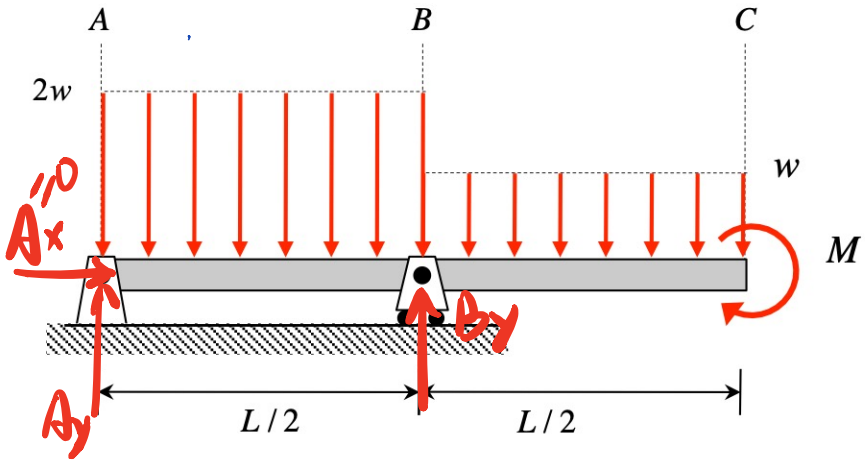
Hints: ① Break up at discontinuities: e.g., concentrated load, between 2 distributions.
② For each distributed load: use slopes, orders of "x", and area under curve.
③ For each concentrated load: decide shear / bending moment jump "up" or "down"

Homework H30.A

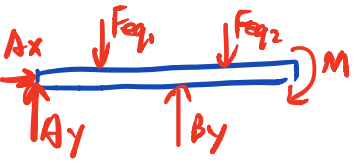
Given: Beam with the line loading w acting over its length and a concentrated couple M acting at end C.

Find: Construct the shear force and bending moment diagrams for this beam.

For this problem, use the following parameters: w = 8 kips/ft, M = 60 kips-ft and L = 6 ft.



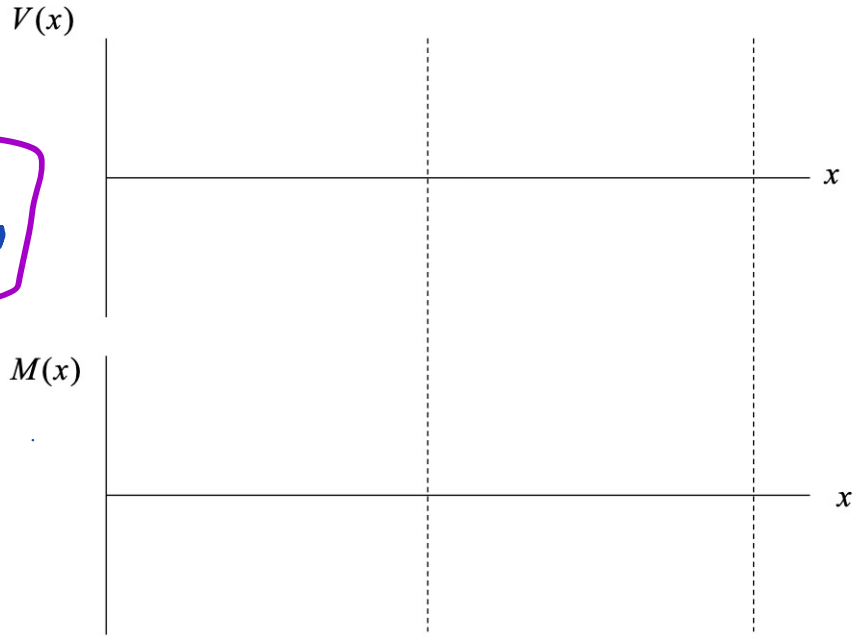
★ Always solve for reactions first



$\Sigma M_A = 0 \Rightarrow B_y$

$\Sigma F_y = 0 \Rightarrow A_y$

★ Recommend to start from "left"

