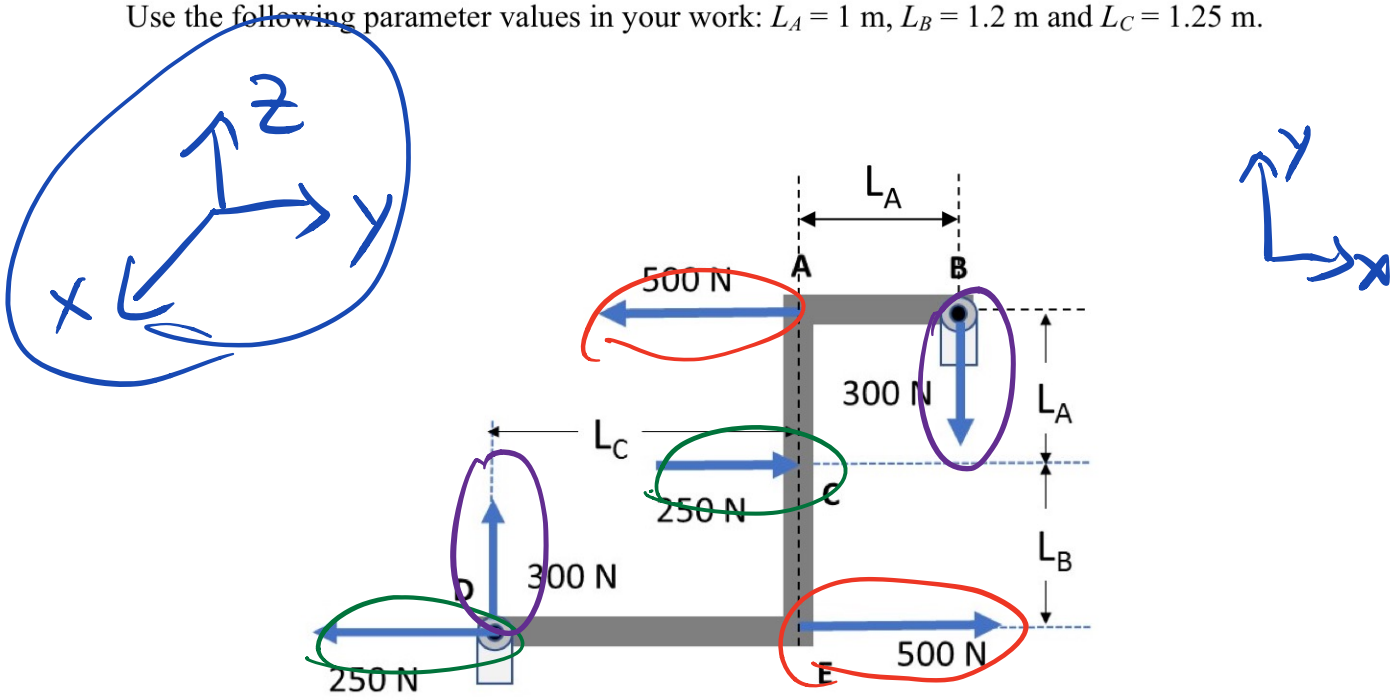
Homework H7.A
Given: Loading on the member shown.

Find:

a) Determine the magnitude and direction of each force-couple pair.
b) Determine the magnitude and direction of the combination of all the force-couple pairs.

equivalent moment

Use the following parameter values in your work: L_A = 1 m, L_B = 1.2 m and L_C = 1.25 m.



Homework H7.B

Given: Pole AB is fixed into the ground. Cables BC and BD carry tensions of T_{BC} and T_{BD}. A load F is applied at location E.

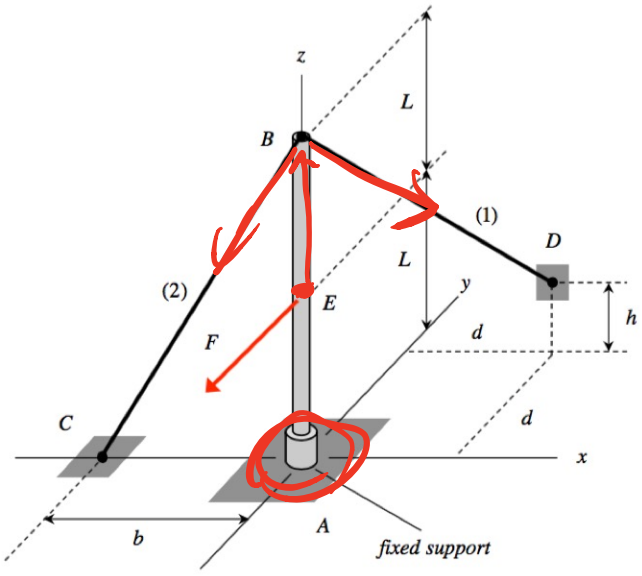
Find: Determine an equivalent force-couple pair for all loads at point E.

equivalent force & equivalent moment.

