 NAME (Last, First): ________________________________

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ______________________________________

Instructor's Name and Section: (Circle Your Section)

Sections: J Jones 9:30-10:20 AM K Zhao 1:30-2:20 PM F Semperlotti 4:30-5:20 PM

J Jones Distance Learning

Please review and sign the following statement:

Purdue Honor Pledge – "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – We are Purdue."

Signature: ______________________________________

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, use the white lined paper provided to you.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented. Also, please make note of the following instructions.

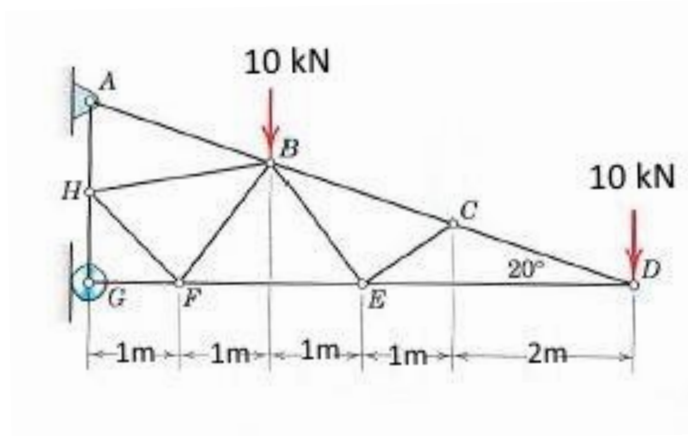
- The only authorized exam calculators are the TI-30XIIS or the TI-30Xa.
- The allowable exam time for the Final Exam is 120 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.
- Please use a **black pen** for the exam.
- Do not write on the back side of your exam paper.

If the solution does not follow a logical thought process, it will be assumed in error.

When handing in the test, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of every page that you wish to have graded.

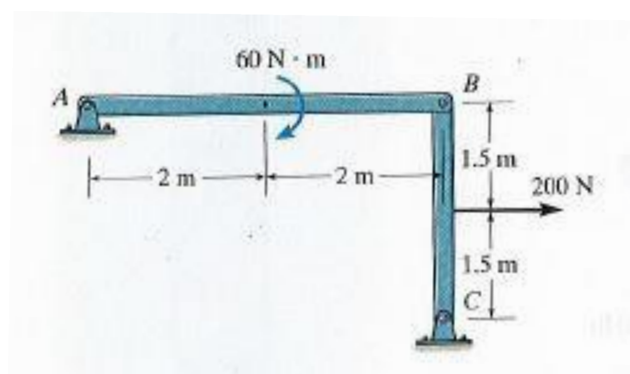
PROBLEM 1 (20 points)

1A. For the truss shown, identify all zero-force members and determine the magnitude of the load in member CD and whether it is in tension or compression or zero.



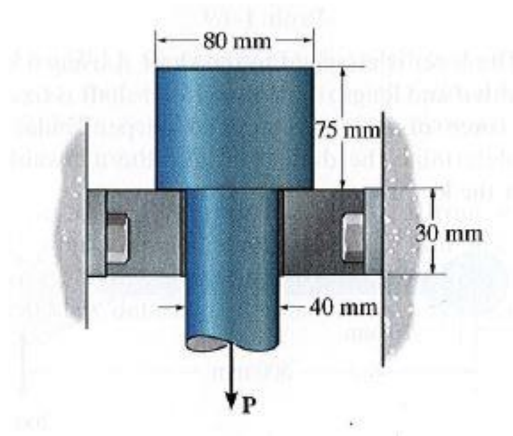
$\boldsymbol{Zero\ Force\ Members} \quad =$						(3 pts)
$\boldsymbol{F_{CD}} \quad =$	**kN**	**Tension**	**Zero**	**Compression**	**(Circle One)**	(2 pts)

1B. For the frame shown, determine the forces acting at pin B on both members AB and BC. Express both forces in vector form.