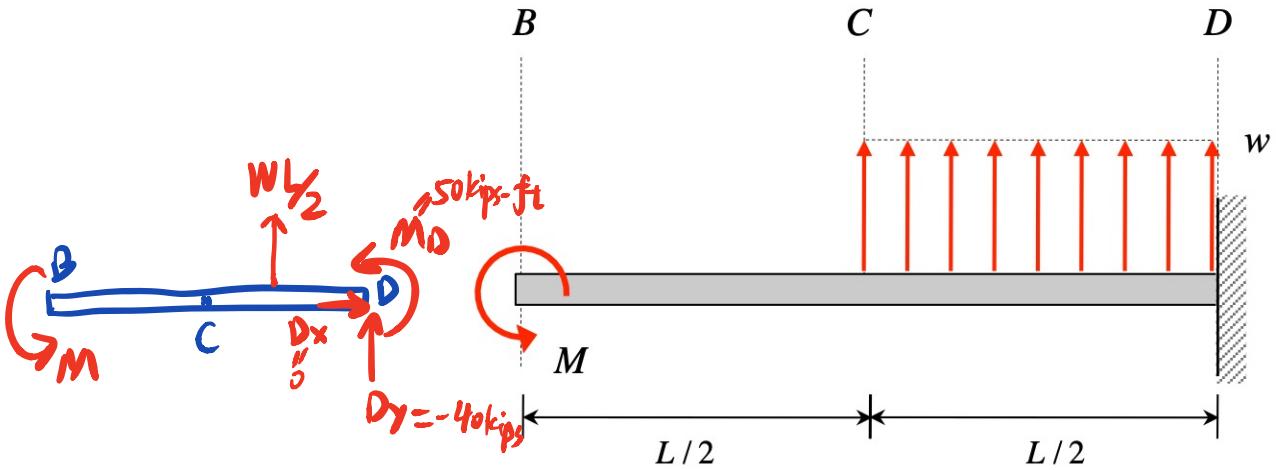


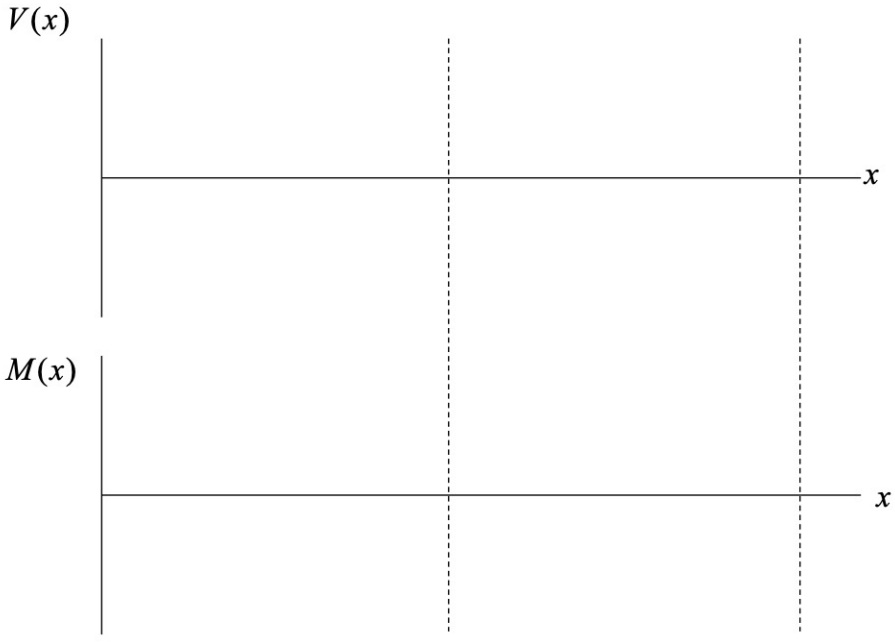
Homework H30.B

Given: Cantilevered beam with the line loading w acting over its length and a concentrated couple M acting at C.

Find: Construct the shear force and bending moment diagrams for this beam.

For this problem, use the following parameters: w = 8 kips/ft, M = 50 kips-ft and L = 10 ft.





Example 9.B.13

Given: Simply-supported beam loaded as shown.

Find: For this problem, use graphical methods to draw the shear force and bending moment diagrams.

Overall analysis:

$F_{eq} = 5(5) = 25\ kN$

$\Sigma M_B = 0$

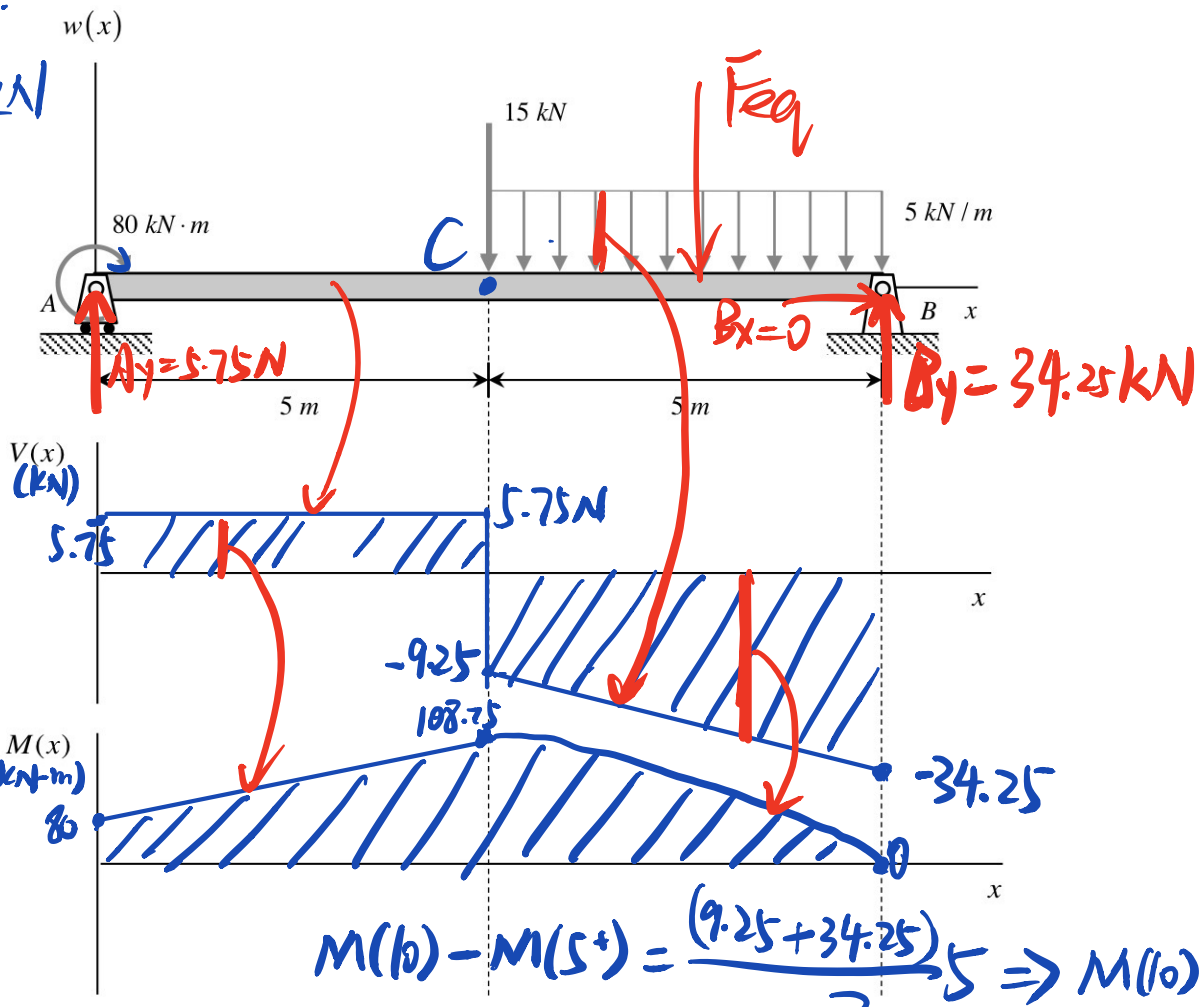
$= F_{eq}(2.5) - A_y(10)$

$\Rightarrow \boxed{A_y = 5.75\ N}$

$\Sigma F_y = 0$

$= A_y + B_y - F_{eq} - 15$

$\Rightarrow \boxed{B_y = 34.25\ N}$



$M(10) - M(5^+) = \frac{(9.25 + 34.25)}{2} 5 \Rightarrow M(10) = 0$

AC: $w(x) = 0 \rightarrow V(x)$: (+) constant $\rightarrow M(x)$: (+) constant slope

