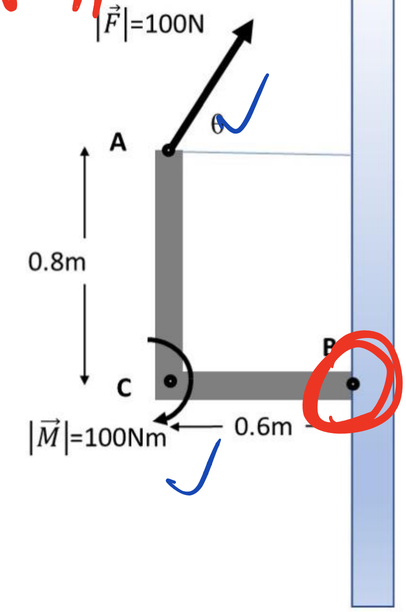
Homework H8.A

Given: The bar BCA is loaded as shown and $\theta = 30°$.

Find:

a) Determine the reactions at B.

b) Determine the angle θ so that the resultant moment at B for the combined loading is zero.



2 forces 1 moment

sum of applied force

★ Determine the type of support & related unknowns

★ FBD (draw separately)

★ $\Sigma \bar{F} = \Sigma \bar{F}_{applied} + \Sigma \bar{F}_{reaction} = 0$

$\Sigma \bar{M} = \Sigma \bar{M}_{applied} + \Sigma \bar{M}_{reaction} = 0$

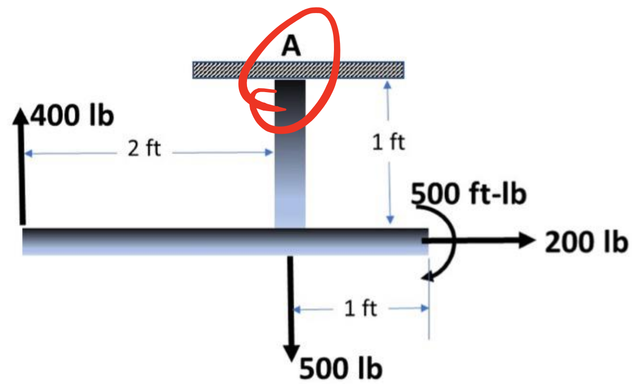
$f(\theta) = 0$ → 0

Homework H8.B
Given: The bar is fixed into the wall at point A.

2 forces 1 moment

Find:

a) Determine the reactions at A.
b) If the 500 lb force is moved to the left, how does that change influence the reactions at A?



ME 27000
BASIC MECHANICS I

Course Outcomes [Related ME Program Outcomes in brackets]

1. Develop an understanding of *static equilibrium* and *stresses in statically determinate structures* and how to apply them to engineering systems. [A1, A2]
2. Learn a systematic approach to *problem solving*. [A2]
3. Foster effective mathematical and graphical *communication skills*. [B1]

Statics of Rigid Bodies (9 wks)

Fundamentals (2 wks)

1. Newton's Laws
2. Vector algebra; vector components
3. Position, unit and force vectors
4. Dot product
5. Cross product
6. Moment of a force about a point

Static Equilibrium (5 wks)

1. Equilibrium of a particle
2. Support reactions and free body diagrams
3. Static indeterminacy and partial constraints
4. 2-D and 3-D static equilibrium
5. Trusses
   - method of joints
   - method of sections
6. Frames and machines
7. Dry friction
   - Coulomb's Laws
   - Systems with friction
   - Sliding or tipping
   - Wedges

Equivalent Systems (2 wks)

1. Determination of the resultant of concurrent forces
2. Equivalent force/couple systems
3. Centroid and center of mass
   - by composite parts
   - by integration
4. Surface loadings
   - line loads
   - pressure distributions
5. Fluid statics
   - rectangular surfaces

Stresses in Statically-Determinate Structures (6 wks)

Introduction of Stress and Strain in Materials (1 wk)

1. Basic definitions of stress and strain
2. Mechanical properties of materials
3. Shear stress and strain

Stress Analysis (5 wks)

1. Stress due to axial loading
2. Shear stress due to torsion
3. Shear force and moment diagrams
4. Second area moments for beams
5. Flexural stresses in beams
6. Stress analysis of beams

