ME 270 – Fall 2015 Final Exam **NAME (Last, First):** ________________________________

Please review the following statement: **Group Number** (if Applicable):_________

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: ____________________________________

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, use the white lined paper provided to you.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented, i.e.

- The only authorized exam calculator is the TI-30IIS
- The allowable exam time for the Final Exam is 120 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.

If the solution does not follow a logical thought process, it will be assumed in error.

When handing in the test, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of every page that you wish to have graded.

Instructor's Name and Section:

Sections:			
	J Jones 9:30-10:20AM	I Bilionis 12:30-1:20PM	Yangfan Liu 4:30-5:20PM
	J Jones Distance Learning	J Gilbert 2:30-3:20PM	M Murphy 10:30-11:45AM
	E Nauman 8:30-9:20AM	KM Li 11:30AM-12:20PM	

Problem 1 __________

Problem 2 __________

Problem 3 __________

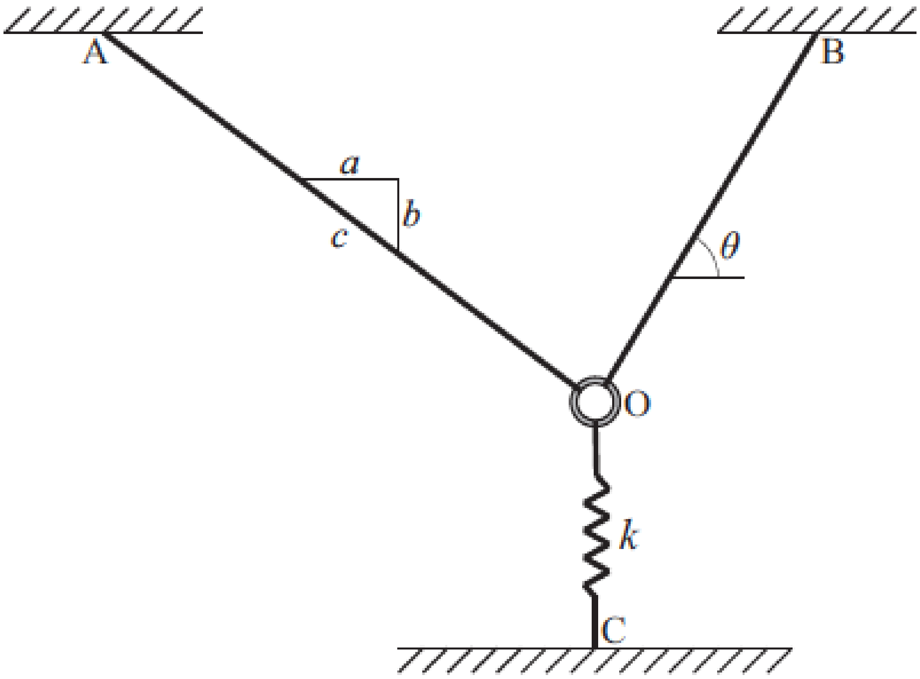
Problem 4 __________

Problem 5 __________

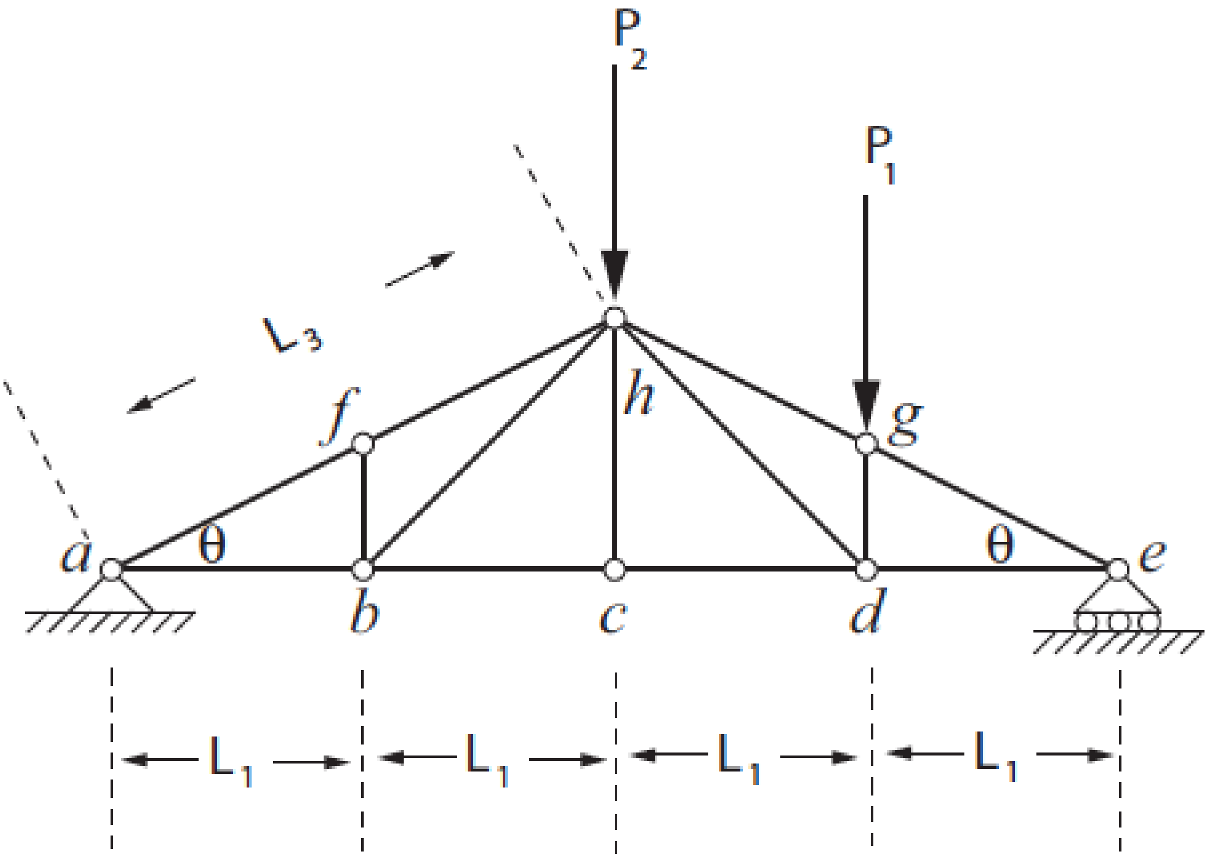
Total ______________

PROBLEM 1.

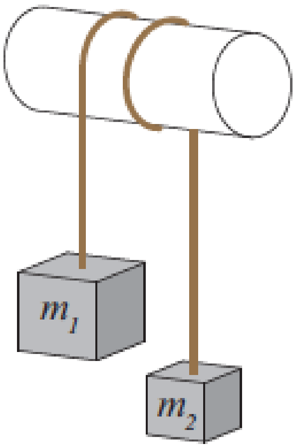
1a. Equilbrium. The ring is held in equilibrium by two cables and a spring. At equilibrium the spring is stretched 0.15 m and the spring constant, k = 1,000 N/m. In order to maintain equilibrium, the cable OB makes an angle, $\theta = 60^o$ and a, b, c makes a 4, 3, 5 triangle. Draw a free body diagram and determine the forces in the two cables. (7 points)



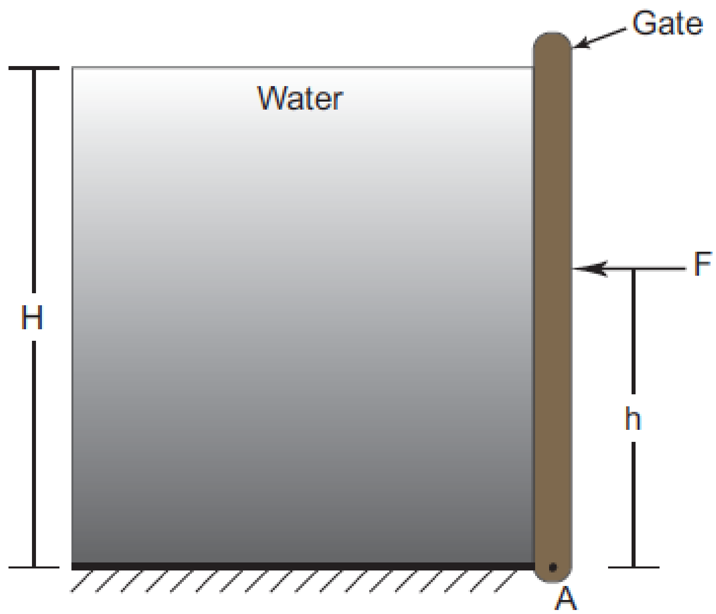
1b. Find the zero force members. Place a “0” on each zero force member in the truss below. (4points)