CB: $w(x)$: (−) constant $\rightarrow V(x)$: (−) constant slope
(−) increasing value $\rightarrow M(x)$: (−) increasing slope

A: $V(0) = A_y = 5.75\ kN$, $M(0) = 80\ kN \cdot m$

" ": area under curves

C: $V(5) = V(0) + 0 = 5.75\ kN$, $V(5^+) = V(5^-) - 15 = -9.25\ kN$

$M(5^-) = M(0) + 5.75(5) = 108.75\ kN \cdot m$, $M(5^+) = M(5^-) = 108.75\ kN \cdot m$

B: $V(10) = V(5^+) - 5(5) = -34.25\ kN$, $M(10) = M(5^+) - \frac{(9.25 + 34.25)5}{2} = 0$

Example 9.B.14

Given: Consider the beam shown below that is supported by a pin joint at A and a roller support at B, with the loading shown.

Find: For this problem, use graphical methods to draw the shear force and bending moment diagrams.

Overall analysis:

$F_{eq} = 4(2) = 8\ kN$

$\Sigma M_A = 0$

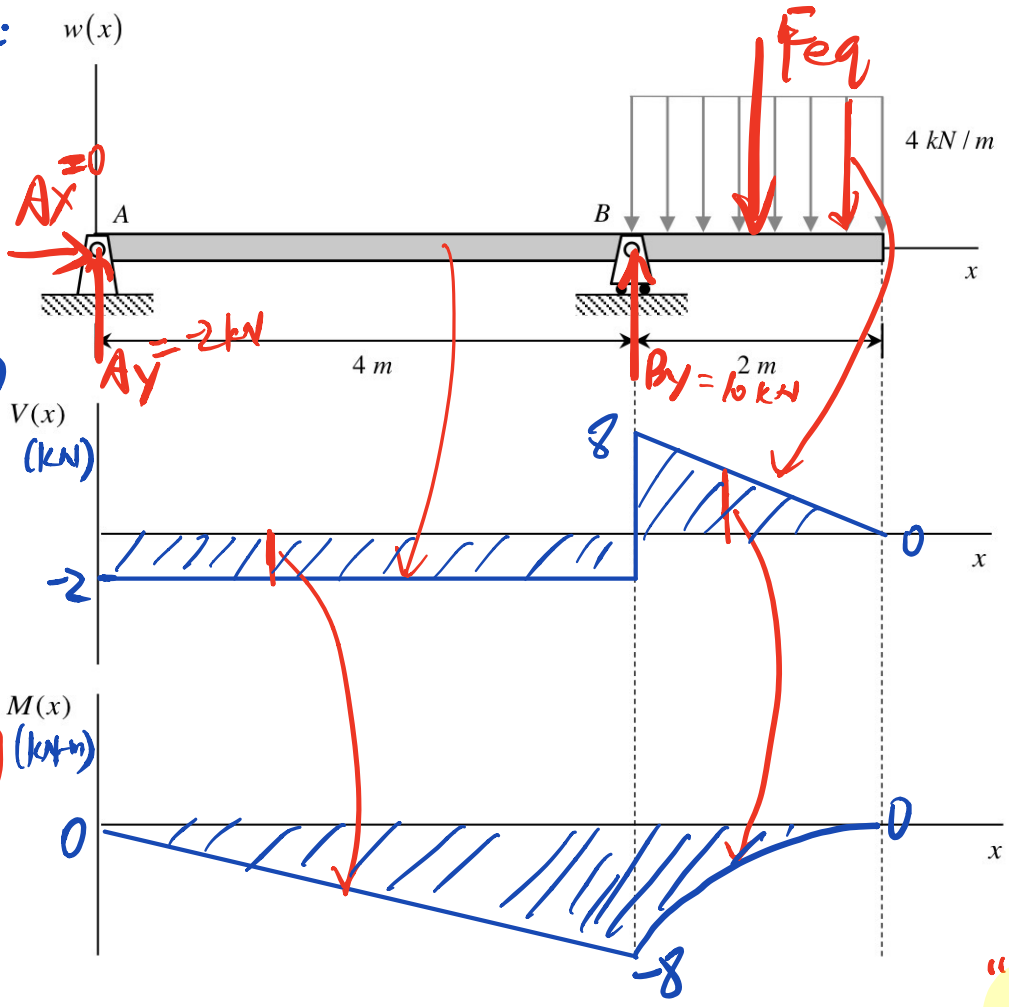
$= B_y(4) - F_{eq}(4+1)$

$\Rightarrow \boxed{B_y = 10\ kN}$

$\Sigma F_y = 0$

$= A_y + B_y - F_{eq}$

$\Rightarrow \boxed{A_y = -2\ kN}$



" " : area under curves

AB: $w(x) = 0 \rightarrow V(x)$: (-) constant $\rightarrow M(x)$: (-) constant slope

BC: $w(x)$: (-) constant $\rightarrow V(x)$: (-) constant slope

(+) decreasing value $\rightarrow M(x)$: (+) decreasing slope

A: $V(0) = A_y = -2kN$ $\quad M(0) = 0$

B: $V(4^-) = V(0) + 0 = -2kN$, $\quad V(4^+) = V(4^-) + B_y = 8\ kN$

$M(4^-) = M(0) + 2(4) = -8\ kN\text{-}m$, $\quad M(4^+) = M(4^-) = -8\ kN\text{-}m$

