x} = \frac{\int x_c\,dA}{\int dA}$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA}$$

$$\bar{x} = \frac{\sum_i x_{ci}A_i}{\sum_i A_i}$$

$$\bar{y} = \frac{\sum_i y_{ci}A_i}{\sum_i A_i}$$

In 3D, $\bar{x} = \frac{\sum_i x_{ci}V_i}{\sum_i V_i}$