1c. Friction: If m_1 = 100 kg and μ = 0.35, what is the range of masses for m_2 that will keep the system in equilibrium? (4 points)

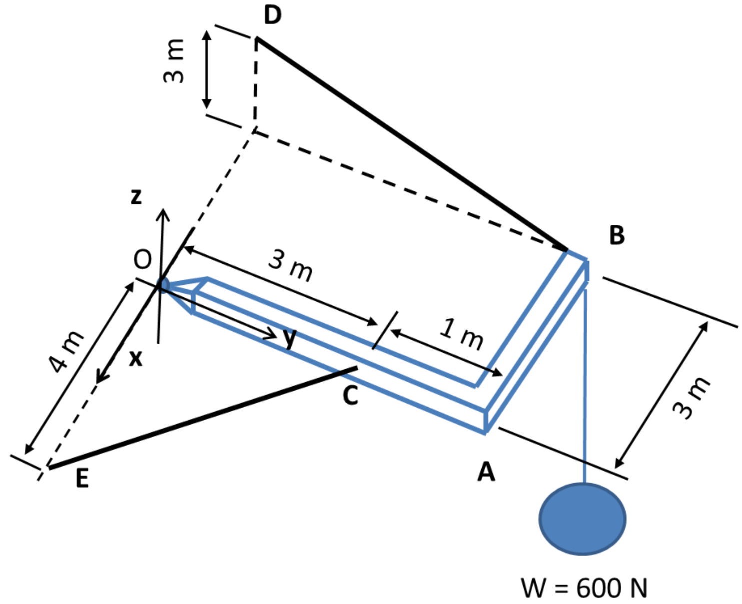


1d. Fluids: If H = 10 m, h = 3 m, the gate extends 1 m into the page, and the density of the water is 1,000 kg/m3, determine the force, F, required to hold the gate in place. (5 points)



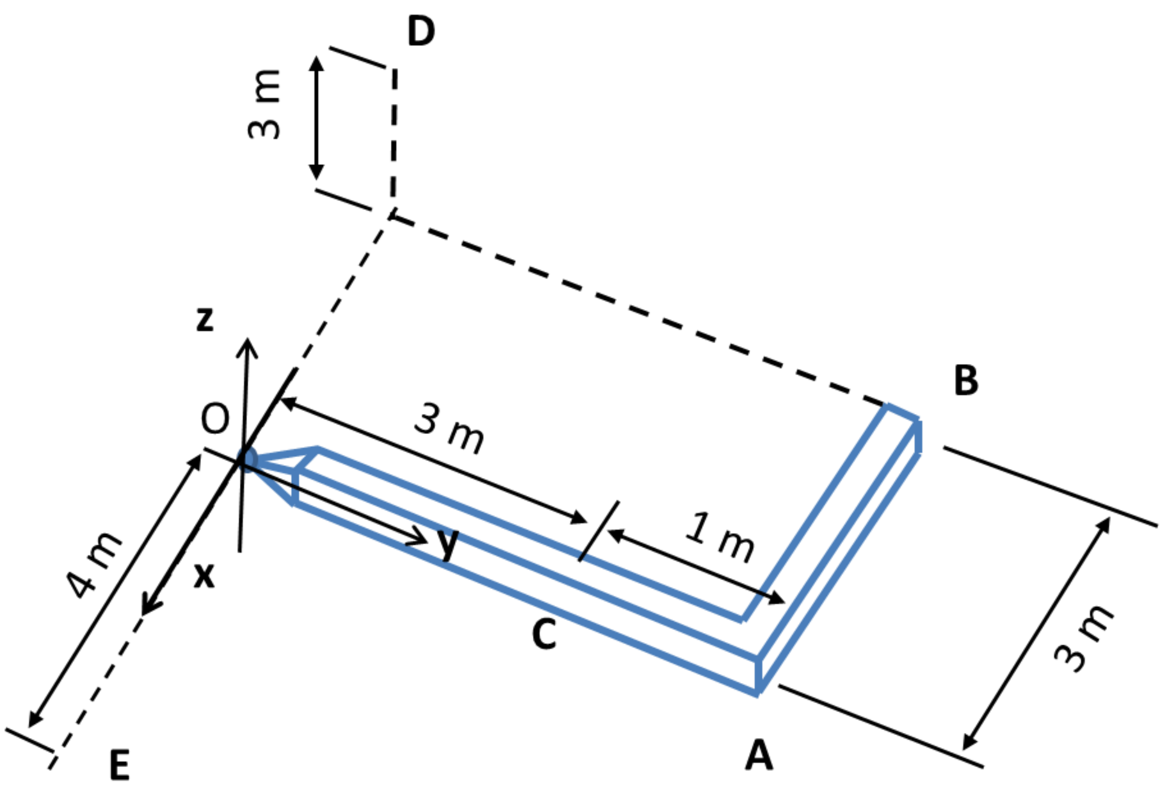
PROBLEM 2. (20 points)