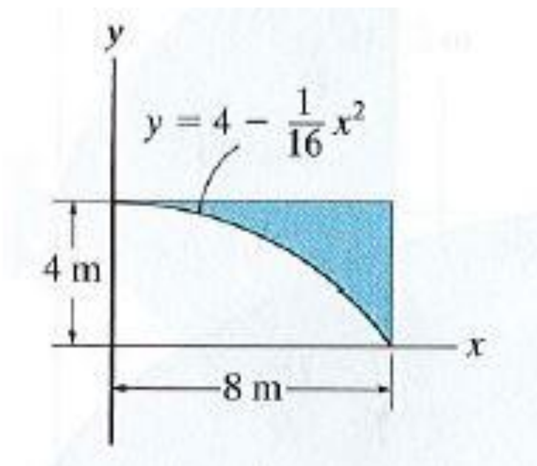
$(\overline{\boldsymbol{F}}_{\boldsymbol{B}})_{\boldsymbol{on\ AB}} = (\qquad\qquad)\bar{\boldsymbol{\imath}} + (\qquad\qquad)\,\bar{\boldsymbol{\jmath}} \quad \boldsymbol{N}$	(3 pts)
$(\overline{\boldsymbol{F}}_{\boldsymbol{B}})_{\boldsymbol{on\ BC}} = (\qquad\qquad)\bar{\boldsymbol{\imath}} + (\qquad\qquad)\,\bar{\boldsymbol{\jmath}} \quad \boldsymbol{N}$	(2 pts)

1C. A bolt head is made of a material having a shear failure of $\tau = 120MPa$. Using a factor of safety of F.S. = 2.5 against shear failure, determine the allowable shear stress (τ_{allow}) and the maximum allowable force P that can be applied to the bolt so that it does not pull through the rigid plate.



$\boldsymbol{\tau_{allow}}$ =		**MPa**	(2 pts)
$\boldsymbol{P_{max}}$ =		**N**	(3 pts)

1D. Determine the second moment of area about the y-axis (I_y) of the shaded shape. Qualitatively, would you expect I_x to be greater than, equal to or less than I_y? (No calculations are required).



I_y =					(3 pts)
I_x =	**Greater Than**	**Equal To**	**Less Than**	**(Circle One)**	(2 pts)

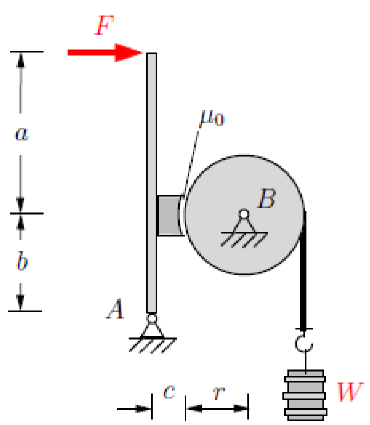
PROBLEM 2 - (20 points) Partial credit will not be given unless the solution procedure is clearly detailed.

2A. Consider the system in figure where a weight *W* is suspended by an inextensible cable connected to a pulley. The pulley is free to rotate without friction around the pinned joint B. A lever is hinged at A and can be actuated by a force of magnitude *F* so that the breaking pad can come into contact with the pulley. The coefficient of static friction between the breaking pad and the pulley is μ_0.

- **Draw** a complete free body diagram of both the bar and the pulley on the schematic provided below on the right.
- **Determine** the minimum magnitude of the applied force *F* that will prevent the pulley from rotating under the effect of the weight *W*.

Assume W = 10 N, a = 0.5 m, b = 1 m, c = 0.1 m, μ_0 = 0.3.

Hint: note that the point of contact between the breaking pad and the pulley is at a distance c from the point A. Assume the friction force between the pad and the pulley to be solely in the vertical direction (i.e. the curvature of the pad does not have any effect).