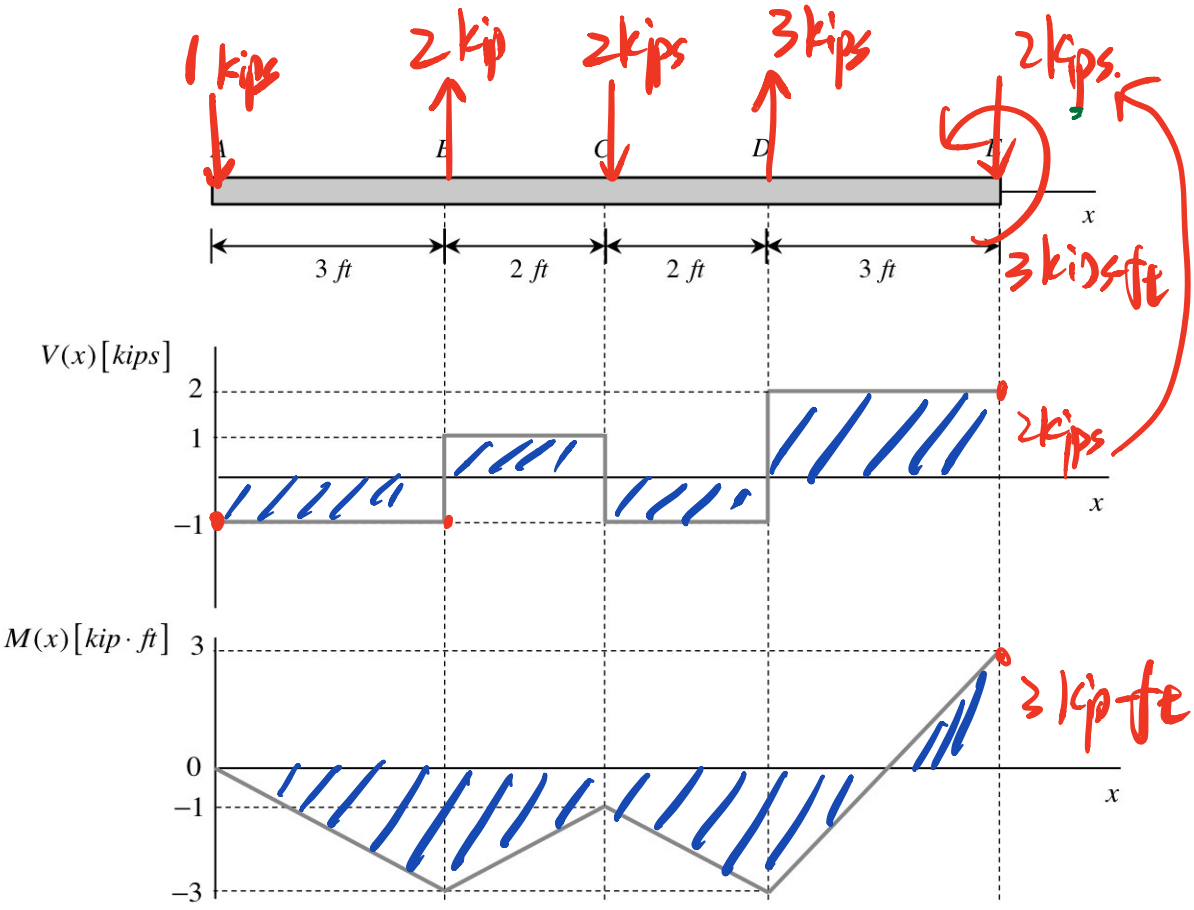
C: $V(6) = V(4^+) - 4(2) = 0$, $\quad M(6) = M(4^+) + \frac{1}{2}(2)(8) = 0$

Inverse Problem

Example 9.B.16

Given: The shear force and bending moment diagrams for a loaded beam are provided below.

Find: On the beam drawing provided, sketch a valid set of loads that would result in the shear force and bending moment diagrams provided.



ME 27000
BASIC MECHANICS I

Course Outcomes [Related ME Program Outcomes in brackets]

1. Develop an understanding of *static equilibrium* and *stresses in statically determinate structures* and how to apply them to engineering systems. [A1, A2]
2. Learn a systematic approach to *problem solving*. [A2]
3. Foster effective mathematical and graphical *communication skills*. [B1]

Statics of Rigid Bodies (9 wks)

Stresses in Statically-Determinate Structures (6 wks)

Fundamentals (2 wks)

1. Newton's Laws
2. Vector algebra; vector components
3. Position, unit and force vectors
4. Dot product
5. Cross product
6. Moment of a force about a point

Static Equilibrium (5 wks)

1. Equilibrium of a particle
2. Support reactions and free body diagrams
3. Static indeterminacy and partial constraints
4. 2-D and 3-D static equilibrium
5. Trusses – 0-force member
   - method of joints
   - method of sections
6. Frames and machines
7. Dry friction
   - Coulomb's Laws
   - Systems with friction
   - Sliding or tipping
   - Wedges

Equivalent Systems (2 wks)

1. Determination of the resultant of concurrent forces
2. Equivalent force/couple systems
3. Centroid and center of mass
   - by composite parts
   - by integration
4. Surface loadings
   - line loads
   - pressure distributions
5. Fluid statics
   - rectangular surfaces
   - curved surface

Introduction of Stress and Strain in Materials (1 wk)

1. Basic definitions of stress and strain
2. Mechanical properties of materials
3. Shear stress and strain

4. Internal force/couple system
   - General topic
   - Shear force & Bending moment

→ extension of rigid body S.E.
→ prepare for stress-strain analysis

Stress Analysis (5 wks)

1. Stress due to axial loading
2. Shear stress due to torsion
3. Shear force and moment diagrams
4. Second area moments for beams
5. Flexural stresses in beams
6. Stress analysis of beams