GIVEN: A L-shaped beam, OAB, (ignore its weight) is supported at O with a ball joint, a weight of 600 N (acting in the negative z direction) is attached at B, and two cables BD and CE are used to connect the beam to the wall. Use the dimensions as shown in the left figure.



FIND:

a) Complete the **free body diagram** in the provided figure below (4 pts).



b) Express the tensions in the cables in terms of their unit vector and unknown magnitude (4 pts).

$\bar{T}_{BD}$= ____ (4 pts)

$\bar{T}_{CE}$= ____

c) Determine the magnitudes of the tensions in each cable (6 pts).

T_{BD}= ______________________________ (6 pts)

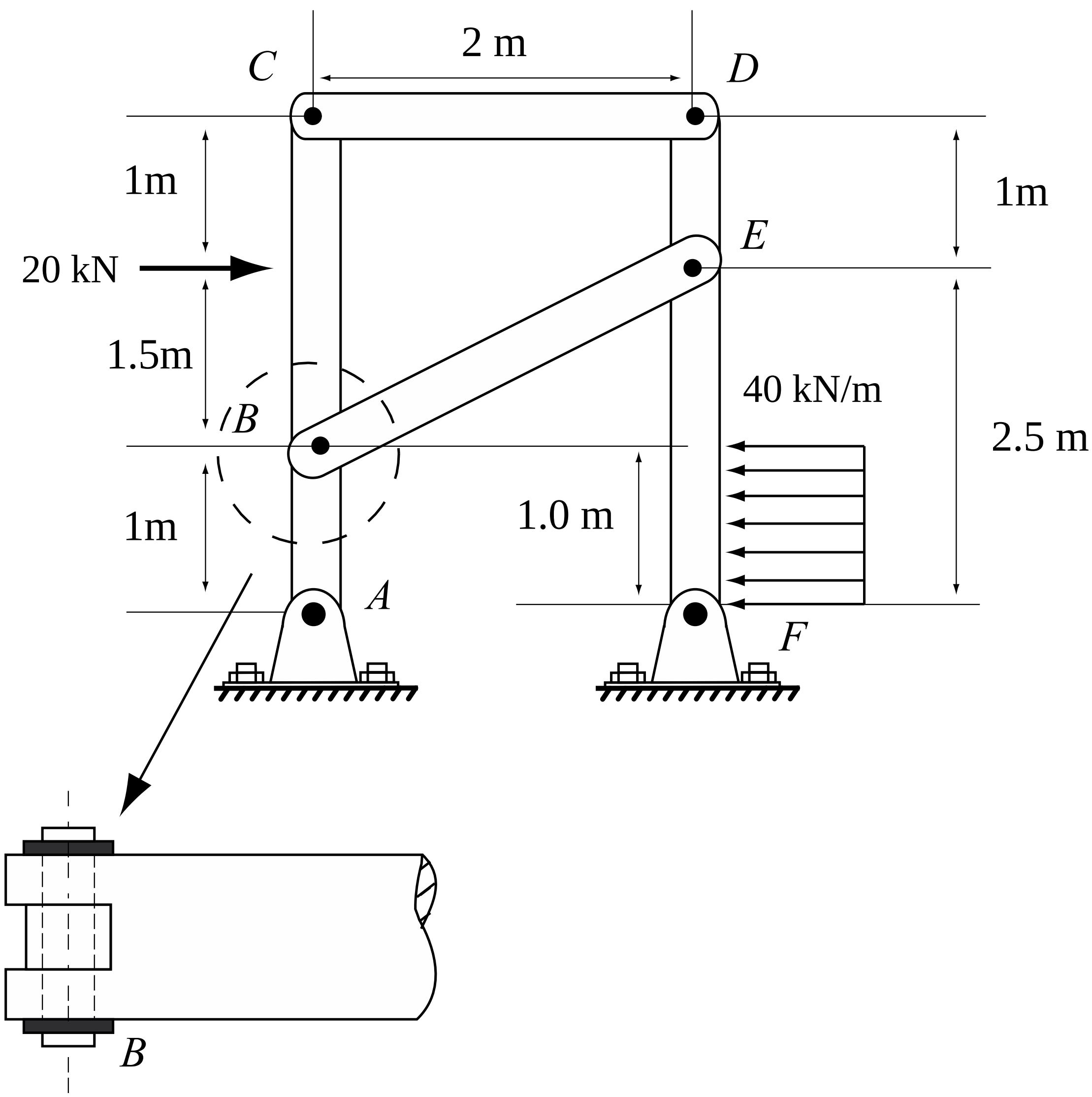
T_{CE}= ______________________________

d) Determine the reaction forces at the ball joint O and express it in the vector form (6 pts).

Reaction force at O: ____ (6 pts)

PROBLEM 3. (20 points)

GIVEN: Shown below is a frame of negligible weight that is pinned at *A* and *F*. A 20 kN force is applied to member *AC* of the frame and a rectangular distributed load with intensity of 40 kN//m is applied to member *DF*.

