Homework H19.A

Given: An automobile is attempting to tow a sled having a weight of W and whose contact with the ground has a coefficient of static friction of μ_S. The center of mass of the automobile and driver is located at point G on the automobile, with the total weight of the automobile and driver being W_C. The automobile has *rear wheel drive*, with a coefficient of static friction of $2\mu_S$ between the rear tires and the ground and with the front wheels being free to roll.

Find: For this problem:

means impending motion

a) Determine the maximum weight W of the sled that the automobile can initiate the towing of the sled without slipping between the tires and ground, and without the front wheels losing contact with the ground. Express your answer in terms of W_C.

$f = \mu_S N$

impending slipping

OR

impending tipping

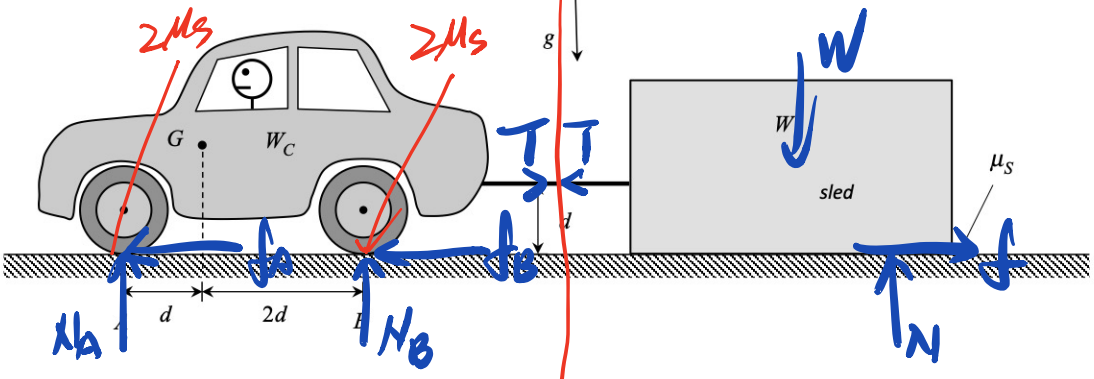
b) For the sled weight found in a) above, is the automobile on the verge of having the tires slip on the ground or of having the front wheels losing contact with the ground? Explain.

$f \neq \mu_S N$ $(< \mu_S N)$

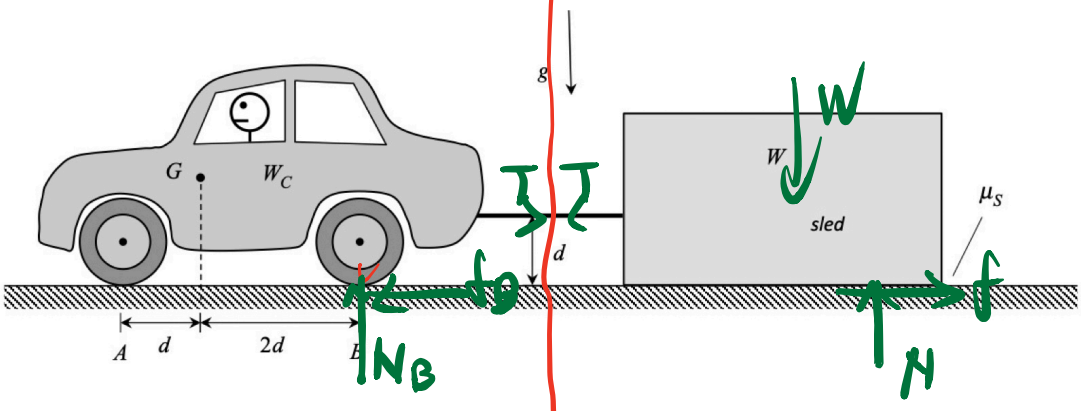
For this problem, use the following parameter value: $\mu_S = 0.50$.

Look 2 rigid bodies separately

impending slipping



impending tipping



Homework H19.B

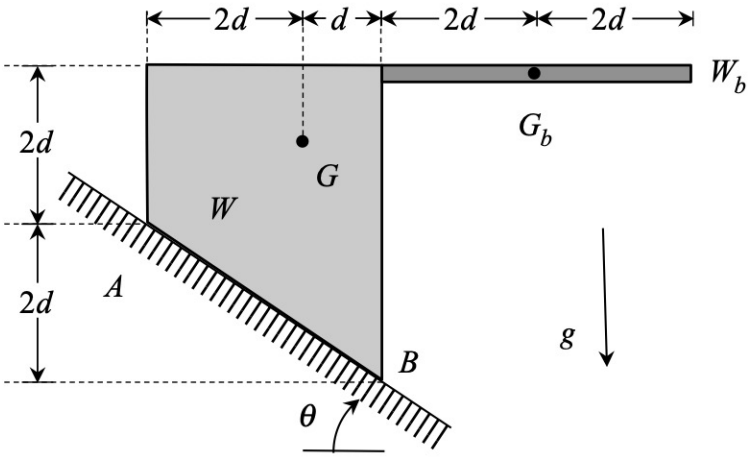
Given: An inhomogeneous, trapezoidal-shaped block has a weight of W and its center of mass at point G. The block is at rest on a rough, inclined surface, with the coefficient of static friction between the block and the inclined surface being μ_s. A homogeneous bar with a weight of W_b is then attached to the right side of the block, as shown below.

Find: For this problem:

a) Determine the maximum weight of the bar for which the system remains in equilibrium. Express you answer in terms of W.

b) For the weight found above in a), is the block in a state of impending slipping or tipping? Explain.

For this problem, use the following parameter value: $\theta = 30°$ and $\mu_s = 0.70$.