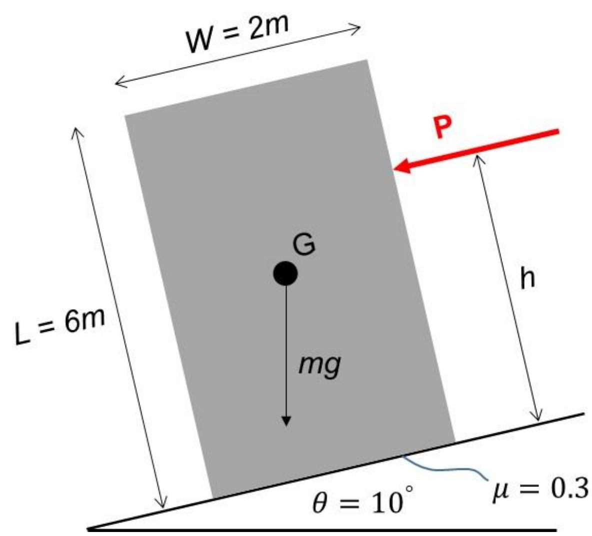


FBD (5pts)

$F\ =$	(5 pts)

2B. A rectangular cabinet of weight *mg* rests on an incline. The coefficient of static friction between the incline and the cabinet is $\mu = 0.3$. A force P is applied to the cabinet as in figure. **Find**:

- The magnitude of the force P and the height *h* (which indicates the point of application of the force P) such that the cabinet will simultaneously slip and tip.
- How would the value of *h* change if the plane was horizontal (θ = 0)?

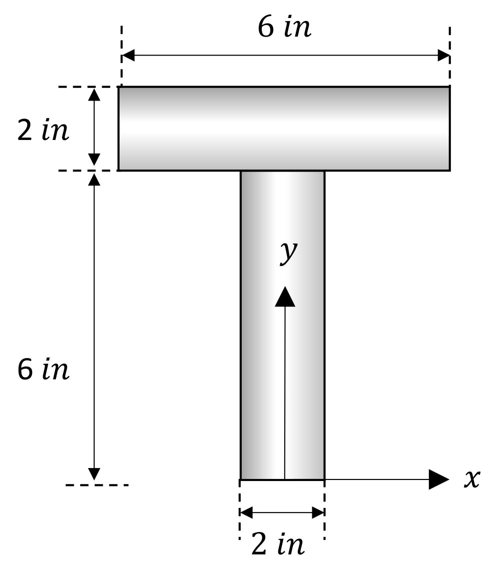


$P =$				(4 pts)
$h =$				(4 pts)
h when $\theta = 0$ (circle one)	Increase	Decrease	Remain the same	(2 pts)

PROBLEM 3. (20 points)

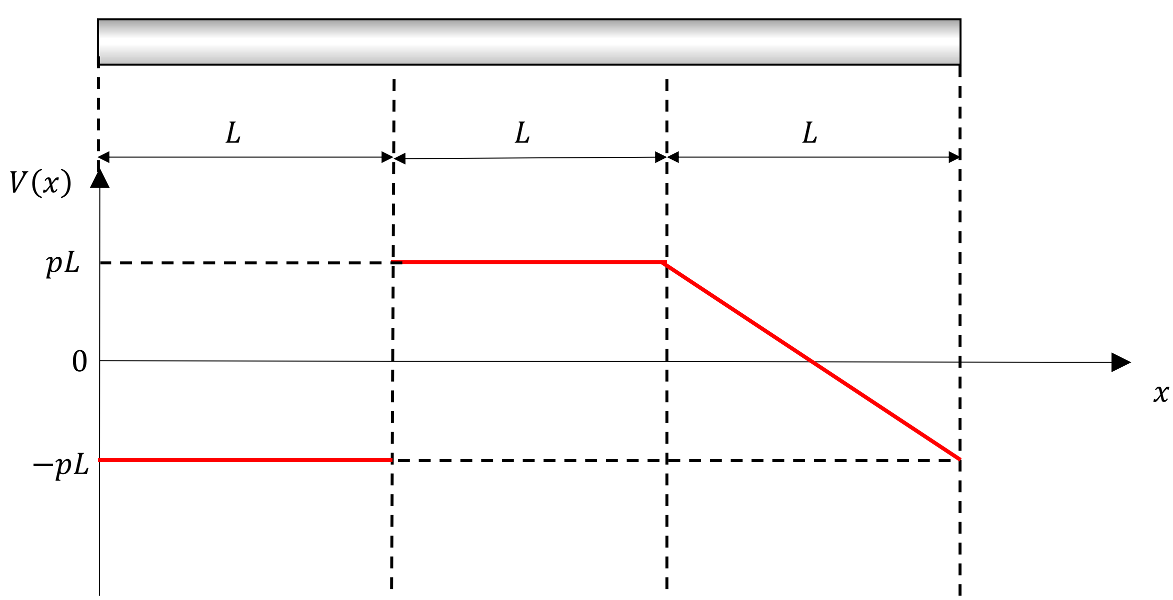
3A. A T-shape cross section is shown right. Determine:

- The location of the centroid (x_c, y_c).
- The second area moment about the horizontal axis which passes its centroid I_c.

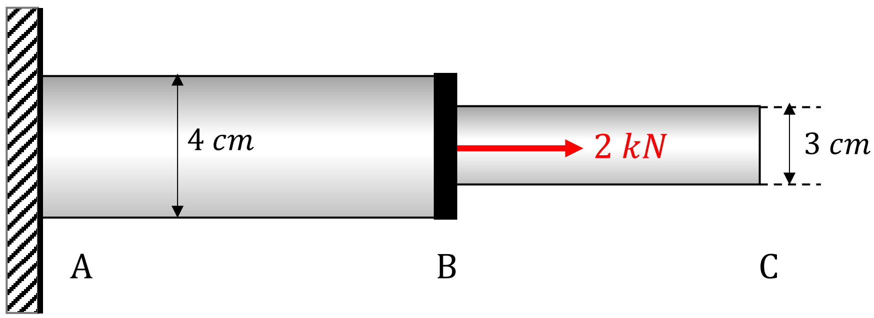


$(x_{c,}\, y_c) =$	**(3pts)**
$I_c =$	**(3pts)**

3B. A beam is loaded only by concentrated and distributed forces (no external moment applied). Given the shear force diagram shown below, draw the external loading (both concentrated and distributed forces) and lab their magnitudes on the figure of the beam below. (**4pts**)



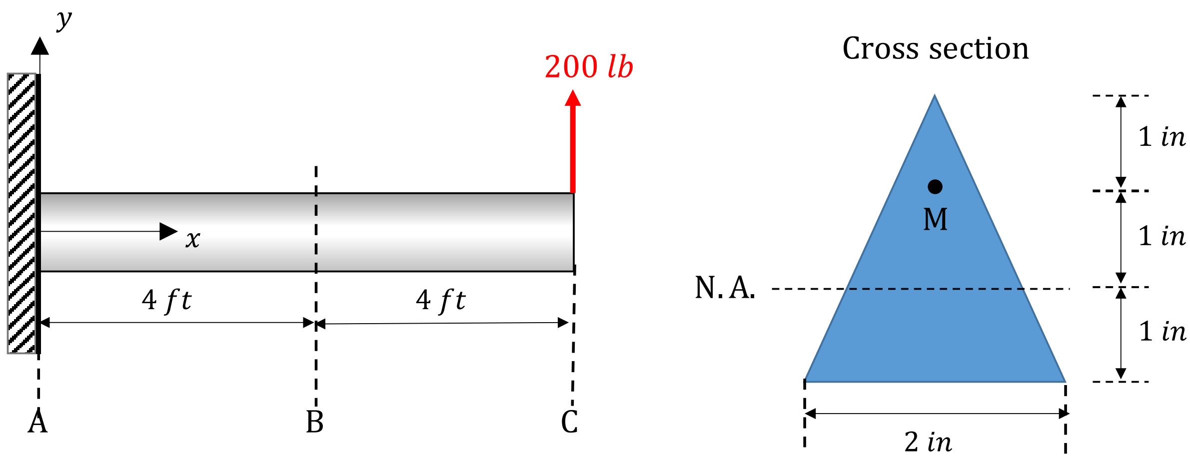
3C. A stepped shaft composed of components AB and BC is shown below. AB and BC are joined by a rigid connector at B. Both AB and BC have a circular cross section, and their diameters are 4 cm and 3 cm, respectively. An external force 2 kN is applied at B. Determine the internal axial stress in AB and BC.