Use the following parameter values in your work: T_{BC} = 125 N, T_{BD} = 150 N, F = 200 N, h = 0.5 m., $b = d$ = 2 m and L = 3 m.

$\bar{F}_{eq} = \Sigma \bar{F}$

$(\bar{M}_{eq})_E = \Sigma \bar{M}$



Moment about a Point
Group Quiz 1

Group #: _____ Group Members: 1) _____
(Present Only)

Date: _____ Period: _____ 2) _____

3) _____

4) _____

Given: A force of 800N acts on a bracket as shown.

Find:

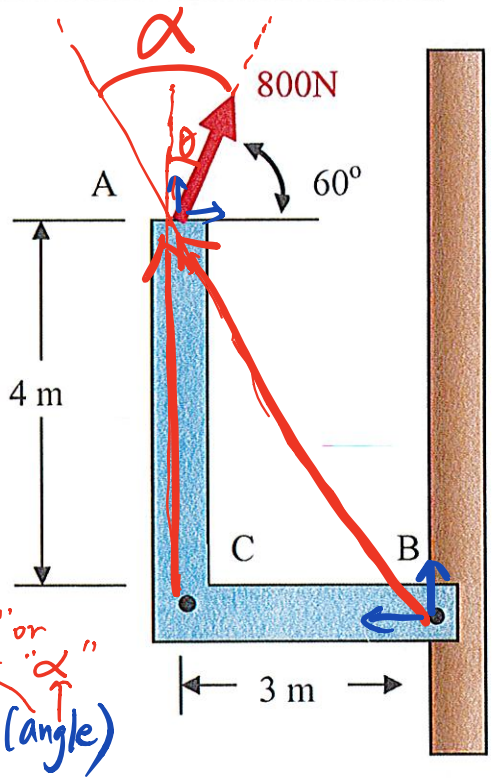
a) Estimate M_C.

b) Calculate M_C.

c) Estimate M_B.

d) Calculate M_B.

e) What is M_A?



Solution:

- Identify angles
- magnitude $|\bar{M}|$: $|\bar{F}||\bar{r}|\sin(\text{angle})$ "θ" or "α"
- Vector form of $\bar{M}$: RHH to decide deriction: "$-\bar{k}$"

Causing CW rotation: "−"

★ Break up force & arm (position vector) into x & y components

(equivalent to "trig" + RHH)

→ Cross product $= (\bar{F}_x\bar{i} + \bar{F}_y\bar{j}) \times (r_x\bar{i} + r_y\bar{j})$

$$= -\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ F_x & F_y & 0 \\ r_x & r_y & 0 \end{vmatrix} = (F_x r_y - F_y r_x)\,\bar{k}$$

$F_x r_y$: moment due to F_x on B

$F_y r_x$: moment due to F_y on B.

Shortcuts

$\bar{i} \to \bar{j}$: +

$\bar{j} \to \bar{i}$: −

$\bar{i} \to \bar{k}$: −

$\bar{k} \to \bar{j}$: −

ME 27000
BASIC MECHANICS I

Course Outcomes [Related ME Program Outcomes in brackets]

1. Develop an understanding of *static equilibrium* and *stresses in statically determinate structures* and how to apply them to engineering systems. [A1, A2]
2. Learn a systematic approach to *problem solving*. [A2]
3. Foster effective mathematical and graphical *communication skills*. [B1]

Statics of Rigid Bodies (9 wks)

Stresses in Statically-Determinate Structures (6 wks)

Fundamentals (2 wks)

1. Newton's Laws
2. Vector algebra; vector components
3. Position, unit and force vectors
4. Dot product
5. Cross product
6. Moment of a force about a point

Static Equilibrium (5 wks)