ME 270 – Basic Mechanics I

Fall 2019

Period		Date	Topic	Reading	Homework
			STATICS		
1	M	Aug. 19	Introduction, Unit Conversions	1.A-F	H1.A, H1.B
2	W	Aug. 21	Position, Unit, and Force Vectors	2.A-B	H2.A, H2.B
3	F	Aug. 23	Dot Product	2.A-E	H3.A, H3.B
4	M	Aug. 26	Particle Equilibrium (2-D)	3.A-F	H4.A, H4.B
5	W	Aug. 28	Particle Equilibrium (3-D)	3.A-F	H5.A, H5.B
6	F	Aug. 30	Moment About a Point	1.D,4.A-B	H6.A, H6.B
B	M	Sep. 2	**Labor Day**		
7	W	Sep. 4	Force Couples, Equivalent Systems	5.A-B	H7.A, H7.B
8	F	Sep. 6	Free Body Diagrams; Equilibrium of Rigid Bodies (2-D)	4.C-D	H8.A, H8.B
9	M	Sep. 9	Equilibrium of Rigid Bodies (2-D)	4.E-G	H9.A, H9.B
10	W	Sep. 11	Equilibrium of Rigid Bodies (3-D)	4.E-G	H10.A, H10.B
11	F	Sep. 13	Equilibrium of Rigid Bodies (3-D)	4.E-G	H11.A, H11.B
12	M	Sep. 16	Distributed Loading	5.D	H12.A, H12.B
13	W	Sep. 18	Centers of Mass of Centroids: By Composite Parts	5.C	H13.A, H13.B
14	F	Sep. 20	Centers of Mass of Centroids: By Integration	5.C	H14.A, H14.B
15	M	Sep. 23	Fluid Statics: Buoyancy	5.E-G	H15.A, H15.B
R	W	Sep. 25	**Review for Exam 1**		
E	Th	Sep. 26	**EXAM 1 (6:20 – 7:30 PM); (Covers Lectures 1-14)**	Ch. 1-5	None assigned
B	F	Sep. 27	**NO LECTURE**		
16	M	Sep. 30	Fluid Statics: Hydrostatic Loads (flat surface)	5.E-F	H16.A, H16.B
17	W	Oct. 2	Fluid Statics: Hydrostatic Loads (curved surface)	5.E-G	H17.A, H17.B
18	F	Oct. 4	Friction: General	6.A-B	H18.A, H18.B
B	M	Oct. 7	**OCTOBER BREAK**		
19	W	Oct. 9	Friction: Slipping-Tipping	6.C	H19.A, H19.B
20	F	Oct. 11	Friction: Flat Belts	6.D	H20.A, H20.B
21	M	Oct. 14	Friction: Wedges	6.E-G	H21.A, H21.B
22	W	Oct. 16	Trusses: Method of Joints	7.A-C	H22.A, H22.B
23	F	Oct. 18	Trusses: Method of Sections	7.E	H23.A, H23.B

Period		Date	Topic	Reading	Homework
24	M	Oct. 21	Trusses: Zero-Force Members	7.C-I	H24.A, H24.B
25	W	Oct. 23	Frames and Machines: Simple Systems	8.A-D	H25.A, H25.B
26	F	Oct. 25	Frames and Machines: Simple Systems & Review	8.A-D	H26.A, H26.B
R	M	Oct. 28	**Review for Exam 2**	Ch. 6-8	None assigned
E	Tu	Oct. 29	**EXAM 2 (6:20 – 7:30 PM); (Covers Lectures 15-26)**		
B	W	Oct. 30	**NO LECTURE**		
27	F	Nov. 1	Internal Force/Couple Analysis	9.A	H27.A, H27.B
28	M	Nov. 4	Shear Force and Bending Moment Diagrams (Pt. Loads)	9.B	H28.A, H28.B
29	W	Nov. 6	Shear Force and Bending Moment Diagrams (Dist. Loads)	9.B	H29.A, H29.B
30	F	Nov. 8	Shear Force and Bending Moment Diagrams (Graph. Meth.)	9.B-E	H30.A, H30.B
31	M	Nov. 11	Stress-Strain Curves; Axial Stress and Strain	10.A-F	H31.A, H31.B
32	W	Nov. 13	Axial Stress and Strain; Factor of Safety	10.A-F	H32.A, H32.B
33	F	Nov. 15	Shear Stress and Strain; Direct Shear	11.A-D	H33.A, H33.B
34	M	Nov. 18	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E	H34.A, H34.B
35	W	Nov. 20	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E, F	H35.A, H35.B
36	F	Nov. 22	Flexural Stresses in Beams	12.A	H36.A, H36.B
37	M	Nov. 25	Flexural Stresses in Beams	12.A	H37.A, H37.B
B	W	Nov. 27	**THANKSGIVING**		
B	F	Nov. 29	**THANKSGIVING**		
38	M	Dec. 2	Second Moments of Area: By Composite Parts	12.B	H38.A, H38.B
39	W	Dec. 4	Second Moments of Area: By Integration	12.B, D	H39.A, H39.B
R	F	Dec. 6	**Review for Final Exam**	Ch. 1-12	Practice Exams
E	Tu	Dec. 10	**FINAL EXAM (10:30 AM – 12:30 PM); (Covers Lectures 1-39)**		

Coding: Integer = Lecture number; B = Break; E = Exam; R = Review lecture.
Homework numbers correspond to lecture numbers. Review lectures do not increase counter. Homework is due the class period after it is assigned.

TEXTS
ME 270 textbook ("Statics: A Lecturebook", 2nd Edition, Fall 2019).

ME 270 BLOG
You will automatically be added to the blog once you have registered for ME 270. Once you have access to the blog, you can adjust your email settings to receive all, some or none of the posting, as according to your preference.

Lec 31. 11/11/2019 (Mon.)

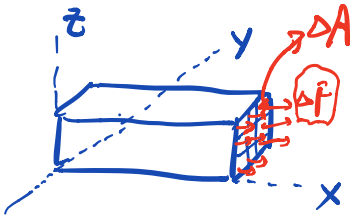
Normal Stress and Axial Strains

Normal Stress σ - The stress acting normal to a surface along an axis of a member.