ME 270 – Basic Mechanics I

Fall 2019

Period		Date	Topic	Reading	Homework
			STATICS		
1	M	Aug. 19	Introduction, Unit Conversions	1.A-F	H1.A, H1.B
2	W	Aug. 21	Position, Unit, and Force Vectors	2.A-B	H2.A, H2.B
3	F	Aug. 23	Dot Product	2.A-E	H3.A, H3.B
4	M	Aug. 26	Particle Equilibrium (2-D)	3.A-F	H4.A, H4.B
5	W	Aug. 28	Particle Equilibrium (3-D)	3.A-F	H5.A, H5.B
6	F	Aug. 30	Moment About a Point	1.D,4.A-B	H6.A, H6.B
B	M	Sep. 2	**Labor Day**		
7	W	Sep. 4	Force Couples, Equivalent Systems	5.A-B	H7.A, H7.B
8	F	Sep. 6	Free Body Diagrams; Equilibrium of Rigid Bodies (2-D)	4.C-D	H8.A, H8.B
9	M	Sep. 9	Equilibrium of Rigid Bodies (2-D)	4.E-G	H9.A, H9.B
10	W	Sep. 11	Equilibrium of Rigid Bodies (3-D)	4.E-G	H10.A, H10.B
11	F	Sep. 13	Equilibrium of Rigid Bodies (3-D)	4.E-G	H11.A, H11.B
12	M	Sep. 16	Distributed Loading	5.D	H12.A, H12.B
13	W	Sep. 18	Centers of Mass of Centroids: By Composite Parts	5.C	H13.A, H13.B
14	F	Sep. 20	Centers of Mass of Centroids: By Integration	5.C	H14.A, H14.B
15	M	Sep. 23	Fluid Statics: Buoyancy	5.E-G	H15.A, H15.B
R	W	Sep. 25	**Review for Exam 1**		
E	Th	Sep. 26	**EXAM 1 (6:30 – 7:30 PM); (Covers Lectures 1-15)**	Ch. 1-5	None assigned
B	F	Sep. 27	**NO LECTURE**		
16	M	Sep. 30	Fluid Statics: Hydrostatic Loads	5.E-F	H16.A, H16.B
17	W	Oct. 2	Fluid Statics: Hydrostatic Loads	5.E-G	H17.A, H17.B
18	F	Oct. 4	Friction: General	6.A-B	H18.A, H18.B
B	M	Oct. 7	**OCTOBER BREAK**		
19	W	Oct. 9	Friction: Slipping-Tipping	6.C	H19.A, H19.B
20	F	Oct. 11	Friction: Flat Belts	6.D	H20.A, H20.B
21	M	Oct. 14	Friction: Wedges	6.E-G	H21.A, H21.B
22	W	Oct. 16	Trusses: Method of Joints	7.A-C	H22.A, H22.B
23	F	Oct. 18	Trusses: Method of Sections	7.E	H23.A, H23.B

→ Due tonight (H8.A, H8.B)

Both due Fri. 9/13 (H9.A, H9.B; H10.A, H10.B)

Period		Date	Topic	Reading	Homework
24	M	Oct. 21	Trusses: Zero-Force Members	7.C-I	H24.A, H24.B
25	W	Oct. 23	Frames and Machines: Simple Systems	8.A-D	H25.A, H25.B
26	F	Oct. 25	Frames and Machines: Simple Systems	8.A-D	H26.A, H26.B
R	M	Oct. 28	**Review for Exam 2**	Ch. 6-8	None assigned
E	Tu	Oct. 29	**EXAM 2 (6:30 – 7:30 PM); (Covers Lectures 16-26)**		
B	W	Oct. 30	**NO LECTURE**		
27	F	Nov. 1	Internal Force/Couple Analysis	9.A	H27.A, H27.B
28	M	Nov. 4	Shear Force and Bending Moment Diagrams (Pt. Loads)	9.B	H28.A, H28.B
29	W	Nov. 6	Shear Force and Bending Moment Diagrams (Dist. Loads)	9.B	H29.A, H29.B
30	F	Nov. 8	Shear Force and Bending Moment Diagrams (Graph. Meth.)	9.B-E	H30.A, H30.B
31	M	Nov. 11	Stress-Strain Curves; Axial Stress and Strain	10.A-F	H31.A, H31.B
32	W	Nov. 13	Axial Stress and Strain; Factor of Safety	10.A-F	H32.A, H32.B
33	F	Nov. 15	Shear Stress and Strain; Direct Shear	11.A-D	H33.A, H33.B
34	M	Nov. 18	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E	H34.A, H34.B
35	W	Nov. 20	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E, F	H35.A, H35.B
36	F	Nov. 22	Flexural Stresses in Beams	12.A	H36.A, H36.B
B	M	Nov. 25	Flexural Stresses in Beams	12.A	H37.A, H37.B
B	W	Nov. 27	**THANKSGIVING**		
37	F	Nov. 29	**THANKSGIVING**		
38	M	Dec. 2	Second Moments of Area: By Composite Parts	12.B	H38.A, H38.B
39	W	Dec. 4	Second Moments of Area: By Integration	12.B, D	H39.A, H39.B
R	F	Dec. 6	**Review for Final Exam**	Ch. 1-12	Practice Exams
E			**FINAL EXAM (details to be announced); (Covers Lectures 1-39)**		

Coding: Integer = Lecture number; B = Break; E = Exam; R = Review lecture.
Homework numbers correspond to lecture numbers. Review lectures do not increase counter. Homework is due the class period after it is assigned.