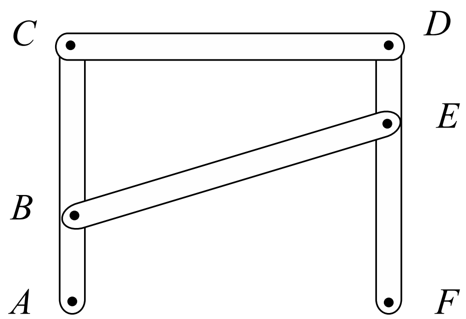


Joint Connection between Member *BE* and *AC*

FIND:

a) Circle the two force members (1 points): *AC*, C*D*, *BE*, *DF*

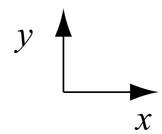
b) (10 points) Draw the FBD of the frame/ and member below. Fill in the blanks for the summation of forces and moments. **Indicate in the blank behind the summation of moment expression the point that you are taking the moments about**. In the FBD's be sure to reduce any distributed load to an equivalent force and given the appropriate location. **If the member is a two-force member leave the moment expression blank.**

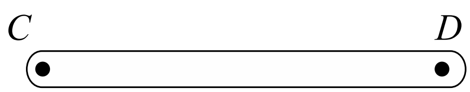


$\sum F_x$:

$\sum F_y$:

$\sum M$ __ :

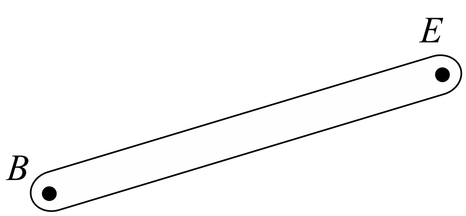




$\sum F_x$:

$\sum F_y$:

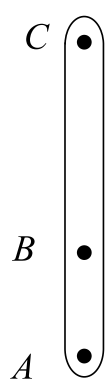
$\sum M$ __ :



$\sum F_x$:

$\sum F_y$:

$\sum M$ __ :



$\sum F_x$:

$\sum F_y$:

$\sum M$ __ :

c) (5 points) The force carried by member *BE* is ________ kN. What is the sign of the force + or - (circle one) .You may use the space below for your work.