1. Equilibrium of a particle
2. Support reactions and free body diagrams
3. Static indeterminacy and partial constraints
4. 2-D and 3-D static equilibrium
5. Trusses
   - method of joints
   - method of sections
6. Frames and machines
7. Dry friction
   - Coulomb's Laws
   - Systems with friction
   - Sliding or tipping
   - Wedges

Equivalent Systems (2 wks)

1. Determination of the resultant of concurrent forces
2. Equivalent force/couple systems
3. Centroid and center of mass
   - by composite parts
   - by integration
4. Surface loadings
   - line loads
   - pressure distributions
5. Fluid statics
   - rectangular surfaces

Introduction of Stress and Strain in Materials (1 wk)

1. Basic definitions of stress and strain
2. Mechanical properties of materials
3. Shear stress and strain

Stress Analysis (5 wks)

1. Stress due to axial loading
2. Shear stress due to torsion
3. Shear force and moment diagrams
4. Second area moments for beams
5. Flexural stresses in beams
6. Stress analysis of beams

ME 270 – Basic Mechanics I

Fall 2019

Period		Date	Topic	Reading	Homework
			STATICS		
1	M	Aug. 19	Introduction, Unit Conversions	1.A-F	H1.A, H1.B
2	W	Aug. 21	Position, Unit, and Force Vectors	2.A-B	H2.A, H2.B
3	F	Aug. 23	Dot Product	2.A-E	H3.A, H3.B
4	M	Aug. 26	Particle Equilibrium (2-D)	3.A-F	H4.A, H4.B
5	W	Aug. 28	Particle Equilibrium (3-D)	3.A-F	H5.A, H5.B
6	F	Aug. 30	Moment About a Point	1.D,4.A-B	H6.A, H6.B
B	M	Sep. 2	**Labor Day**		
7	W	Sep. 4	Force Couples, Equivalent Systems	5.A-B	H7.A, H7.B
8	F	Sep. 6	Free Body Diagrams; Equilibrium of Rigid Bodies (2-D)	4.C-D	H8.A, H8.B
9	M	Sep. 9	Equilibrium of Rigid Bodies (2-D)	4.E-G	H9.A, H9.B
10	W	Sep. 11	Equilibrium of Rigid Bodies (3-D)	4.E-G	H10.A, H10.B
11	F	Sep. 13	Equilibrium of Rigid Bodies (3-D)	4.E-G	H11.A, H11.B
12	M	Sep. 16	Distributed Loading	5.D	H12.A, H12.B
13	W	Sep. 18	Centers of Mass of Centroids: By Composite Parts	5.C	H13.A, H13.B
14	F	Sep. 20	Centers of Mass of Centroids: By Integration	5.C	H14.A, H14.B
15	M	Sep. 23	Fluid Statics: Buoyancy	5.E-G	H15.A, H15.B
R	W	Sep. 25	**Review for Exam 1**		
E	Th	Sep. 26	**EXAM 1 (6:30 – 7:30 PM); (Covers Lectures 1-15)**	Ch. 1-5	None assigned
B	F	Sep. 27	**NO LECTURE**		
16	M	Sep. 30	Fluid Statics: Hydrostatic Loads	5.E-F	H16.A, H16.B
17	W	Oct. 2	Fluid Statics: Hydrostatic Loads	5.E-G	H17.A, H17.B
18	F	Oct. 4	Friction: General	6.A-B	H18.A, H18.B
B	M	Oct. 7	**OCTOBER BREAK**		
19	W	Oct. 9	Friction: Slipping-Tipping	6.C	H19.A, H19.B
20	F	Oct. 11	Friction: Flat Belts	6.D	H20.A, H20.B
21	M	Oct. 14	Friction: Wedges	6.E-G	H21.A, H21.B
22	W	Oct. 16	Trusses: Method of Joints	7.A-C	H22.A, H22.B
23	F	Oct. 18	Trusses: Method of Sections	7.E	H23.A, H23.B