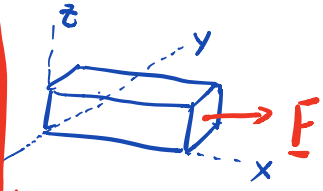
- due to axial loading
- normal to cross-sectional surface

$$\sigma = \text{Normal Stress} = \lim_{\Delta A \to 0} \left(\frac{\Delta F}{\Delta A} \right) = \frac{dF}{dA}$$

$$F = \int_A \sigma(y,z)\, dA = \sigma_{AVG}\, A$$



$$\sigma_{AVG} = \frac{F}{A} = \frac{\text{Axial Load}}{\text{Cross-Sectional Area}}$$



where the units are given by:

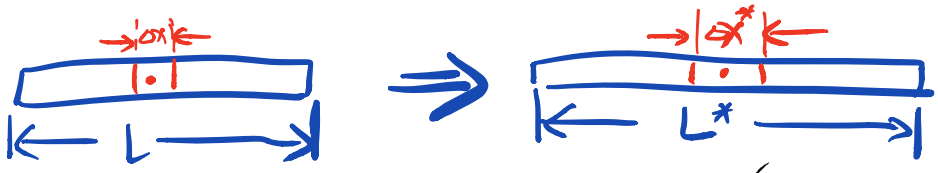
$$\sigma_{AVG} = \frac{\text{lbs}}{\text{in}^2} = \text{psi} \quad \text{or} \quad \frac{\text{N}}{\text{m}^2} = \text{Pa}$$

$1\ \text{ksi} = 10^3\ \text{si}$

Note:

$\sigma_{AVG} > 0$ (Tension); $\sigma_{AVG} < 0$ (Compression)

Axial Strain (ε) – The elongation (or shortening) of a part along an axis.



$$\varepsilon_x = \text{Axial Strain} = \lim_{\Delta x \to 0} \left(\frac{\Delta x^* - \Delta x}{\Delta x} \right)$$

$$(\varepsilon_x)_{AVG} = \text{Avg. Axial Strain} = \frac{L^* - L}{L} = \frac{L^*}{L} - 1$$

$$\varepsilon_{avg} = \frac{L^*}{L} - 1 \ ,$$

Hooke's Law

"usually used to evaluate ε when σ is known"

Hooke's Law – The nearly linear relationship between normal stress and axial strain in the proportional region (typically under low strains)

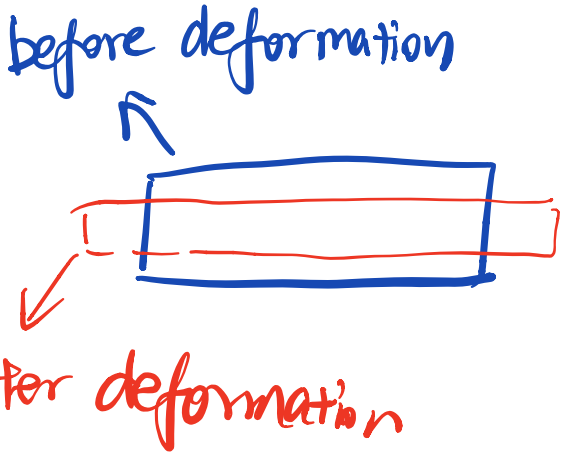
$$\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E}$$

where

E = Modulus of Elasticity or Young's Modulus, which is a measure of the stiffness of a material (E has units of psi or Pa).

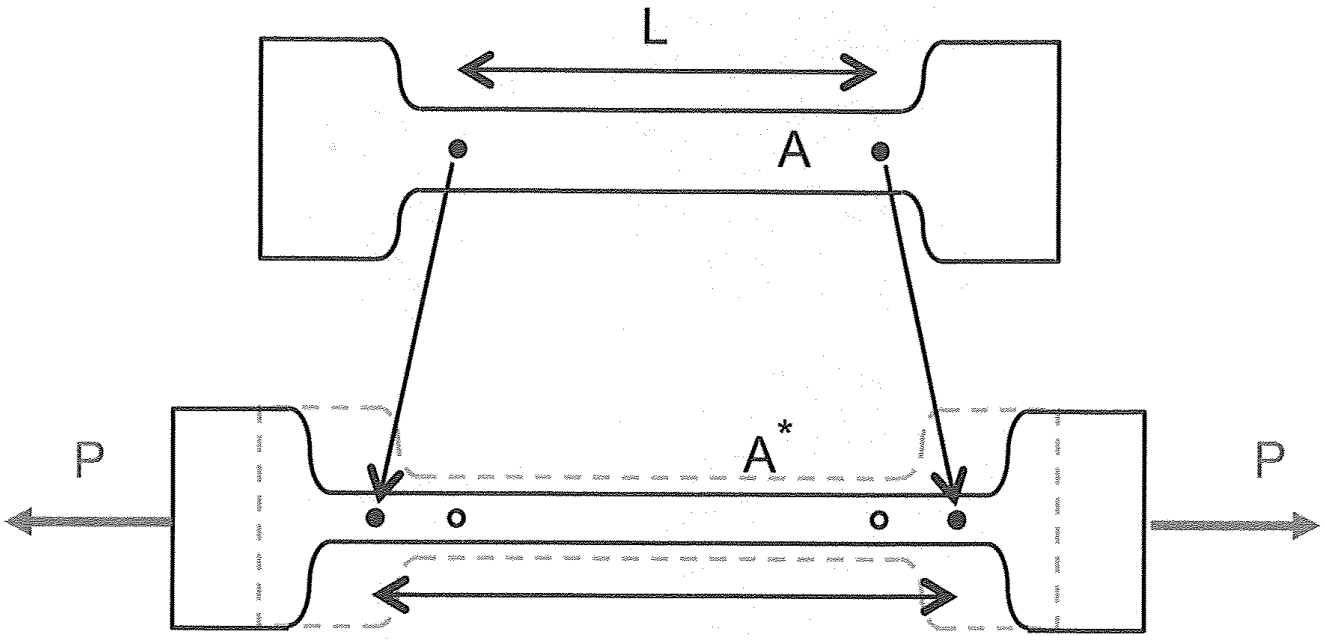
Elongation is the axial direction causes contraction in transverse directions.