TEXTS
ME 270 textbook ("Statics: A Lecturebook", 2nd Edition, Fall 2019).

ME 270 BLOG
You will automatically be added to the blog once you have registered for ME 270. Once you have access to the blog, you can adjust your email settings to receive all, some or none of the posting, as according to your preference.

Lec 9. 09/09/2019 (Mon.)

STATIC EQUILIBRIUM OF RIGID BODIES (2-D)

Learning Objectives

1). To evaluate the *unknown reactions* holding a rigid body in equilibrium by solving the *equations of static equilibrium*.

2). To recognize situations of *partial* and *improper constraint*, as well as *static indeterminacy*, on the basis of the solvability of the equations of static equilibrium.

Newton's First Law

& no net moment

Given *no net force*, a body at rest will remain at **rest** (and a body moving at a constant velocity will continue to do so along a straight path).

Definitions

Zero-Force Members: structural members that support no loading but aid in the stability of the truss.

Two-Force Members: structural members that are: a) subject to no applied or reaction moments, and b) are loaded only at two pin joints along the member.

Multi-Force Members: structural members that have a) applied or reaction moments, or b) are loaded at more than two points along the member.

to be covered in the future

• Equivalent system: net force & net moment of applied force & moment.

vs.

• Static equilibrium: net force & net moment of applied & reaction force & moment = 0.

Vector Equations

$$\overline{F}_R = \sum \overline{F} = \overline{0}$$

$$\overline{M}_{R_O} = \sum \overline{M}_O = \overline{0}$$ where O is any arbitrary point

Component Equations

There are three alternate forms of equilibrium equations for 2-D problems.

(i) Two component force equations (x and y) are one moment equation (z).

most common set of Eq's.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0$$

Additional moment equilibrium Eq. compared to particle cases.

(ii) One component force equation (x or y) and two moment equations (both about different points in the z direction).

$$\sum F_x = 0 \qquad \sum M_A = 0 \qquad \sum M_B = 0$$

(iii) Three moment equations (points A, B and C cannot be collinear).

$$\sum M_A = 0 \qquad \sum M_B = 0 \qquad \sum M_C = 0$$

Static Determinacy/Partial and Improper Constraints

Static Indeterminacy: occurs when a system has *more* constraints than is necessary to hold the system in equilibrium (i.e., the system is *overconstrained* and thus has *redundant* reactions).

Static Determinancy: occurs when a system has a *sufficient* number of constraints to prevent motion without any redundancy.

Partial Constraint: occurs when there is an *insufficient* number of reaction forces to prevent motion of the system (i.e., the system is *partially constrained*).

Improper Constraint: occurs when a system has a *sufficient* number of reaction forces but one or more are *improperly applied* so as not to prevent motion of the system (i.e., the system is *improperly constrained*).

Comments: 2 force Eqn's + 1 moment Eqn

2D Rigid Body Static Eq.

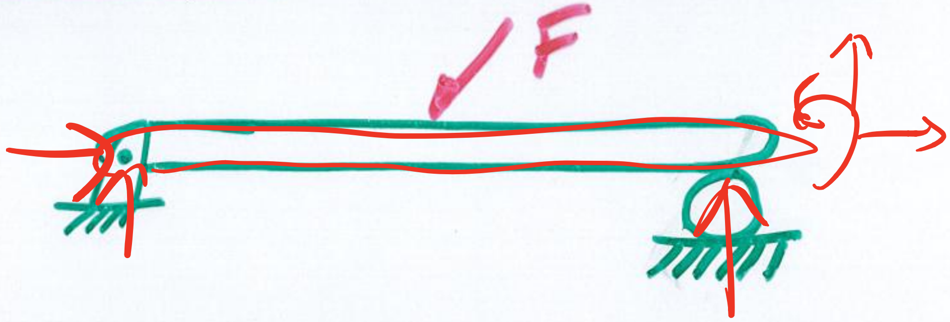
1). Equations (i) are the equilibrium eqns most commonly used.

2). NEVER attempt to use MORE THAN THREE equilibrium equations from a single planar FBD. Only three independent equations can exist for a single planar FBD.