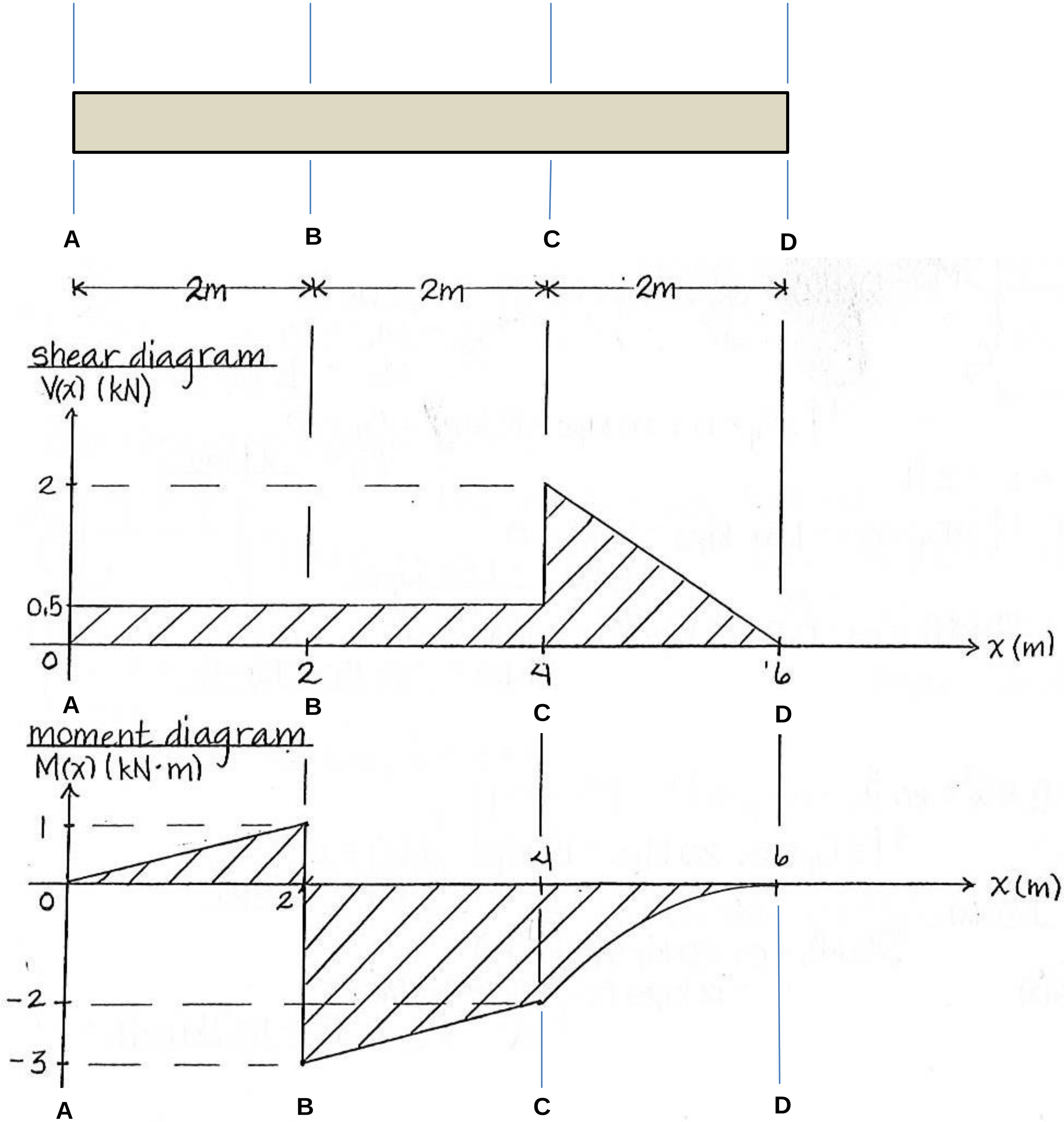
d) (2 points) Pin *B* has a diameter of 1 mm. Calculate the shear stress on pin *B* ,

t_B = ______________ Pa. You may use the space below for your work.

e) (2 points) If member *BE* is made of Aluminum (E = 73 x 10^9 Pa and σ_y= 410 x 10^6 Pa) and its cross section is square. Using a factor of safety of 2, the width/height of the beam is ____________ m to prevent failure from **yielding**. You may use the space below for your work.

PROBLEM 4

4a. Given the shear-force and bending-moment diagrams provided below, sketch the equivalent loading condition on the beam provided below. Make sure you indicate both the magnitude and the direction of each load. (5 pts)



PROBLEM 4 (cont.)