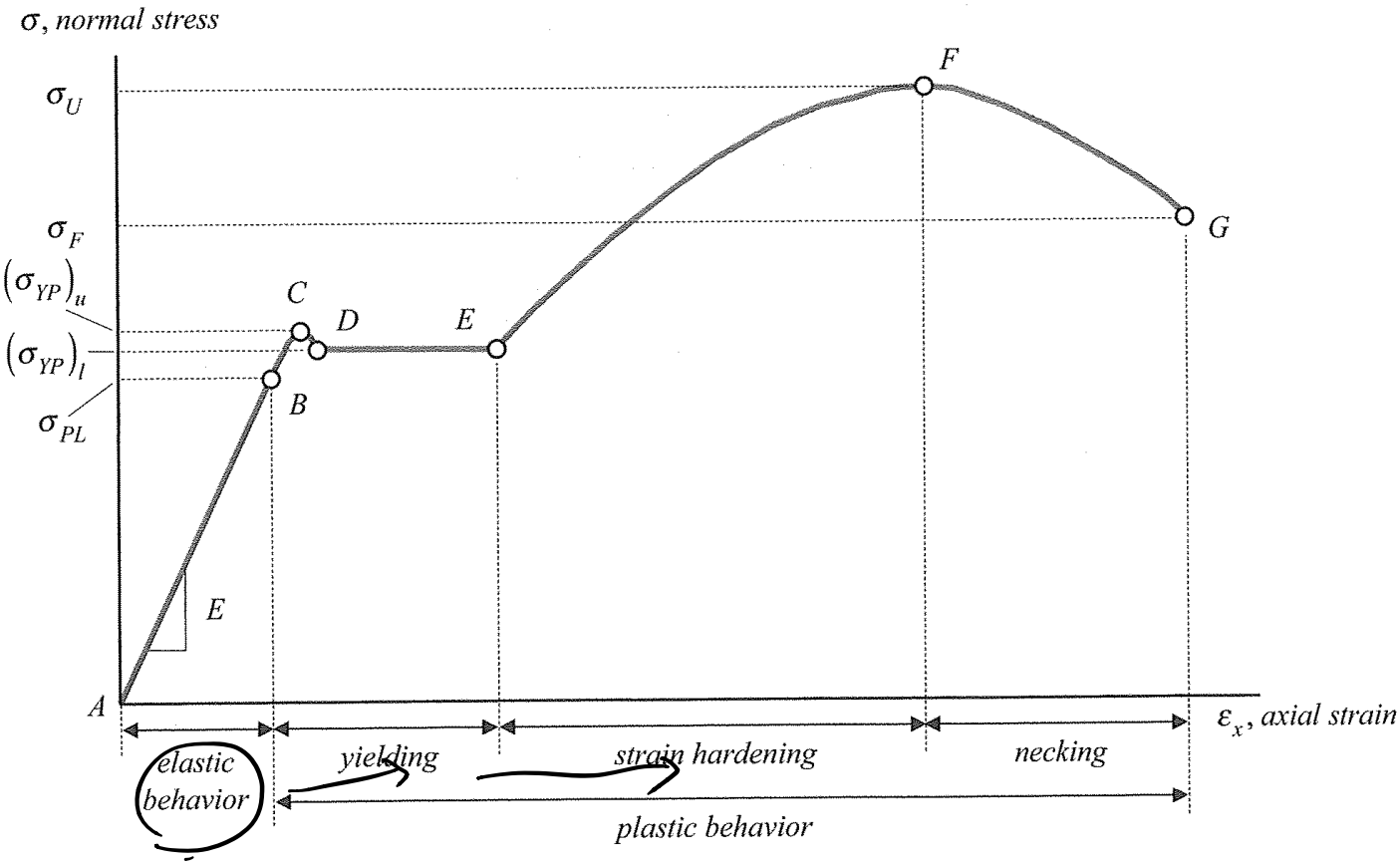
$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x$$



where

ν = Poisson's ratio (dimensionless)


L
A
A*
P
P


σ, normal stress
σU
σF
(σYP)u
(σYP)l
σPL
A
B
C
D
E
F
G
E
εx, axial strain
elastic behavior
yielding
strain hardening
necking
plastic behavior

Stress-Strain Regions:

Proportional Region (A-B): Nearly linear relationship between σ and ε, whose slope is E. Hooke's Law is valid up to the proportional limit (σ_{PL}).

Elastic Region (A-C): Similar to the proportional region except between B and C the material is still behaving elastically up to the upper yield point $(\sigma\gamma_P)_u$, but is not linear (Hooke's Law is not valid).

Yielding Regions (C-E): Plastic deformation begins. A slight material "softening" occurs from C-D, down to the lower yield point $(\sigma\gamma_P)_l$. Perfectly plastic behavior occurs from D-E as the material strain increases without an increase in stress.

Strain Hardening Region (E-F): Strain hardening occurs from E-F up to the "ultimate stress" (σ_U).

Necking Region (F-G): Above σ_U, the material "necks down" resulting in a significant reduction in cross-sectional area and ultimately in fracture (G) at the fracture stress (σ_F).

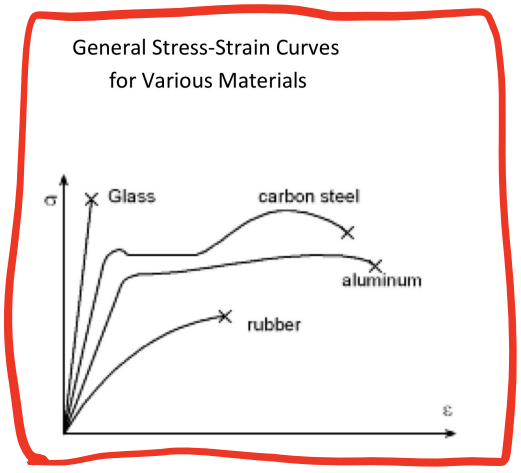


Table 14.1: Experimentally-Determined Values of Modulus and Poisson's Ratio

Material	Young's Modulus (10^3 ksi)	Young's Modulus (GPa)	Poisson's Ratio
Alumnium alloy 2014-T6	10.6	73	0.33
Aluminum alloy 6061-T6	10.0	70	0.33
Brass, cold-rolled	15	100	0.34
Brass, annealed	15	100	0.34
Cast iron, gray	10	70	0.22
Steel, ASTM-A36 structural	29	200	0.29
Steel, AISI 302 stainless	29	195	0.30
Titanium, alloy	16.5	115	0.33
Wood, Douglas Fir	1.75	12	–
Wood, Southern Pine	1.75	12	–