ME 27000
BASIC MECHANICS I

Course Outcomes [Related ME Program Outcomes in brackets]

1. Develop an understanding of *static equilibrium* and *stresses in statically determinate structures* and how to apply them to engineering systems. [A1, A2]
2. Learn a systematic approach to *problem solving*. [A2]
3. Foster effective mathematical and graphical *communication skills*. [B1]

Statics of Rigid Bodies (9 wks)

Stresses in Statically-Determinate Structures (6 wks)

Fundamentals (2 wks)

1. Newton's Laws
2. Vector algebra; vector components
3. Position, unit and force vectors
4. Dot product
5. Cross product
6. Moment of a force about a point

Static Equilibrium (5 wks)

1. Equilibrium of a particle
2. Support reactions and free body diagrams
3. Static indeterminacy and partial constraints
4. 2-D and 3-D static equilibrium
5. Trusses
   - method of joints
   - method of sections
6. Frames and machines
7. Dry friction
   - Coulomb's Laws
   - Systems with friction
   - Sliding or tipping
   - Wedges

Equivalent Systems (2 wks)

1. Determination of the resultant of concurrent forces
2. Equivalent force/couple systems
3. Centroid and center of mass
   - by composite parts
   - by integration
4. Surface loadings
   - line loads
   - pressure distributions
5. Fluid statics
   - rectangular surfaces
   - curved surface

Introduction of Stress and Strain in Materials (1 wk)

1. Basic definitions of stress and strain
2. Mechanical properties of materials
3. Shear stress and strain

Stress Analysis (5 wks)

1. Stress due to axial loading
2. Shear stress due to torsion
3. Shear force and moment diagrams
4. Second area moments for beams
5. Flexural stresses in beams
6. Stress analysis of beams

ME 270 – Basic Mechanics I

Fall 2019

Period		Date	Topic	Reading	Homework
			STATICS		
1	M	Aug. 19	Introduction, Unit Conversions	1.A-F	H1.A, H1.B
2	W	Aug. 21	Position, Unit, and Force Vectors	2.A-B	H2.A, H2.B
3	F	Aug. 23	Dot Product	2.A-E	H3.A, H3.B
4	M	Aug. 26	Particle Equilibrium (2-D)	3.A-F	H4.A, H4.B
5	W	Aug. 28	Particle Equilibrium (3-D)	3.A-F	H5.A, H5.B
6	F	Aug. 30	Moment About a Point	1.D,4.A-B	H6.A, H6.B
B	M	Sep. 2	**Labor Day**		
7	W	Sep. 4	Force Couples, Equivalent Systems	5.A-B	H7.A, H7.B
8	F	Sep. 6	Free Body Diagrams; Equilibrium of Rigid Bodies (2-D)	4.C-D	H8.A, H8.B
9	M	Sep. 9	Equilibrium of Rigid Bodies (2-D)	4.E-G	H9.A, H9.B
10	W	Sep. 11	Equilibrium of Rigid Bodies (3-D)	4.E-G	H10.A, H10.B
11	F	Sep. 13	Equilibrium of Rigid Bodies (3-D)	4.E-G	H11.A, H11.B
12	M	Sep. 16	Distributed Loading	5.D	H12.A, H12.B
13	W	Sep. 18	Centers of Mass of Centroids: By Composite Parts	5.C	H13.A, H13.B
14	F	Sep. 20	Centers of Mass of Centroids: By Integration	5.C	H14.A, H14.B
15	M	Sep. 23	Fluid Statics: Buoyancy	5.E-G	H15.A, H15.B
R	W	Sep. 25	**Review for Exam 1**		
E	Th	Sep. 26	**EXAM 1 (6:20 – 7:30 PM); (Covers Lectures 1-14)**	Ch. 1-5	None assigned
B	F	Sep. 27	**NO LECTURE**		
16	M	Sep. 30	Fluid Statics: Hydrostatic Loads (flat surface)	5.E-F	H16.A, H16.B
17	W	Oct. 2	Fluid Statics: Hydrostatic Loads (curved surface)	5.E-G	H17.A, H17.B
18	F	Oct. 4	Friction: General	6.A-B	H18.A, H18.B
B	M	Oct. 7	**OCTOBER BREAK**		
19	W	Oct. 9	Friction: Slipping-Tipping	6.C	H19.A, H19.B
20	F	Oct. 11	Friction: Flat Belts	6.D	H20.A, H20.B
21	M	Oct. 14	Friction: Wedges	6.E-G	H21.A, H21.B
22	W	Oct. 16	Trusses: Method of Joints	7.A-C	H22.A, H22.B
23	F	Oct. 18	Trusses: Method of Sections	7.E	H23.A, H23.B