Period		Date	Topic	Reading	Homework
24	M	Oct. 21	Trusses: Zero-Force Members	7.C-I	H24.A, H24.B
25	W	Oct. 23	Frames and Machines: Simple Systems	8.A-D	H25.A, H25.B
26	F	Oct. 25	Frames and Machines: Simple Systems	8.A-D	H26.A, H26.B
R	M	Oct. 28	**Review for Exam 2**	Ch. 6-8	None assigned
E	Tu	Oct. 29	**EXAM 2 (6:30 – 7:30 PM); (Covers Lectures 16-26)**		
B	W	Oct. 30	**NO LECTURE**		
27	F	Nov. 1	Internal Force/Couple Analysis	9.A	H27.A, H27.B
28	M	Nov. 4	Shear Force and Bending Moment Diagrams (Pt. Loads)	9.B	H28.A, H28.B
29	W	Nov. 6	Shear Force and Bending Moment Diagrams (Dist. Loads)	9.B	H29.A, H29.B
30	F	Nov. 8	Shear Force and Bending Moment Diagrams (Graph. Meth.)	9.B-E	H30.A, H30.B
31	M	Nov. 11	Stress-Strain Curves; Axial Stress and Strain	10.A-F	H31.A, H31.B
32	W	Nov. 13	Axial Stress and Strain; Factor of Safety	10.A-F	H32.A, H32.B
33	F	Nov. 15	Shear Stress and Strain; Direct Shear	11.A-D	H33.A, H33.B
34	M	Nov. 18	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E	H34.A, H34.B
35	W	Nov. 20	Shear Stress Due to Torsion in Circular and Tubular Shafts	11.E, F	H35.A, H35.B
36	F	Nov. 22	Flexural Stresses in Beams	12.A	H36.A, H36.B
B	M	Nov. 25	Flexural Stresses in Beams	12.A	H37.A, H37.B
B	W	Nov. 27	**THANKSGIVING**		
37	F	Nov. 29	**THANKSGIVING**		
38	M	Dec. 2	Second Moments of Area: By Composite Parts	12.B	H38.A, H38.B
39	W	Dec. 4	Second Moments of Area: By Integration	12.B, D	H39.A, H39.B
R	F	Dec. 6	**Review for Final Exam**	Ch. 1-12	Practice Exams
E			**FINAL EXAM (details to be announced); (Covers Lectures 1-39)**		

Coding: Integer = Lecture number; B = Break; E = Exam; R = Review lecture.
Homework numbers correspond to lecture numbers. Review lectures do not increase counter. Homework is due the class period after it is assigned.

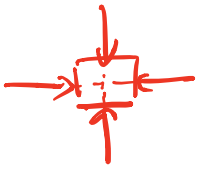
TEXTS
ME 270 textbook ("Statics: A Lecturebook", 2nd Edition, Fall 2019).

ME 270 BLOG
You will automatically be added to the blog once you have registered for ME 270. Once you have access to the blog, you can adjust your email settings to receive all, some or none of the posting, as according to your preference.

Lec 8 09/06/2019 (Fri.)

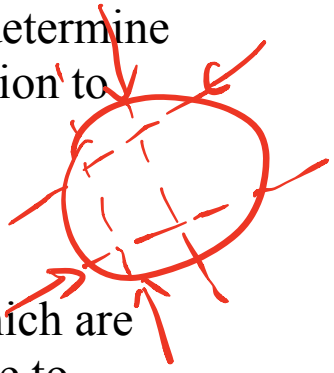
FREE BODY DIAGRAMS (FBDs)

Recall: particle



Learning Objectives

1). To inspect the supports of a rigid body in order to determine the nature of the reactions, and to use that information to draw a *free body diagram* (FBD).



Force/Moment Classifications

Usually known

External Forces/Moments: applied forces/moments which are typically known or prescribed (e.g., forces/moments due to cables springs, gravity, etc.).

usually unknown

Reaction Forces/Moments: constraining forces/moments at supports intended to prevent motion (usually nonexistent unless system is externally loaded).

Free Body Diagram (FBD)

Free Body Diagram (*FBD*): a graphical sketch of the system showing a coordinate system, all external/reaction forces and moments, and key geometric dimensions.

Based on given → solve for unknown

Benefits:

1). Provides a *coordinate system* to establish a solution methodology.

2). Provides a *graphical display* of all forces/moments acting on the rigid body.