$\sigma_{AB} =$	**Pa**	**(2pts)**
$\sigma_{BC} =$	**Pa**	**(2pts)**

3D. A cantilever beam ABC is subjected to a concentrated force 200 lb at C. The beam has a triangular cross section. The second area moment of a triangular cross section about its centroid axis is $I_c = \frac{bh^3}{36}$, where *b* and *h* are the width and height of the shape, respectively. Determine:

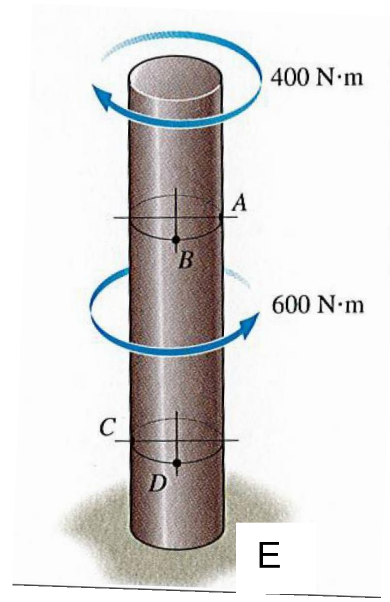
- The normal stress at M within the cross section B, σ_M.
- The maximum tensile stress in the cross section B, σ_{max}.



$\boldsymbol{\sigma_M} =$	**(3pts)**
$\boldsymbol{\sigma_{max}} =$	**(3pts)**

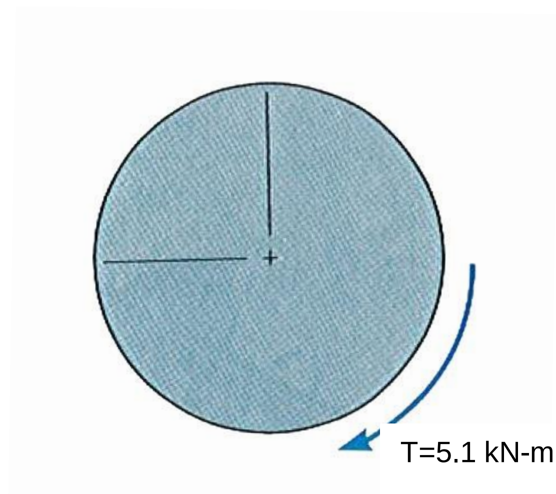
PROBLEM 4. (20 points)

4A A solid circular shaft is loaded as shown and is held in static equilibrium by a fixed support at E. Determine the magnitude of the torque experienced in plane AB. Show your free body diagram.



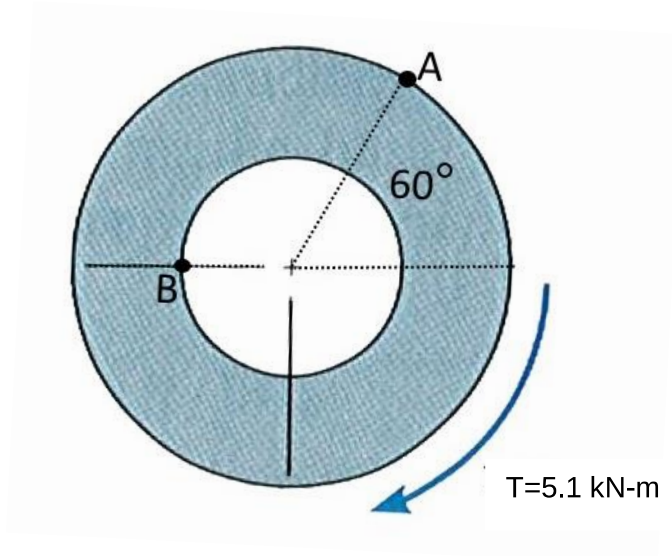
T =	N-m	(2pts)

4B If a solid circular shaft has an applied torque is T = 5.1 kN-m and the allowable stress is τ_{Allow} = 50MPa, what is the minimum diameter shaft that could be used and still insure the allowable stress was not exceeded. Express the minimum shaft size to the nearest whole mm.