Period		Date	Topic	Reading	Homework
24	M	Oct. 21	Trusses: Zero-Force Members	7.C-I	H24.A, H24.B
25	W	Oct. 23	Frames and Machines: Simple Systems	8.A-D	H25.A, H25.B
26	F	Oct. 25	Frames and Machines: Simple Systems	8.A-D	H26.A, H26.B
R	M	Oct. 28	**Review for Exam 2**	Ch. 6-8	None assigned
E	Tu	Oct. 29	**EXAM 2 (6:30 – 7:30 PM); (Covers Lectures 16-26)**		
B	W	Oct. 30	**NO LECTURE**		
27	F	Nov. 1	Internal Force/Couple Analysis	9.A	H27.A, H27.B
28	M	Nov. 4	Shear Force and Bending Moment Diagrams (Pt. Loads)	9.B	H28.A, H28.B
29	W	Nov. 6	Shear Force and Bending Moment Diagrams (Dist. Loads)	9.B	H29.A, H29.B
30	F	Nov. 8	Shear Force and Bending Moment Diagrams (Graph. Meth.)	9.B-E	H30.A, H30.B
31	M	Nov. 11	Stress-Strain Curves; Axial Stress and Strain	10.A-F	H31.A, H31.B
32	W	Nov. 13	Axial Stress and Strain; Factor of Safety	10.A-F	H32.A, H32.B
33	F	Nov. 15	Shear Stress and Strain; Direct Shear	11.A-D	H33.A, H33.B
34	M	Nov. 18	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E	H34.A, H34.B
35	W	Nov. 20	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E, F	H35.A, H35.B
36	F	Nov. 22	Flexural Stresses in Beams	12.A	H36.A, H36.B
B	M	Nov. 25	Flexural Stresses in Beams	12.A	H37.A, H37.B
B	W	Nov. 27	**THANKSGIVING**		
37	F	Nov. 29	**THANKSGIVING**		
38	M	Dec. 2	Second Moments of Area: By Composite Parts	12.B	H38.A, H38.B
39	W	Dec. 4	Second Moments of Area: By Integration	12.B, D	H39.A, H39.B
R	F	Dec. 6	**Review for Final Exam**	Ch. 1-12	Practice Exams
E			**FINAL EXAM (details to be announced); (Covers Lectures 1-39)**		

Coding: Integer = Lecture number; B = Break; E = Exam; R = Review lecture.
Homework numbers correspond to lecture numbers. Review lectures do not increase counter. Homework is due the class period after it is assigned.

TEXTS
ME 270 textbook ("Statics: A Lecturebook", 2nd Edition, Fall 2019).

ME 270 BLOG
You will automatically be added to the blog once you have registered for ME 270. Once you have access to the blog, you can adjust your email settings to receive all, some or none of the posting, as according to your preference.

Lec 20 10/11/2019 (Fri.)

FLEXIBLE FLAT BELTS

"Bad friction" : lubrication system
"Good friction" : tire. walk, belf system

Learning Objectives

1) To analyze the *forces* needed to lift and lower an object with a *cable system*.

2) To determine the *maximum torque* which can be applied to a pulley system without causing the belt to slip.

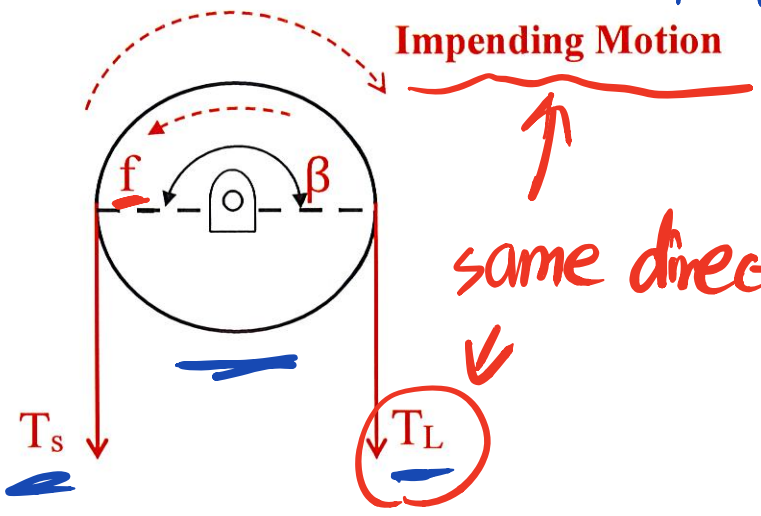
Definition

ideal pulley
smooth

Belts: devices designed to transfer forces between bodies (e.g., pulleys, sheaves, drums, etc.).

T T

Flat Belts



same direction

$$\frac{T_L}{T_s} = e^{\mu\,\beta}$$

where

T_L = larger tension

T_s = smaller tension

β = angle of wrap (in radians)

$\mu = u_s$ (static coefficient of friction) for a *non-slipping belt*

$= u_k$ (kinetic coefficient of friction) for a *slipping belt*

$\frac{T_L}{T_S} = e^{\mu\beta}$

$90^\circ \rightarrow \frac{\pi}{2}$

Remarks

1) The angle of wrap (β), must be measured in radians, not degrees!

2) The larger tension (T_L) is always in the direction of impending motion.

3) The angle of wrap (β) is easier to determine when measured from a common *angle* close to β.

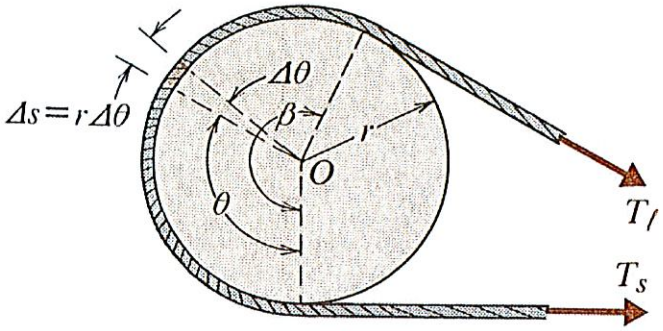
4) The equations above are valid for *impending slipping* or *slipping* only.

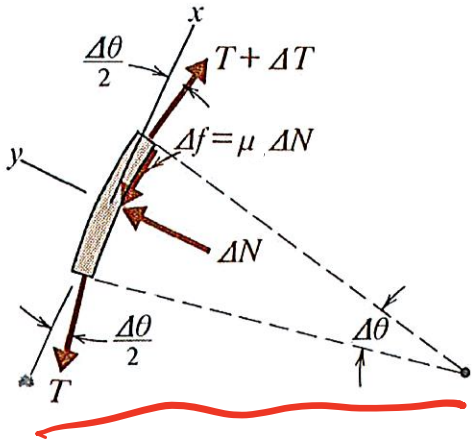
μ_s μ_k

5) As μ and/or β increase, the difference between T_L and T_S increases.