3). Provides a record of *geometric dimensions* needed for establishing moments of the forces.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

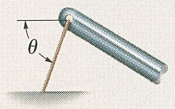
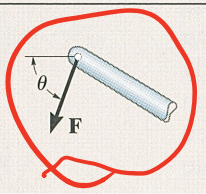
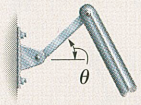
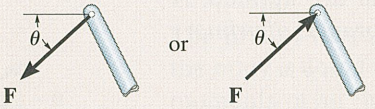
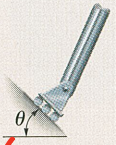
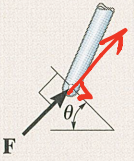
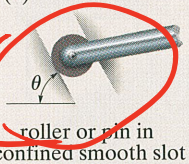
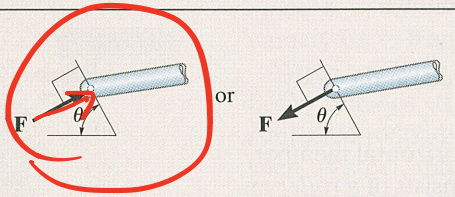
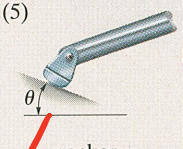
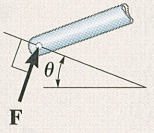
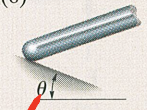
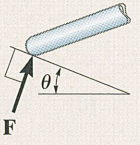
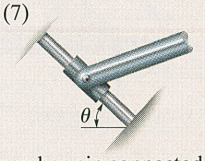
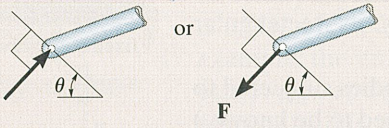
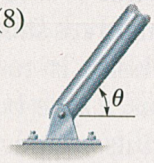
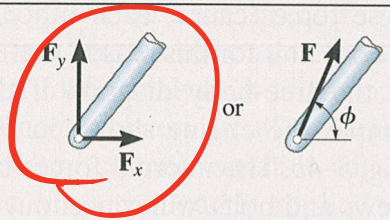
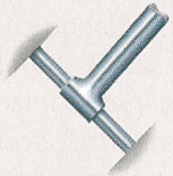
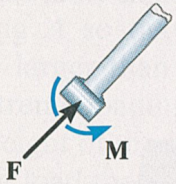
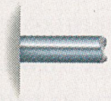
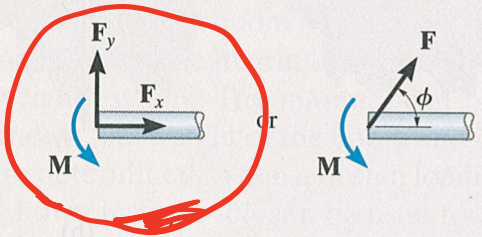
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link		One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

STATIC EQUILIBRIUM OF RIGID BODIES (2-D)

Learning Objectives

1). To evaluate the *unknown reactions* holding a rigid body in equilibrium by solving the *equations of static equilibrium*.
2). To recognize situations of *partial* and *improper constraint*, as well as *static indeterminacy*, on the basis of the solvability of the equations of static equilibrium.

Newton's First Law

& no net moment

Given *no net force*, a body at rest will remain at **rest** (and a body moving at a constant velocity will continue to do so along a straight path).

Definitions

Zero-Force Members: structural members that support no loading but aid in the stability of the truss.

Two-Force Members: structural members that are: a) subject to no applied or reaction moments, and b) are loaded only at two pin joints along the member.

Multi-Force Members: structural members that have a) applied or reaction moments, or b) are loaded at more than two points along the member.

to be covered in the future

- Equivalent system: net force & net moment of applied force & moment.
- Static equilibrium: net force & net moment of applied & reaction force & moment = 0.

vs.

Vector Equations

$$\overline{F_R} = \sum \overline{F} = \overline{0}$$

$$\overline{M_{R_O}} = \sum \overline{M_O} = \overline{0}$$

where O is any arbitrary point

Component Equations

There are three alternate forms of equilibrium equations for 2-D problems.

(i) Two component force equations (x and y) are one moment equation (z).

most common set of Eq's.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0$$

Additional moment equilibrium Eq. compared to Particle cases.