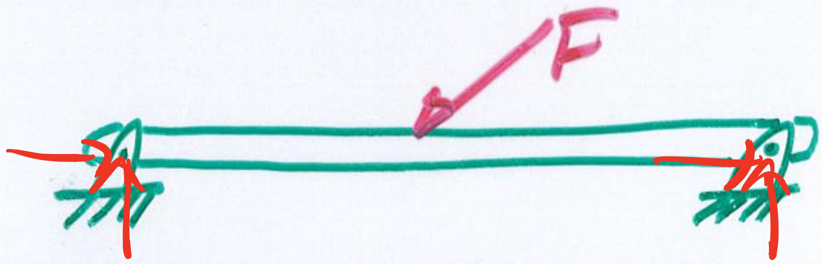
3). If you have more than three unknown forces in your three equations, then consider breaking the system or structure into smaller systems and write down equilibrium equations for each sub-structure. If this is not possible, you may have an indeterminate structure; i.e., the evaluation of member forces requires consideration of deformation of the members resulting from the loading.

→ to be covered in the future

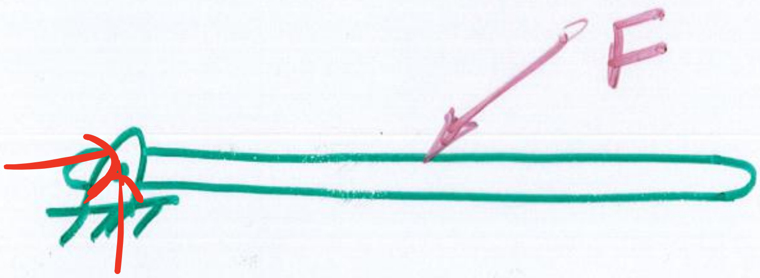
STATIC DETERMINACY ✓



STATIC INDETERMINACY



PARTIAL CONSTRAINT ✓



IMPROPER CONSTRAINT ✓

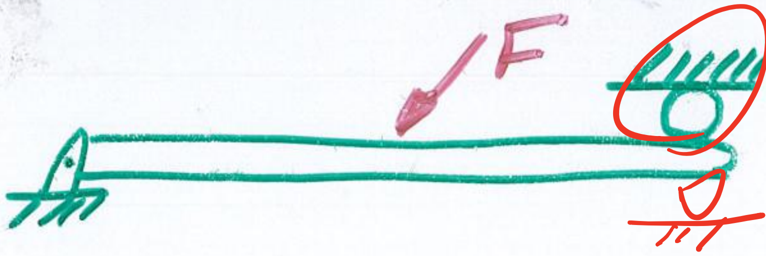


TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

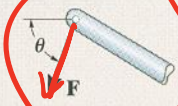
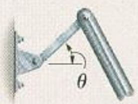
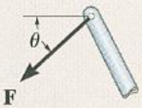
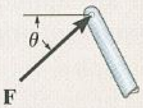
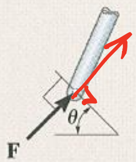
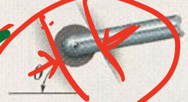
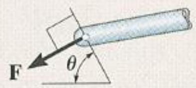
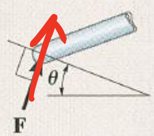
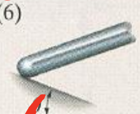
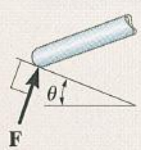
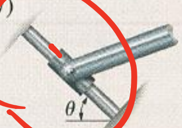
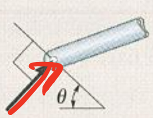
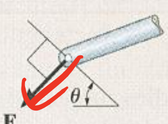
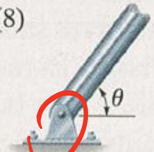
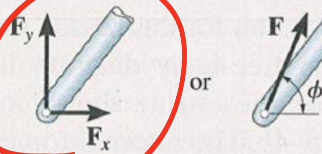
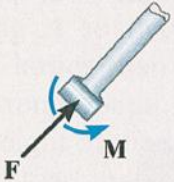
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Example 4.E.8

Given: A traffic-light pole is supported by the fixed end A and support cable BC. The pole is loaded with a traffic light having a weight of W, and cable BC carries a known tension of T_{BC}.

Find: For this loading:

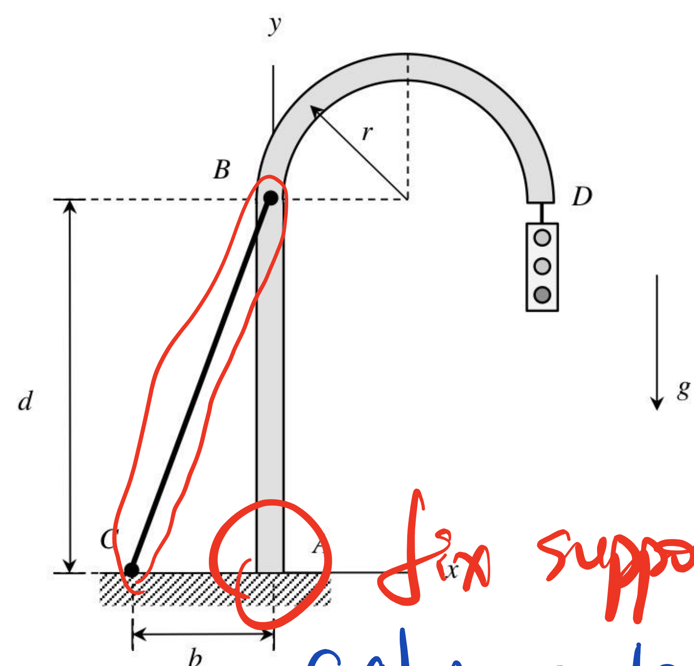
(a) Draw the free body diagram of the traffic-light pole.

(b) Determine the reactions on the pole at the fixed end A.

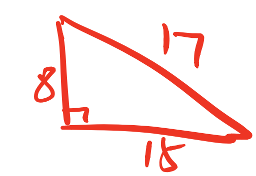
(c) Suppose that cable BC is accidently cut. How are the reactions at A changed by this: do the reaction components go up, go down or remain the same?

neglect weight of pole.

Use the following parameter values in your analysis: $d = 15$ ft, $b = 8$ ft, $r = 4$ ft, $W = 480$ lb and $T_{BC} = 544$ lb.



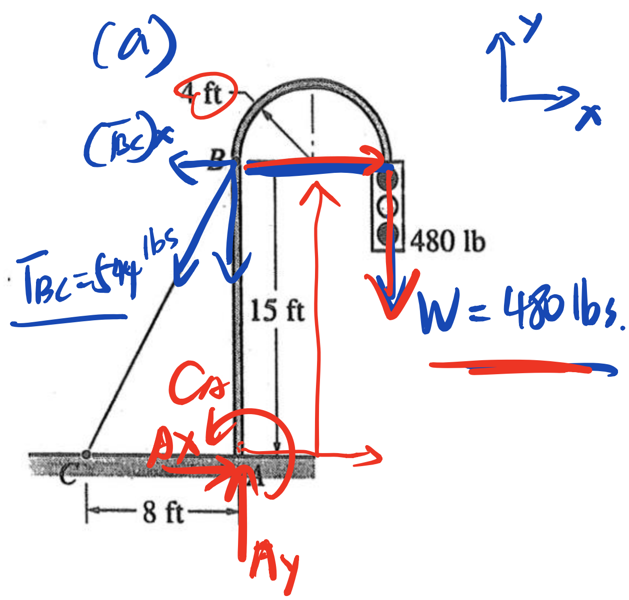
fix support (always decide the type of support first) trig:



(b) $\bar{F}_{BC} = T_{BC}\bar{u}_{BC} = T_{BC}\dfrac{\bar{r}_{BC}}{|\bar{r}_{BC}|}$

$= (T_{BC})\dfrac{-8\bar{i} - 15\bar{j}}{17}$ lbs.

(a)



① $\Sigma M_A = 0 = (T_{BC})\left(\frac{8}{17}\right)(15) - (480)(8) + C_A$

$\Rightarrow C_A = 0$ lb-ft

② $\Sigma F_x = 0 = A_x - T_{BC}\left(\frac{8}{17}\right)$

$\Rightarrow A_x = 256$ lbs

- $\bar{A} = A_x\bar{i} + A_y\bar{j}$
- $\bar{C}_A = C_A\bar{k}$

③ $\Sigma F_y = 0 = A_y - T_{BC}\left(\frac{15}{17}\right) - 480$

$\Rightarrow A_y = 960$ lbs

$\Rightarrow$ $\bar{A} = 256\bar{i} + 960\bar{j}$ lbs

$\bar{C}_A = 0\bar{k}$ lb-ft

(c) T_{BC}: 544 → 0 lbs ↓ $A_x = 0$ lbs (from ②) $A_y = 480$ lbs (from ③)

$C_A = 3840$ lb-ft (from ①)