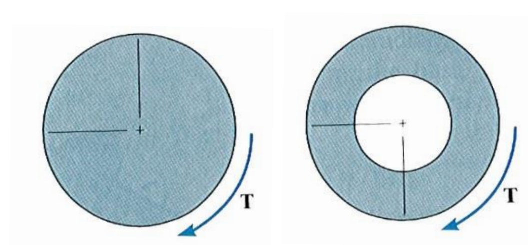
d_{min} =	mm	(4pts)

4C If a tubular shaft has an outside diameter d_o = 200mm and an inside diameter d_i = 100mm, and if the applied torque is T = 5.1 kN-m, determine the polar moment of inertia (J) and the shear stress at points A ($\boldsymbol{\tau_A}$) and B ($\boldsymbol{\tau_B}$) shown on the figure.



J =	**m^4**	**(4pts)**
$\boldsymbol{\tau_A}$ =	**Pa**	**(4pts)**
$\boldsymbol{\tau_B}$ =	**Pa**	**(4pts)**

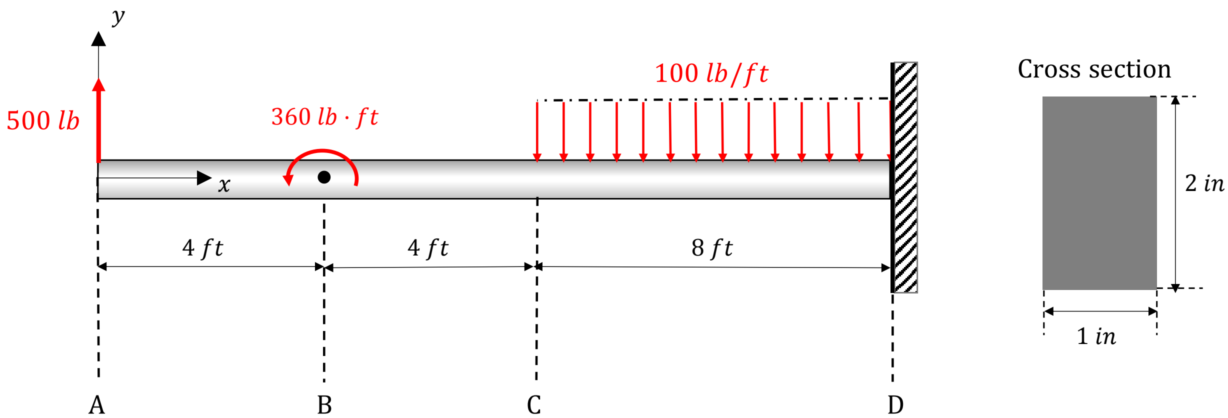
4D If a solid circular shaft is replaced by a tubular shaft of the same outside diameter (d_o), will the maximum shear stress qualitatively increase, remain the same, or decrease given the applied torque is the same for both shafts? No work needs to be shown.



τ_{max} =	**Increase**	**Remain the Same**	**Decrease**	**(Circle One)**	**(2pts)**

PROBLEM 5. (20 points)