6) The tension ratio $\frac{T_L}{T_S}$ is independent of the pulley radius.

Proof: where "$\frac{T_L}{T_S} = e^{\mu\beta}$" comes from





① $$\begin{cases} \sum F_x = -T\cos\dfrac{\Delta\theta}{2} + (T+\Delta T)\cos\dfrac{\Delta\theta}{2} - \mu\ \Delta N = 0 \\ \sum F_y = -T\sin\dfrac{\Delta\theta}{2} - (T+\Delta T)\sin\dfrac{\Delta\theta}{2} + \Delta N = 0 \end{cases}$$

simplify & approximate

② $$\begin{cases} \Delta T\cos\dfrac{\Delta\theta}{2} = \mu\ \Delta N \\ (2T+\Delta T)\sin\dfrac{\Delta\theta}{2} = \Delta N \end{cases}$$

$\cos\frac{\Delta\theta}{2} \to 1$, $\Delta T \to 0$, $\sin\frac{\Delta\theta}{2} \to \frac{\Delta\theta}{2}$

} further approximation when θ is small

③ $$\begin{cases} dT = \mu\, dN \\ T\,d\theta = dN \end{cases}$$

④ $$\frac{dT}{T} = \mu\, d\theta$$

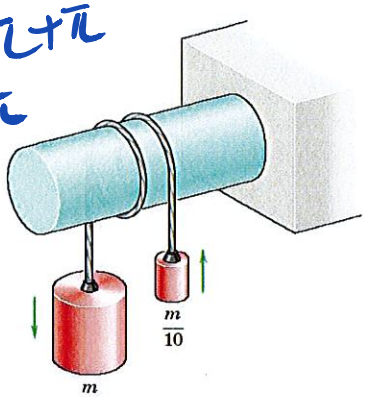
Integral on both sides

⑤ $$\int_{T_s}^{T_L}\frac{dT}{T} = \int_0^{\beta}\mu\, d\theta$$

⑥ $$\frac{T_L}{T_s} = e^{\mu\beta}$$

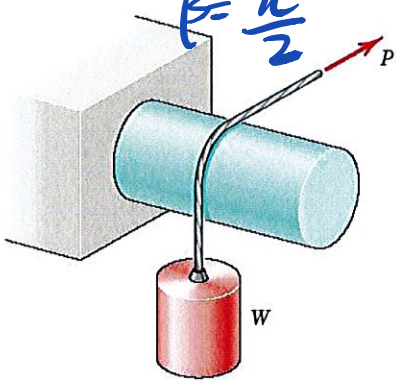
Practice: Identify "β" (in radians!)