(ii) One component force equation (x or y) and two moment equations (both about different points in the z direction).

$$\sum F_x = 0 \qquad \sum M_A = 0 \qquad \sum M_B = 0$$

(iii) Three moment equations (points A, B and C cannot be collinear).

$$\sum M_A = 0 \qquad \sum M_B = 0 \qquad \sum M_C = 0$$

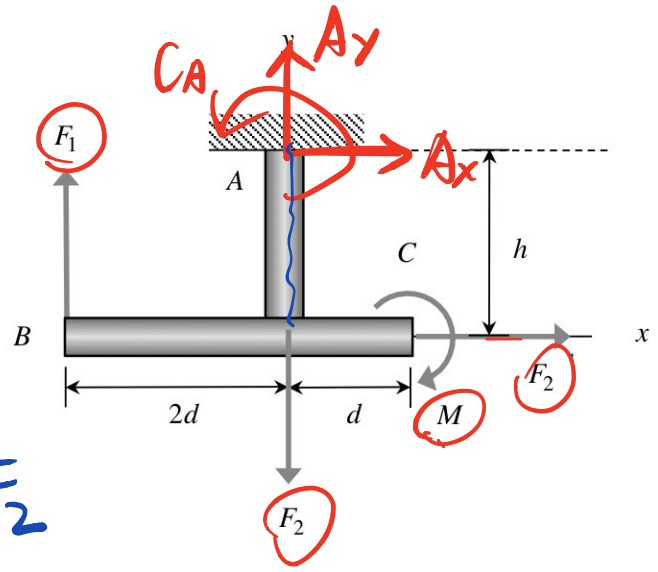
Example 4.E.7

Given: The frame is acted upon the loading shown.

Find: Determine the reactions at support A due to this loading.

Use the following parameter values in your analysis: $d = 3$ ft, $h = 5$ ft, $F_1 = 300$ lb, $F_2 = 200$ lb and $M = 800$ ft·lb.