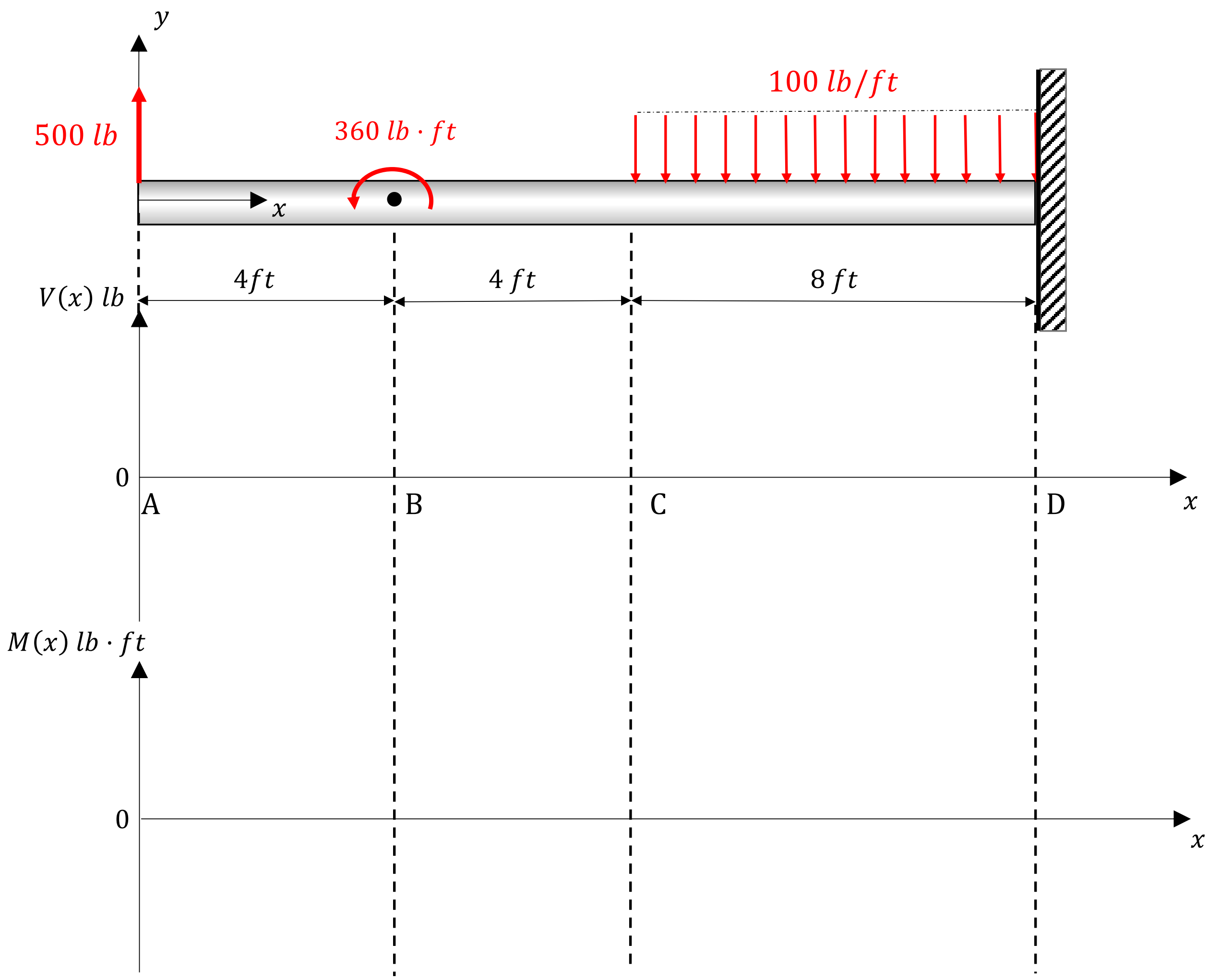
The cantilever beam is subjected to the loading condition as shown below.



a) Draw the Free-Body Diagram of the beam and determine the reactions at the wall. Write your answer in the given box.

$\vec{F}_{wall} = ($	$)\hat{\imath} + ($	$)\hat{\jmath} + ($	$)\hat{k}$	**(2pts)**
$\vec{M}_{wall} = ($	$)\hat{\imath} + ($	$)\hat{\jmath} + ($	$)\hat{k}$	**(2pts)**

b) On the axes provided below, construct the shear force (V vs. x), and the bending moment (M vs. x) diagrams. Specify the shear & moment values at A, B, C, D, and any maximum and minimum values. **(12 pts)**



c) Determine the value of the maximum tensile stress within the beam, and the location of the maximum tensile stress. Write your answer in the given box.