$\beta = 2\pi + \pi = 3\pi$



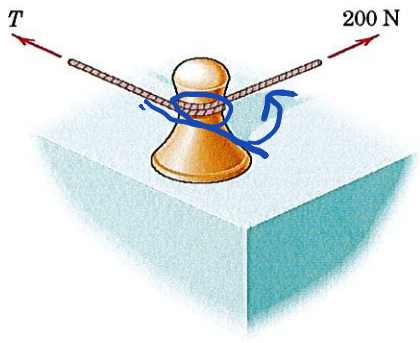
Problem 6/93

$\beta = \frac{\pi}{2}$



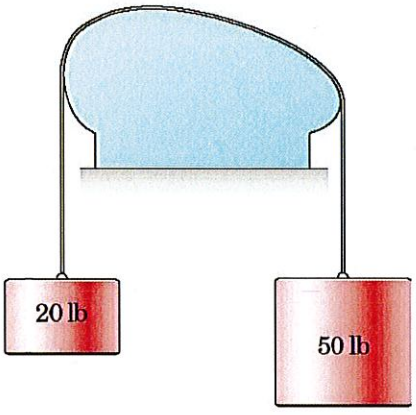
Problem 6/94

$\beta = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$



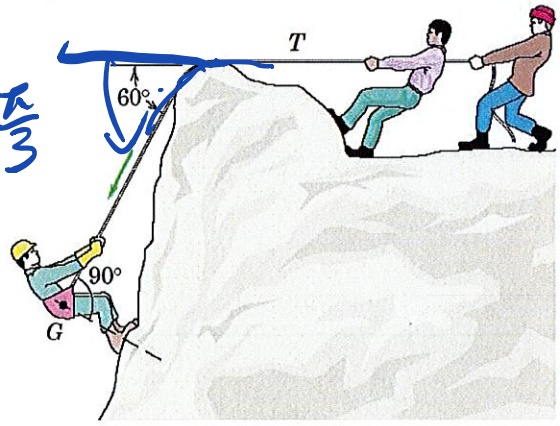
Problem 6/95

$\beta = \pi$



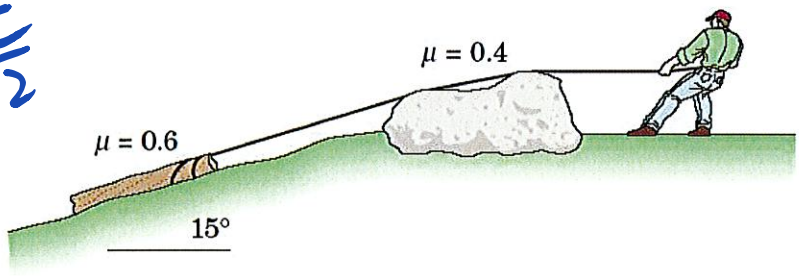
Problem 6/97

$\beta = \frac{\pi}{3}$

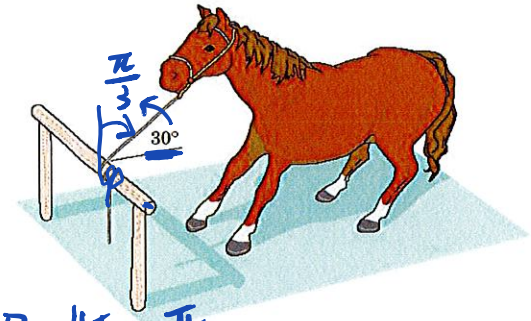


Problem 6/98

$\beta = \frac{\pi}{12}$



Problem 6/99



$\beta = 4\pi + \frac{\pi}{3} = \frac{13\pi}{3}$

Problem 6/100

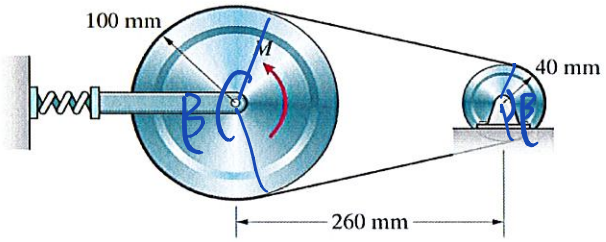


Figure: 09-19-17P9.135 - Problem 9.135

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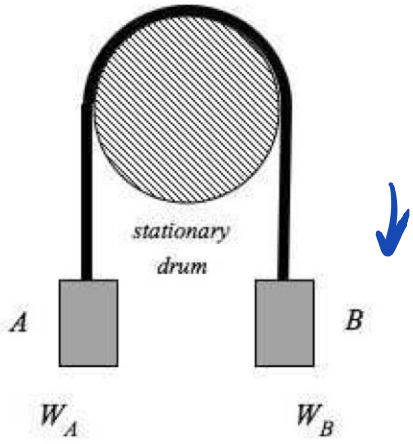
Example 6.D.1

Given: The coefficient of static friction between the rope and the drum is known to be $\mu_s = 0.5$. Block B weighs $W_B = 100$ N.

Find: Determine the range of weights for block A so that the system remains in equilibrium.

$$\frac{T_L}{T_S} = e^{\mu_s \beta} = e^{(0.5)(\pi)} = 4.8$$



① $W_B > W_A$

$T_L = W_B$, $T_S = W_A$

$$\frac{W_B}{W_A} = 4.8 \Rightarrow W_A = W_{A\,min} = 21\,N$$

② $W_B < W_A$

$T_L = W_A$, $T_S = W_B$

$$\frac{W_A}{W_B} = 4.8 \Rightarrow W_A = W_{A\,max} = 480\,N$$

$$21\,N \leq W_A \leq 480\,N$$

Example 6.D.2

Given: During a particular rescue mission, a rescue worker weighing $W = 180$ lb is to be raised and lowered over the edge of a cliff. The rope attached to the worker is wrapped over 90° of the cliff's edge and a one full wrap around a vertical post at the top of the cliff.

Find: For this problem:

(a) Determine the minimum force P required to hold the rescue worker in equilibrium.

(b) Determine the minimum force P required to initiate upward motion of the rescue worker.

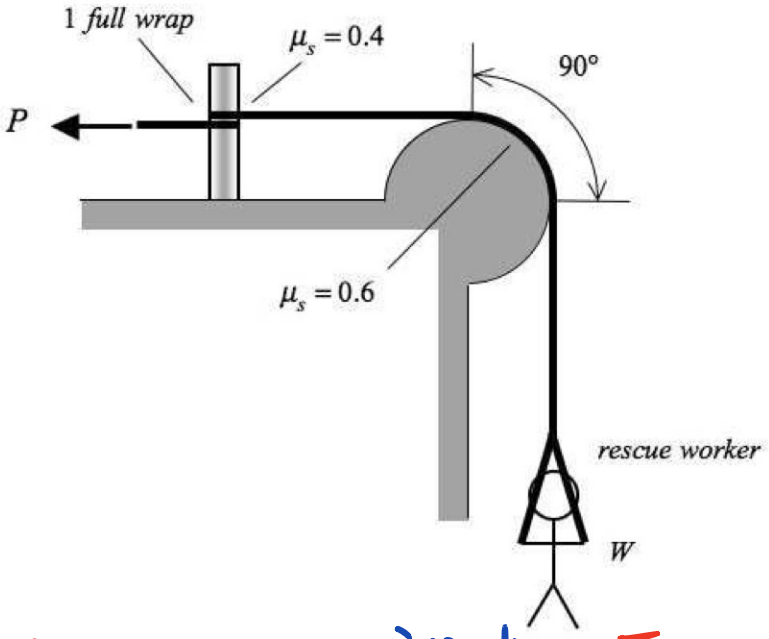
(a) To hold:

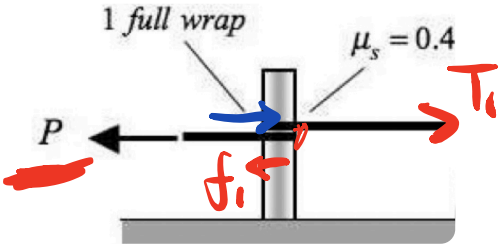
$T_1 > P$

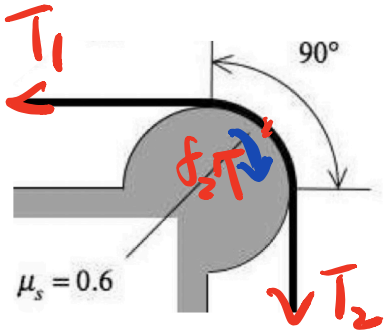
$(T_1 = P + f_1)$

$W = T_2 > T_1$

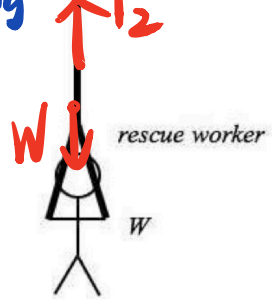
$(T_2 = T_1 + f_2)$







impending

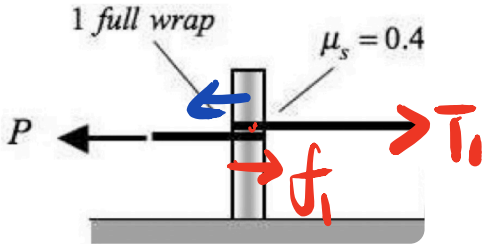


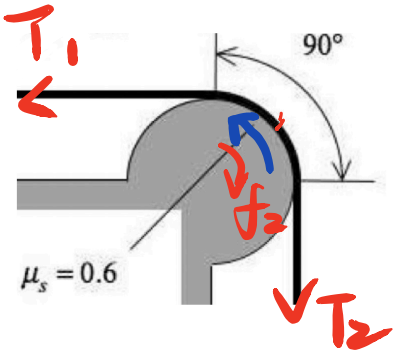
post: $\frac{T_2}{T_s} = \frac{T_1}{P} = e^{\mu_{s1}\beta_1} = e^{(0.4)(2\pi)} = 12.3$

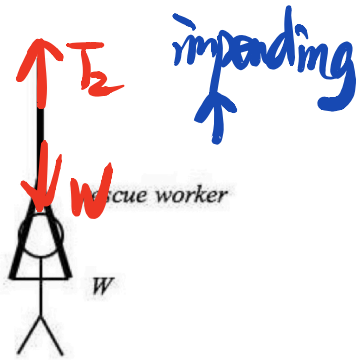
edge: $\frac{T_1}{T_s} = \frac{T_2}{T_1} = \frac{W}{T_1} = e^{\mu_{s2}\beta_2} = e^{(0.6)(\frac{\pi}{2})} = 2.6$

$\Rightarrow T_1 = \frac{W}{2.6} = 69 \text{ lbs} \Rightarrow P = \frac{T_1}{12.3} = \boxed{5.6 \text{ lbs}}$

(b) To initiate. $P > T_1$, $T_1 > T_2 = W$







Post: $\frac{T_2}{T_s} = \frac{P}{T_1} = e^{\mu_{s_1}\beta_1} = 12.3$

edge: $\frac{T_2}{T_s} = \frac{T_1}{T_2} = \frac{T_1}{W} = e^{\mu_{s_2}\beta_2} = 2.6$

$\Rightarrow \quad T_1 = 2.6\,W = 468$ lbs

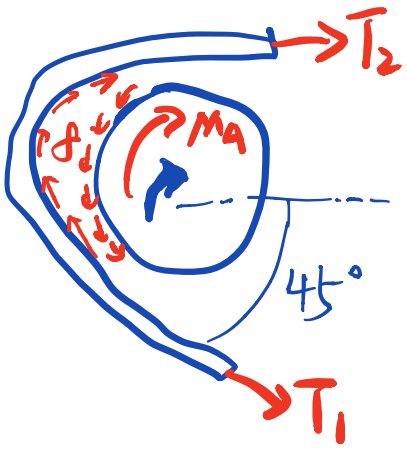
$\Rightarrow P = 12.3\,T_1 =$ **5756 lbs**

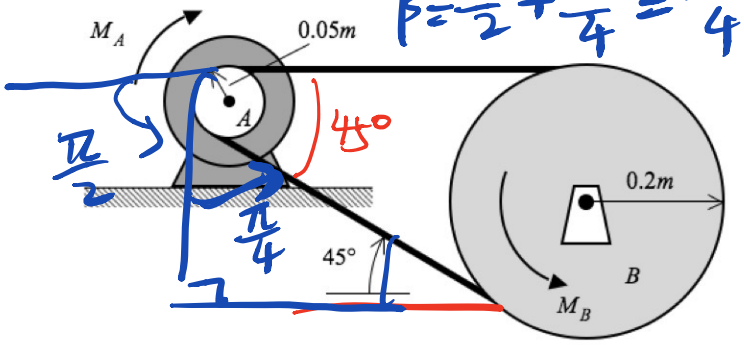
$P_{min}/P_{hold} \approx 1000$

Example 6.D.3

Given: A motor exerts a clockwise torque M_A on pulley A. A counterclockwise torque M_B is applied to pulley B that keeps the system in equilibrium. The coefficient of static friction between the belt and the pulleys is known to be $\mu_s = 0.35$.

Find: Determine the maximum torque M_A that can be applied by the motor such that the tension in the belt does not exceed a maximum allowable value of $2500N$. Determine the corresponding value for the torque M_B.



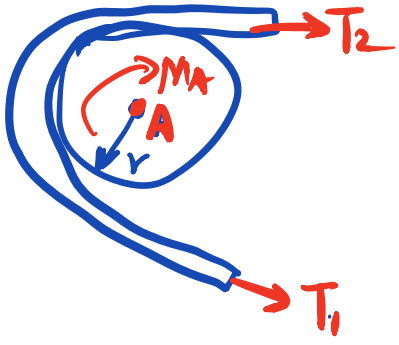


$\beta = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$T_1 = f + T_2 \Rightarrow T_1 > T_2$

$\Rightarrow \frac{T_2}{T_s} = \frac{T_1}{T_2} = e^{\mu_s \beta} = e^{(0.35)(\frac{3\pi}{4})} = 2.3$

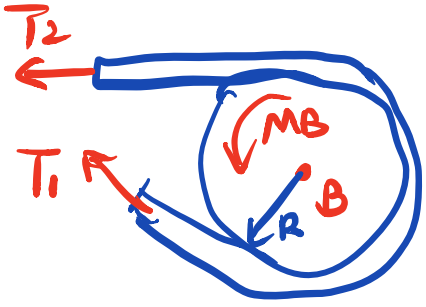
$\Rightarrow T_2 = \frac{T_1}{2.3} = \frac{2500N}{2.3} = 1087N$