magnitudes



(1) $\Sigma F_x = 0 = A_x + F_2$

$\Rightarrow A_x = -200$ lb

(2) $\Sigma F_y = 0 = A_y + F_1 - F_2$

$\Rightarrow A_y = -100$ lb

$\bar{A} = -200\,\bar{i} - 100\,\bar{j}$ lb

(3) $\Sigma M_A = C_A - F_1(2d) + F_2(h) - M = 0$

$C_A - 300(6) + 200(5) - 800 = 0$

$\Rightarrow C_A = 1600$ lb ft

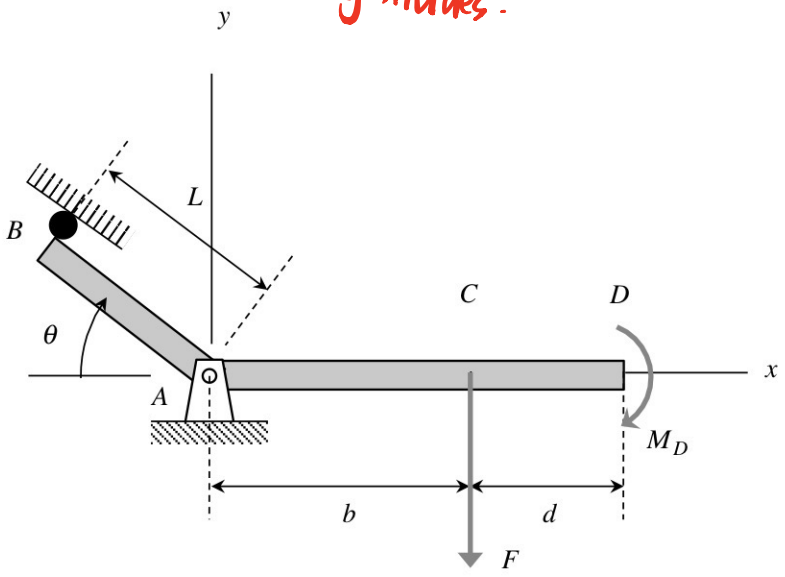
$\bar{C}_A = 1600\,\bar{k}$ lb ft

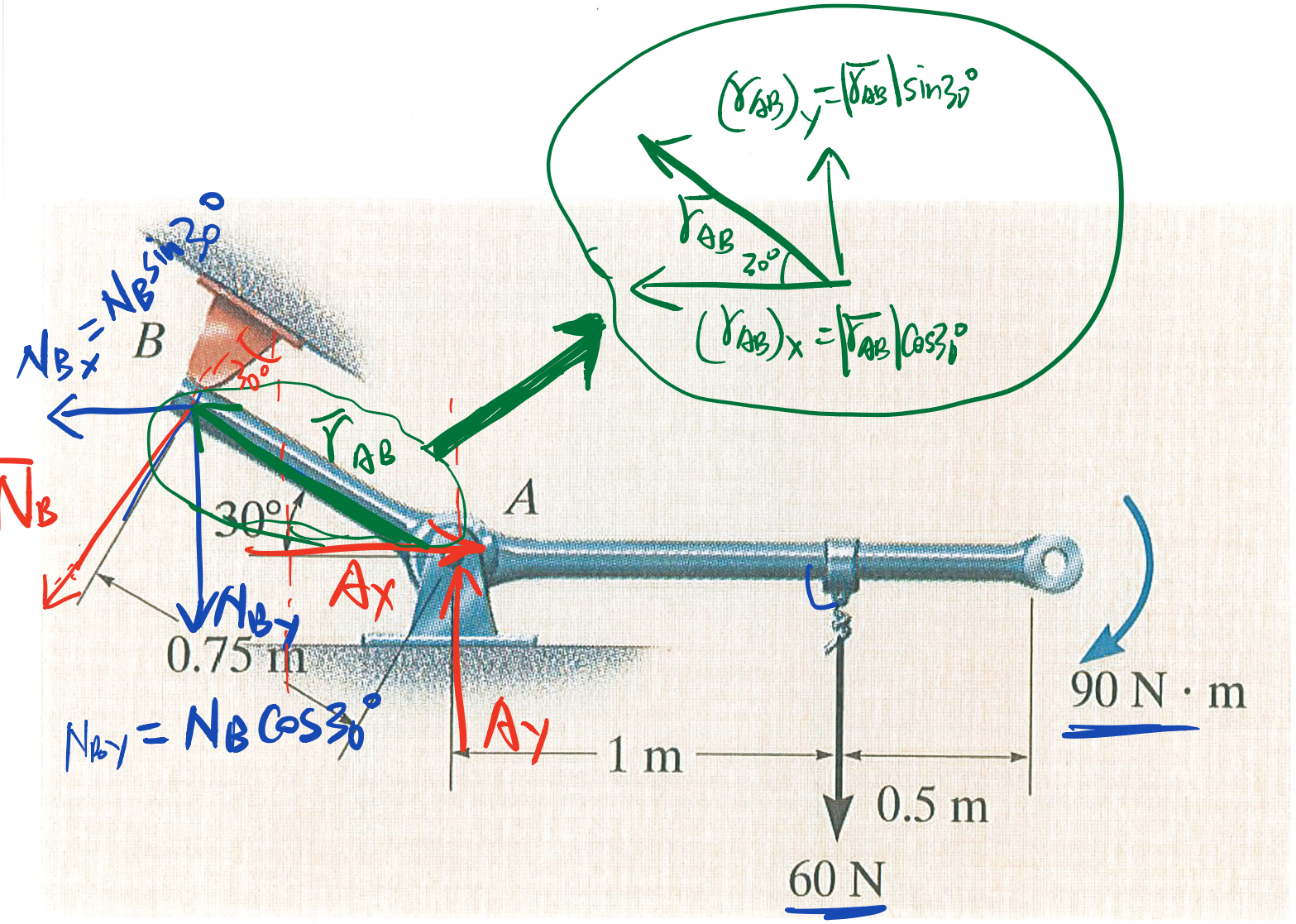
Example 4.E.9

Given: An angled bar is loaded with a force F at C and with a couple M_D at D. The bar is held in equilibrium through the pin support at A and the roller support at B.

Find: For this loading:
(a) Draw the free body diagram of the bar.
(b) Write down the equilibrium equations for the bar.
(c) Solve the equilibrium equations for the reactions on the bar at A.

Use the following parameter values in your analysis: $F = 60$ N, $M_D = 90$ N·m, $d = 0.5$ m, $b = 1$ m, $L = 0.75$ m and $\theta = 30°$.