Example 4.E.11

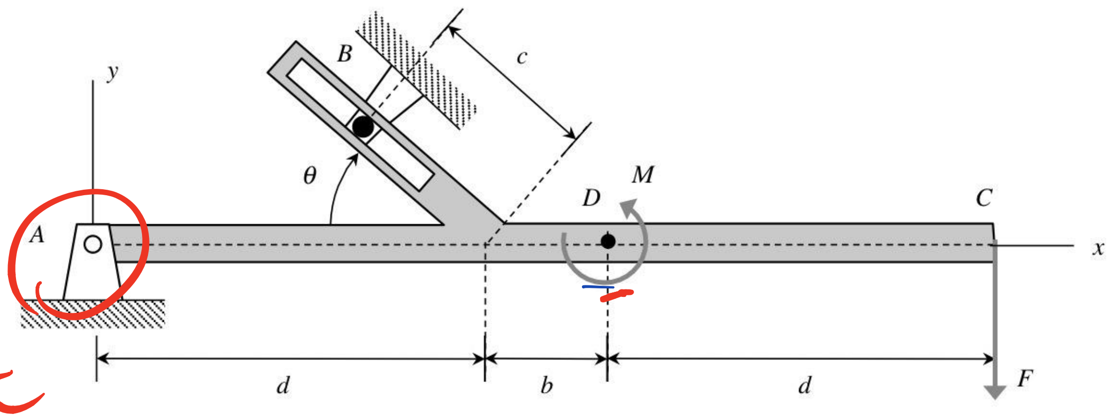
Given: A force F acts at end C of the forked-bar shown, along with a couple M being applied at D. The bar is pinned to ground at A and supported by pin-in-slot connection at B.

Find: For this loading:

neglect weight of bar

(a) Draw the free body diagram of the bracket ABC.
(b) Determine the reactions on the bar at A and B.
(c) If the couple M were moved to a different location on the bar (e.g., if it were moved to end A), how would this change in loading affect the reactions at A and B?
(d) If the couple M were completely removed, how would this change in loading affect the reactions at A and B?

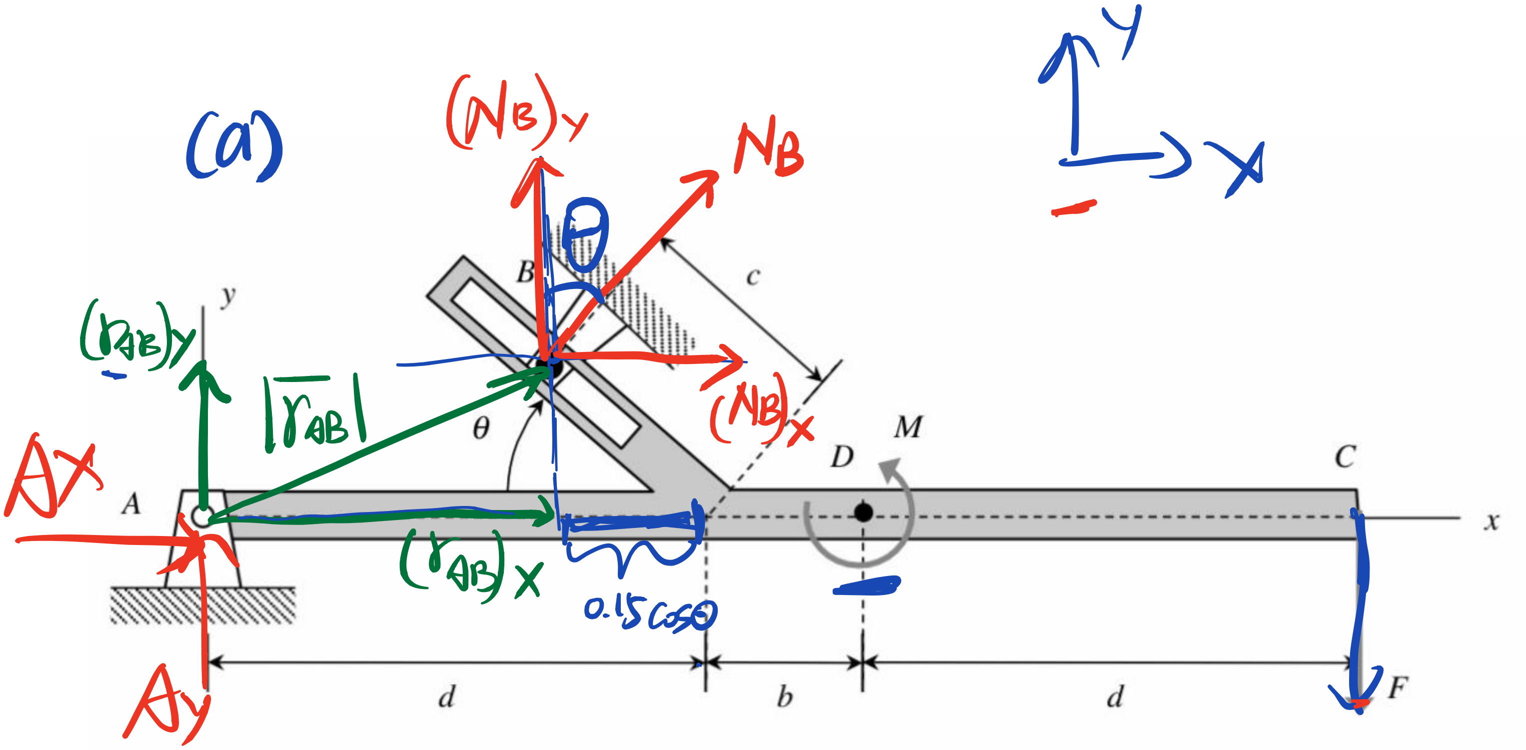
Use the following parameter values in your analysis: $d = 250$ mm, $b = 100$ mm, $c = 150$ mm, $\theta = 36.87°$, $F = 5$ kN and $M = 4$ kN·m.