$\sigma_{max} =$	**(2pts)**
Location of the maximum tensile stress $(x, y) =$	**(2pts)**

1A. Zero Force Members = BE, CE, GH $F_{CD} = 29.2kN$ Tension

1B. $(\bar{F}_B)_{on\ AB} = (100)\bar{\imath} + (15)\bar{\jmath}\ N$ $(\bar{F}_B)_{on\ BC} = (-100)\bar{\imath} + (-15)\bar{\jmath}\ N$

1C. $\tau_{allow} = 48\ MPa$ $P_{max} = 452{,}160\ N$

1D. $I_y = 409.6m^4$ $I_x = Less\ Than$

2A. $F = 22.89\ N$

2B. $h = 3.81\ m$ $h\ when\ \theta = 0$ $Increase$

3A. $(x_{c,}y_c) = (0, 5)\ \ in$ $I_c = 136in^4$

3B. Diagram

3C. $\sigma_{AB} = 1.59x10^6\ Pa$ $\sigma_{BC} = 0\ Pa$

3D. $\sigma_M = -6400\ psi$ $\sigma_{max} = 6400\ psi$

4A. $T = 400\ N - m$

4B. $d_{min} = 81\ \ mm$

4C. $J = 0.000147\ m^4$ $\tau_A = 3.47\ MPa$ $\tau_B = 1.73\ MPa$

4D. Increase

5A. Free Body Diagram $\vec{F}_{wall} = (0)\hat{\imath} + (300)\hat{\jmath} + (0)\hat{k}\ \ lb$

$\vec{F}_{wall} = (0)\hat{\imath} + (0)\hat{\jmath} + (4440)\hat{k}\ \ lb.ft$

5B. Shear-Force and Bending-Moment Diagrams

5C. $\sigma_{max} = 88020\ psi$

Location of the maximum tensile stress (x, y) = (13ft, -1 in)

Spring 2018 Final Exam – Equation Sheet

Normal Stress and Strain

$$\sigma_x = \frac{F_n}{A}$$

$$\sigma_x(y) = \frac{-My}{I}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{\Delta L}{L}$$

$$\varepsilon_y = \varepsilon_z = -\vartheta\varepsilon_x$$

$$\varepsilon_x(y) = \frac{-y}{\rho}$$

$$FS = \frac{\sigma_{fail}}{\sigma_{allow}}$$

Shear Stress and Strain

$$\tau = \frac{V}{A}$$

$$\tau(\rho) = \frac{T\rho}{J}$$

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\vartheta)}$$

$$\gamma = \frac{\delta_s}{L_s} = \frac{\pi}{2} - \theta$$

Second Area Moment

$$I = \int_A y^2 dA$$

$$I = \frac{1}{12}bh^3 \qquad \text{Rectangle}$$

$$I = \frac{\pi}{4}r^4 \qquad \text{Circle}$$

$$I_B = I_O + A{d_{OB}}^2$$

Polar Area Moment

$$J = \frac{\pi}{2}r^4 \qquad \text{Circle}$$

$$J = \frac{\pi}{2}(r_o^4 - r_i^4) \quad \text{Tube}$$

Shear Force and Bending Moment

$$V(x) = V(0) + \int_0^x p(\epsilon)d\epsilon$$

$$M(x) = M(0) + \int_0^x V(\epsilon)d\epsilon$$

Buoyancy

$$F_B = \rho g V$$

Fluid Statics

$$p = \rho g h$$

$$F_{eq} = p_{avg}(Lw)$$

Belt Friction

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

Distributed Loads

$$F_{eq} = \int_0^L w(x)dx$$

$$\bar{x}F_{eq} = \int_0^L x\, w(x)dx$$

Centroids

$$\bar{x} = \frac{\int x_c dA}{\int dA} \qquad \bar{y} = \frac{\int y_c dA}{\int dA}$$

$$\bar{x} = \frac{\sum_i x_{ci}A_i}{\sum_i A_i} \qquad \bar{y} = \frac{\sum_i y_{ci}A_i}{\sum_i A_i}$$

In 3D, $\bar{x} = \dfrac{\sum_i x_{ci}V_i}{\sum_i V_i}$

Centers of Mass

$$\tilde{x} = \frac{\int x_{cm}\rho dA}{\int \rho dA} \qquad \tilde{y} = \frac{\int y_{cm}\rho dA}{\int \rho dA}$$

$$\tilde{x} = \frac{\sum_i x_{cmi}\rho_i A_i}{\sum_i \rho_i A_i} \qquad \tilde{y} = \frac{\sum_i y_{cmi}\rho_i A_i}{\sum_i \rho_i A_i}$$