① $\Sigma M_A = 0 = -90 - (60)(1) + (N_B)(0.75)$

$\Rightarrow N_B = 200\ N$ $+ N_{Bx}(r_{AB})_y + N_{By}(r_{AB})_x$

RHR

② $\Sigma F_x = 0 = A_x - N_B \sin 30°$

$$\Sigma F_x = 0 = A_x - N_B \sin 30^\circ$$

$$\Rightarrow \quad A_x = 200 \sin 30^\circ = 100 N$$

$$\Sigma F_y = 0 = A_y - 60 - N_B \cos 30^\circ$$

$$\Rightarrow \quad A_y = 233 N$$

$$\bar{A} = 100\,\bar{i} + 233\,\bar{j} \ N$$

$$\bar{N}_B = -N_B \sin 30^\circ \bar{i} - N_B \cos 30^\circ \bar{j}$$

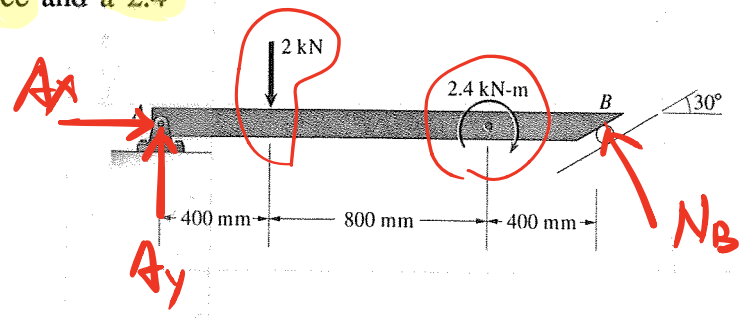
$$= -100\,\bar{i} - 173\,\bar{j} \ N$$

ME 270 - Basic Mechanics I - Group Quiz

Name/Group #: _____ Group Members: 1) _____ 2) _____

Date: _____ Period: _____ 3) _____ 4) _____

Given: Beam AB is loaded with a 2 kN force and a 2.4 kN-m couple as shown.



Find:

(a) Draw a free body diagram of beam AB.

(b) Determine the reaction at the support.

(c) If the reaction at roller B was found to be negative, what would this imply physically about the system.

Solution:

ME 270 - Basic Mechanics I - Group Quiz

Name/Group #: _____ Group Members: 1) _____ 2) _____

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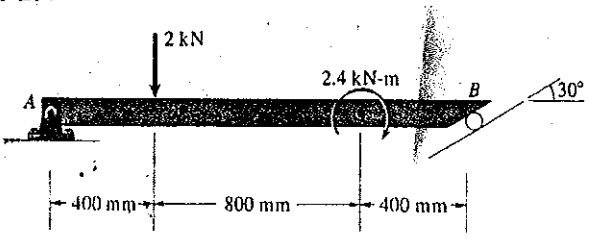


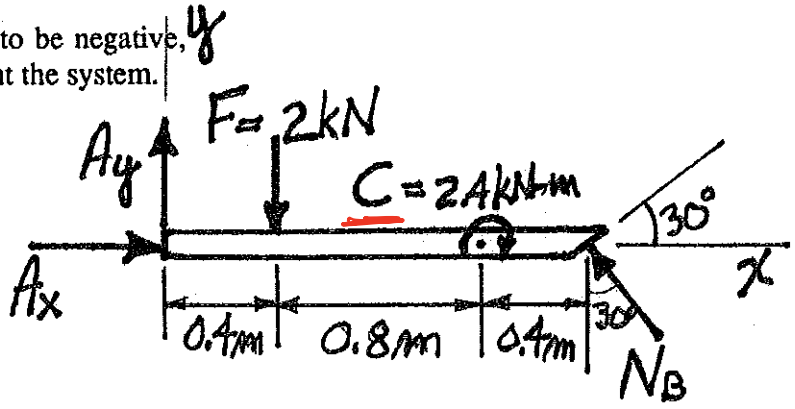
Find:

(a) Draw a free body diagram of beam AB.

(b) Determine the reaction at the support.

(c) If the reaction at roller B was found to be negative, what would this imply physically about the system.

Solution: (a)



(b) $\Sigma M_A = 0 = -r_1(F) - C + r_2(N_B \cos 30°)$

$= -(0.4)(2) - 2.4 + (1.6)(N_B \cos 30°)$

$\therefore N_B = 2.31\,kN$

$\Sigma F_x = 0 = A_x - N_B \sin 30°$

$A_x = N_B \sin 30° \Rightarrow A_x = 1.15\,kN$

$$\Sigma F_y = 0 = A_y - F + N_B \cos 30^\circ$$

$$A_y = F - N_B \cos 30^\circ = 2 - 2.31 \cos 30^\circ$$

$$\therefore A_y = 0.00\ \text{kN}$$

(c) If N_B were negative, it would mean that the force was shown in the wrong direction on the free body diagram.

Note, it would be physically impossible for N_B to be negative for the roller boundary condition shown. If static equilibrium requires N_B to be negative, then it is impossible for the system to be in static equilibrium.