pin joint

(always decide the type of support first)

(a)



(b) ① $\Sigma M_A = 0 = -N_B \sin\theta\,(0.15\sin\theta) + N_B\cos\theta\,(0.25 - 0.15\cos\theta)$

$+ 4 - 5(0.6)$

$-1 \rightarrow +3$

$+1 \rightarrow -3$

$\Rightarrow N_B = -20\ \text{kN}$

② $\Sigma F_x = 0 = A_x + N_B\sin\theta$

$\Rightarrow A_x = 12\ \text{kN}$

③ $\Sigma F_y = 0 = A_y + N_B\cos\theta - 5$

$\Rightarrow A_y = 21\ \text{kN}$

$\bar{N}_B = N_B\sin\theta\,\bar{i} + N_B\cos\theta\,\bar{j}\ \text{kN}$

$\bar{A} = 12\bar{i} + 21\bar{j}\ \text{kN}$

(c) M is a moment due to force couple → acting on the whole system ⇒ so no change

(d) (from ①) $|N_B|$ ↑ 3 times bigger & change direction

(from ②) $|A_x|$ ↑ & change direction

(from ③) $|A_y|$ ↑

Group Practice

Weight is NOT negligible

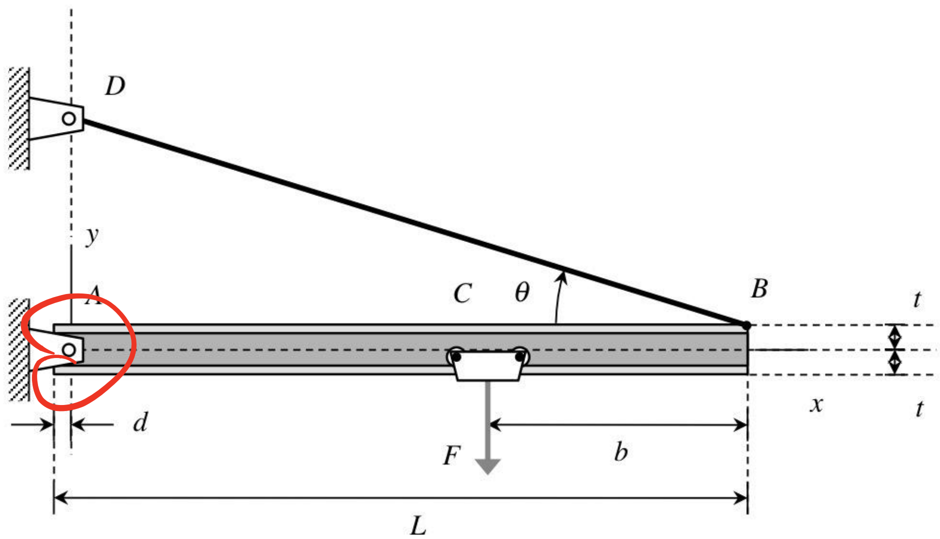
Example 4.E.13

Given: A jib crane AB consists a standard I-beam of length L having a mass per length of ρ. The left end of the crane is pinned to ground, and the right end is supported by a cable attached to end B. The crane supports a load of F at C.