Table 14.2: Experimentally-Determined Values of Failure Properties

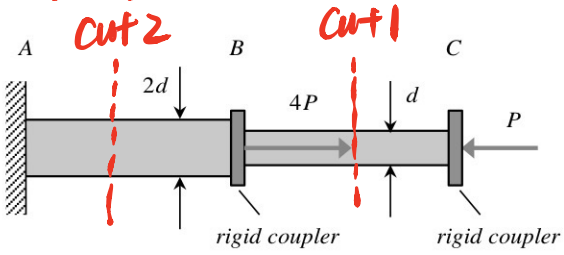
(10^3 ksi)	Yield Strength (ksi)	Yield Strength (MPa)	Ultimate Strength (ksi)	Ultimate Strength (MPa)
Alumnium alloy 2014-T6	60	410	70	480
Aluminum alloy 6061-T6	40	275	45	310
Brass, cold-rolled	60	410	75	520
Brass, annealed	15	100	40	275
Cast iron, gray	–	–	25	170
Steel, ASTM-A36 structural	36	250	58	400
Steel, AISI 302 stainless	75	520	125	860
Titanium, alloy	120	830	130	900
Wood, Douglas Fir	–	–	7.5	60
Wood, Southern Pine	–	–	8.5	60

Example 10.D.1

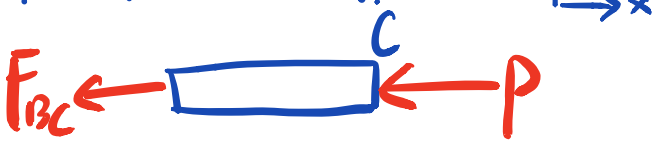
Given: A circular cross-section solid annealed brass rod AB and a circular cross-section solid aluminum alloy 2014-T6 rod BC are connected together by a coupler at B.

Find: Determine the axial stresses in rods AB and BC. Although you were given the material makeup of the two components of the rod, do you really need that information in order to solve the problem?

Use the following parameters in your analysis: $d_1 = 2.5$ in and $d_2 = 2.0$ in.



Based on Cut 1



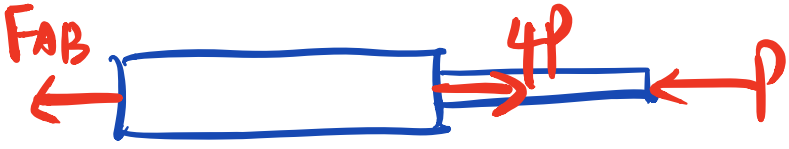
$\Sigma F_x = 0 = -P - F_{BC} \Rightarrow F_{BC} = -P$ (C)

$$\sigma_{BC} = \frac{F_{BC}}{\text{Area}_1} = \frac{-P}{\frac{\pi d^2}{4}} \Rightarrow$$

Based on Table

$$\varepsilon_{BC} = \frac{\sigma_{BC}}{E}$$

Based on Cut 2



★ Be Careful of the unit

$\Sigma F_x = 0 = -P - F_{AB} + 4P \Rightarrow F_{AB} = 3P$ (T)

$$\sigma_{AB} = \frac{F_{AB}}{\text{Area}_2} = \frac{3P}{\frac{\pi (2d)^2}{4}} \Rightarrow$$

Based on Table

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E}$$

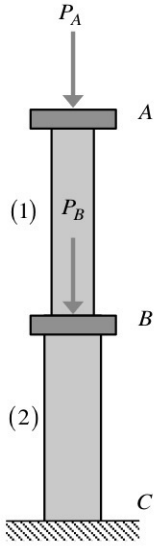
Self-practice at home

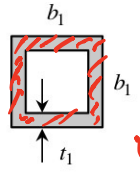
Example 10.E.2

Given: A column used in the construction of a two-story building is made up from square cross-section tubing having dimensions shown below. Axial loads of P_A and P_B act on the column. Neglect the weight of the column.

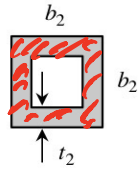
(a) Determine the axial stress σ_1 in segment (1) of the column.
(b) Determine the axial stress σ_2 in segment (2) of the column.

Use the following parameters in your analysis: $b_1 = 150$ mm, $b_2 = 200$ mm, $t_1 = 10$ mm, $t_2 = 12$ mm, $P_A = 200$ kN and $P_B = 350$ kN.





column cross section for (1)



column cross section for (2)

★ Be careful of Cross-sectional area calculation

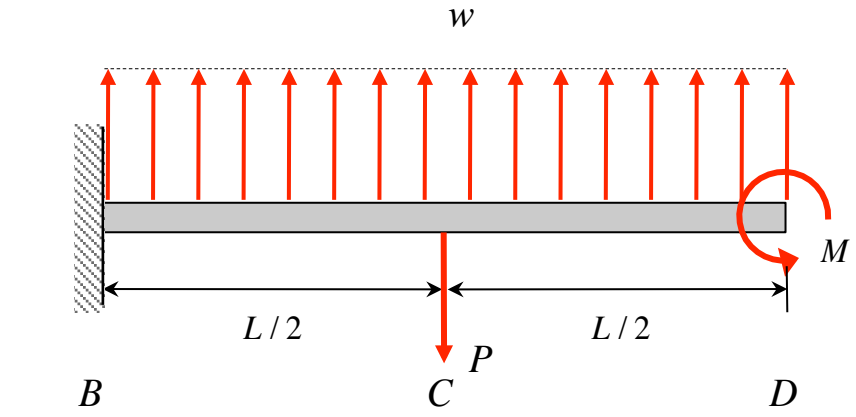
★ Be careful of the unit → $\frac{N}{m^2} \sim Pa$

→ $\frac{lb}{in^2} \sim psi$

(ksi = 10^3 psi)

Group Quiz **Group #:**_____ **Names:** _____ _____ _____ _____

Given: Cantilevered beam with the line loading w acting over its length, a concentrated force P acting at C and a concentrated couple M acting at end D.

Find: Construct the shear force and bending moment diagrams for this beam.

For this problem, use the following parameters: $w = 10$ kN/m, $P = 40$ kN, $M = 80$ kN-m and $L = 4$ m.