$\Sigma(M_A) = 0 = -M_A + T_1 r - T_2 r$

$\Rightarrow M_A = (T_1 - T_2) r = (2500 - 1087)(0.05)$

$= 70.7 \ N \cdot m$



$\Sigma(M)_B = 0 = M_B - T_1 R + T_2 R$

$\Rightarrow M_B = (T_1 - T_2) R = 282.6 N \cdot m$

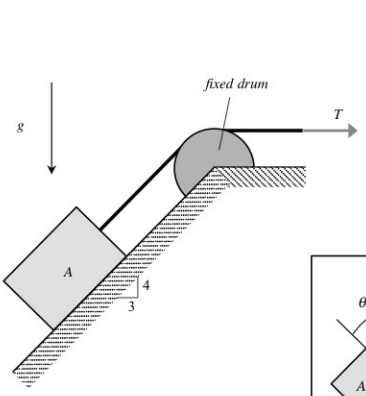
Example 6.D.4 - Solved

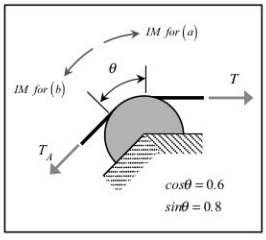
Given: Block A, having a weight of W, rests on a rough incline (coefficient of static friction μ_S), and is held in place by a cable that is passed over a rough, stationary cylinder (coefficient of static friction μ_S) with a force T acting on the free end of the cable. Use $\mu_S = 0.3$.

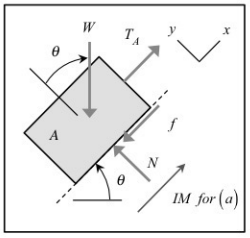
Find: For this problem:

(a) Determine the *largest* force T_{max} that can be applied and have the system remain in static equilibrium.

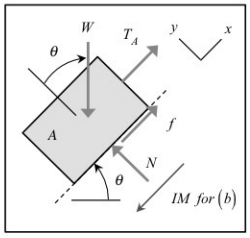
(b) Determine the *smallest* force T_{min} that can be applied and have the system remain in static equilibrium.







FBD (a)



FBD (b)

Part (a)

For the *largest* force T, the impending motion of the block is UP the incline. Therefore, the friction force on the block points down the incline and $T > T_A$.

Equilibrium:

$$\sum F_y = N - W\cos\theta = 0 \;\Rightarrow\; N = W\cos\theta$$

$$\sum F_x = T_A - f - W\sin\theta = 0 \;\Rightarrow$$

$$T_A = W\left(\mu_S\cos\theta + \sin\theta\right)$$

Belt equation:

$$\frac{T}{T_A} = e^{\mu_S\theta} \;\Rightarrow$$

$$T = W\left(\mu_S\cos\theta + \sin\theta\right)e^{\mu_S\theta}$$

$$= W\left[\left(0.3\right)\left(0.6\right) + \left(0.8\right)\right]e^{(0.3)(53.13\pi/180)}$$

$$= 1.29W$$

Part (b)

For the *smallest* force T, the impending motion of the block is DOWN the incline. Therefore, the friction force on the block points up the incline and $T < T_A$.

Equilibrium:

$$\sum F_y = N - W\cos\theta = 0 \;\Rightarrow\; N = W\cos\theta$$

$$\sum F_x = T_A + f - W\sin\theta = 0 \;\Rightarrow$$

$$T_A = W\left(-\mu_S\cos\theta + \sin\theta\right)$$

Belt equation:

$$\frac{T_A}{T} = e^{\mu_S\theta} \;\Rightarrow$$

$$T = W\left(-\mu_S\cos\theta + \sin\theta\right)e^{-\mu_S\theta}$$

$$= W\left[-\left(0.3\right)\left(0.6\right) + \left(0.8\right)\right]e^{-(0.3)(53.13\pi/180)}$$

$$= 0.47W$$

Group Practice.

Example 6.D.5

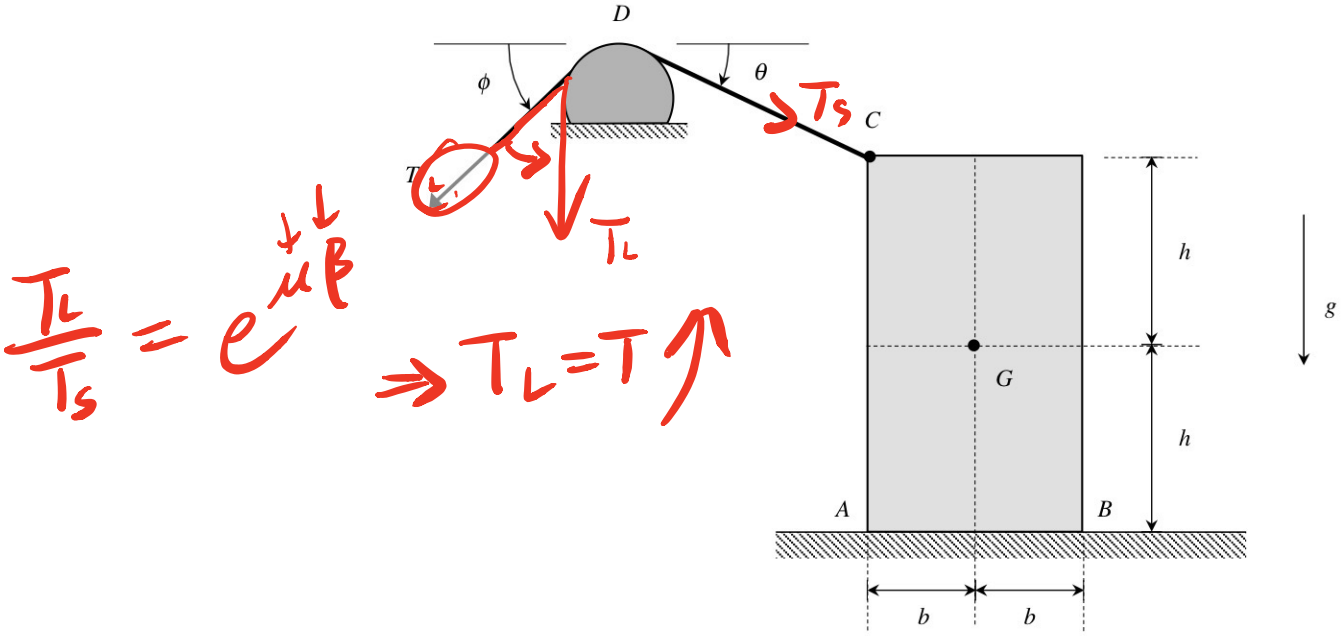
Given: A block weighing W is to be moved using a rope with the rope being pulled over a stationary drum. The coefficients of static friction between the block and ground, and between the rope and the drum, are μ_{s1} and μ_{s2}, respectively. A force T acts on the free end of the cable.