pin joint

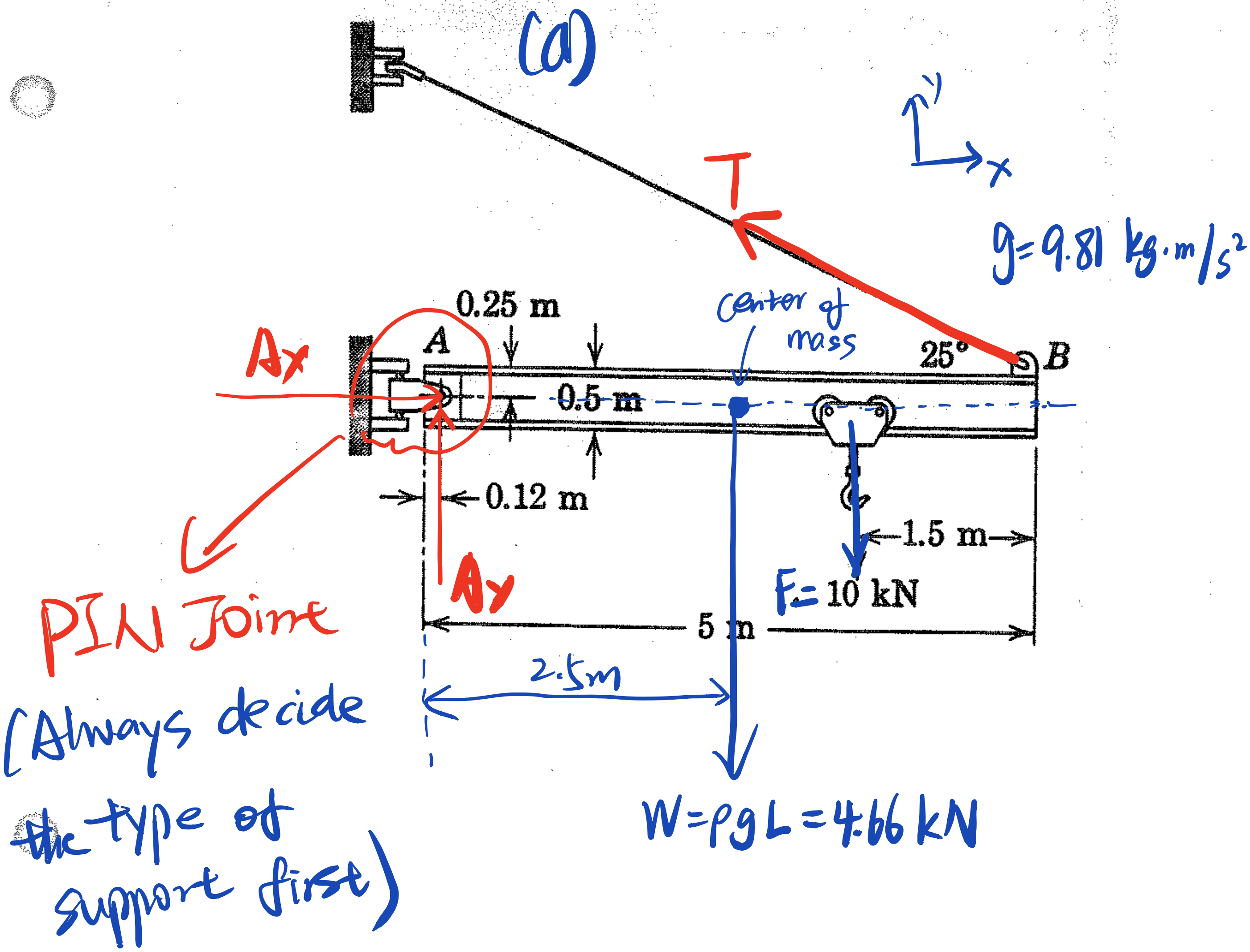
Find: For this loading:
(a) Draw the free body diagram of crane AB.
(b) Determine the reactions on the crane at end A (as a vector) and the tension in cable DB.
(c) If the load F is moved toward end B, what influence will this have on the tension load in the cable and on the reactions on the crane at A?

Use the following parameter values in your analysis: $L = 5$m, $d = 0.12$ m, $b = 1.5$ m, $t = 0.25$ m, $\theta = 25°$, $F = 10$ kN and $\rho = 95$ kg/m.



pin joint

(always decide the type of support first)



(a)
y
x
T
g = 9.81 kg·m/s²
0.25 m
A
center of mass
25°
B
Ax
0.5 m
0.12 m
1.5 m
Ay
F = 10 kN
PIN Joint
5 m
2.5m
(Always decide the type of support first)
W = ρgL = 4.66 kN

$$\underline{\underline{\Sigma M_A = 0}} = -(2.5-0.12)W - (5-1.5-0.12)F + (5-0.12)T\sin 25° + (0.25)T\cos 25°$$

$$\boxed{T = 19.6\ kN}$$

$$\underline{\underline{\Sigma F_x = 0}} = A_x - T\cos 25°$$

$$\therefore A_x = T\cos 25° = (19.6)\cos 25° = \boxed{17.8\ kN \rightarrow}$$

$$\Sigma F_y = 0 = A_y - W - F + T\sin 25°$$

$$\therefore A_y = W + F - T\sin 25° = 4.66 + 10 - 19.6\sin 25°$$

$$\boxed{A_y = 6.38\ kN \uparrow}$$

$\therefore$ In vector form, $\boxed{\overline{A} = 17.8\ \overline{i} + 6.38\ \overline{j}\ kN}$

(c) If the 10kN load were shifted toward end B,

$T \rightarrow$ increase
$A_x \rightarrow$ increase
$A_y \rightarrow$ decrease (initially)