Find: For this problem:

(a) Determine the smallest force T required to move the block. Does the block tip or slip with this force?

(b) Would the minimum force T need to be increased, decreased or kept the same if the rope were pulled straight down on the left side of the drum?

Use the following parameter values in your analysis: $h = 2$ ft, $b = 1$ ft, $W = 40$ lb, $\theta = 30°$, $\phi = 60°$, $\mu_{s1} = 0.35$ and $\mu_{s2} = 0.5$.



$\frac{T_L}{T_s} = e^{\mu\beta}$ $\Rightarrow T_L = T$

ME 270 - Basic Mechanics I - Group Quiz

Name/Group #: _____ Group Members: 1) _____ 2) _____

Date: _____ Period: _____ 3) _____ 4) _____

Given: A 40 lb block is being moved using a rope and a drum. The coefficients of friction are 0.5 between the rope and drum and 0.35 between the block and the plane.

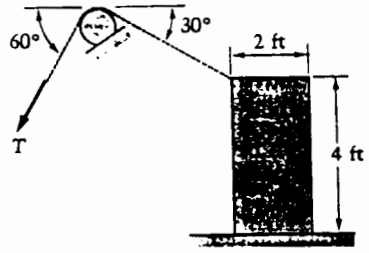


Figure P6.162

Find:

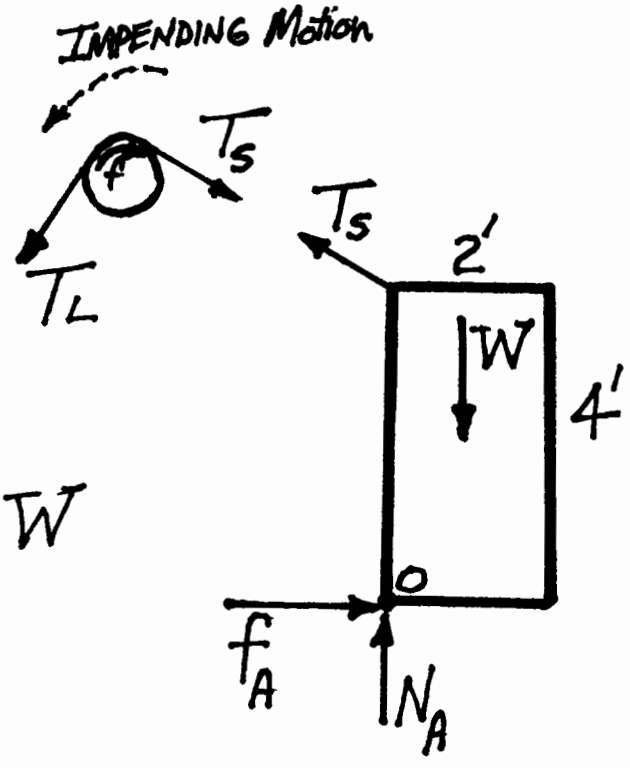
(a) Determine the smallest force T required to cause impending motion. Will the block slip or tip?

(b) Would the minimum force T (increase, decrease or remain the same) if the cable were pulled straight down?

Solution:

(a) ASSUME IMPENDING TIPPING

- f and N act at tipping edge
- $f \leq f_{MAX}$



$$\Sigma M_O = 0 = (4)\,T_S \cos 30° - (1)W$$

$$\therefore \boxed{T_S = 11.5 \text{ lbs}}$$

CHECK TIPPING ASSUMPTION

$\Sigma F_x = 0 = f_A - T_S \cos 30^\circ \Rightarrow \boxed{f_A = 10.0 \text{ lbs}}$

$\Sigma F_y = 0 = N_A - W + T_S \sin 30^\circ \Rightarrow \boxed{N_A = 34.3 \text{ lbs}}$

$f_{MAX} = \mu_s N_A = (0.35)(34.3) = \boxed{12.0 \text{ lbs}}$

Since $f_A < f_{A_{MAX}}$, tipping will occur first

$\frac{T_L}{T_S} = e^{\mu_s \beta}$ $\quad \mu_s = 0.5 \quad \beta = \frac{\pi}{2}$

$\therefore T_L = T_S e^{\mu_s \beta} = 11.5\, e^{0.5\left(\frac{\pi}{2}\right)} = \boxed{25.2 \text{ lbs.}}$