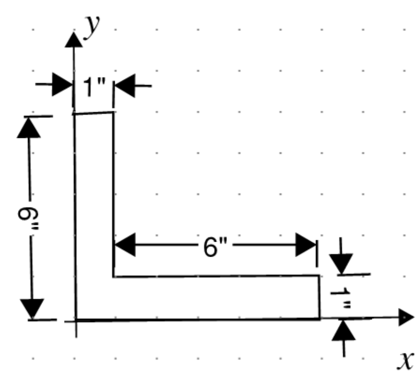
4b. A hollow shaft with an external diameter of 150 mm is required to provide a torque of 12 kN-m. The yield shear stress of the material is 140 MN/m^{2}. Assuming a safety factor (FS) of 2, calculate

(i) the maximum allowable shear stress τ_{max}, and

(ii) a suitable internal diameter (d_i) of the shaft if the shear stress is not to exceed this allowable shear stress. (Hint: Write down the polar moment of the shaft in terms of the internal diameter).

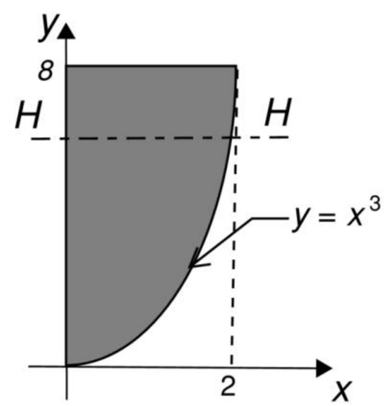
τ_{max} = __________________ MN/m^2 **(2 pts)** d_i = ______________________ mm **(3 pts)**

4c. A beam has a constant L-shaped cross section shown in the diagram. Find the location of the centroid y_c from the x-axis. Determine the second moment of area I_G corresponding to the neutral axis of the beam.



y_c = ___________________________ in **(2 pts)** I_G = _________________________ in^4 **(3 pts)**

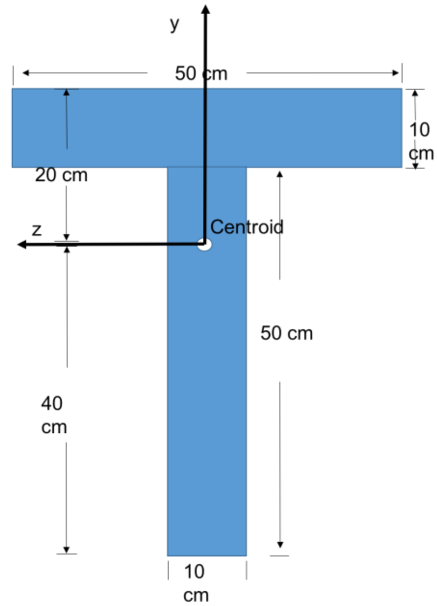
4d. Determine the second moment area of the shaded region about the x-axis, I_x by integration. Then, use the parallel axis theorem to calculate the second moment area, I_H about the axis H-H at y = 6. For the shaded region, you may take its area as 12 and the location of the centroid (measured from the x-axis) as 4.571, respectively. (Hint: the axis H-H does not pass through the neutral axis of the shaded region.)

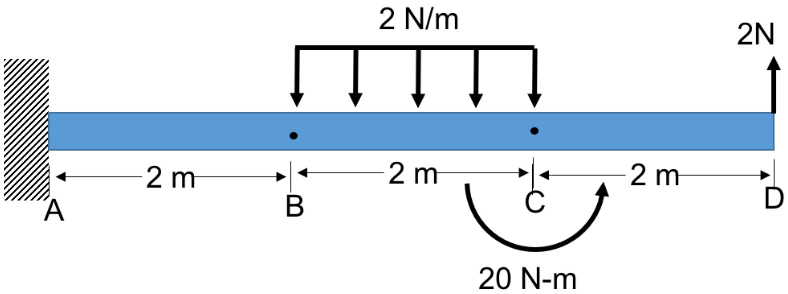


I_x = ____________________________ (3 pts) I_H = ______________________________(2 pts)

PROBLEM 5. (20 points)

GIVEN: Consider the ASTM-A36 structural steel T-beam ABCD with the given external loads shown in the figure below. The beam is cantilevered at A. The cross section of the T-beam is shown in the figure on the right. The second area moment about the centroid of the T cross section is I = 33.33 cm^4. The mechanical properties of ASTM-A36 are: Young modulus = 200GPa, Poisson's ratio = 0.29, Yield strength = 250MPa.





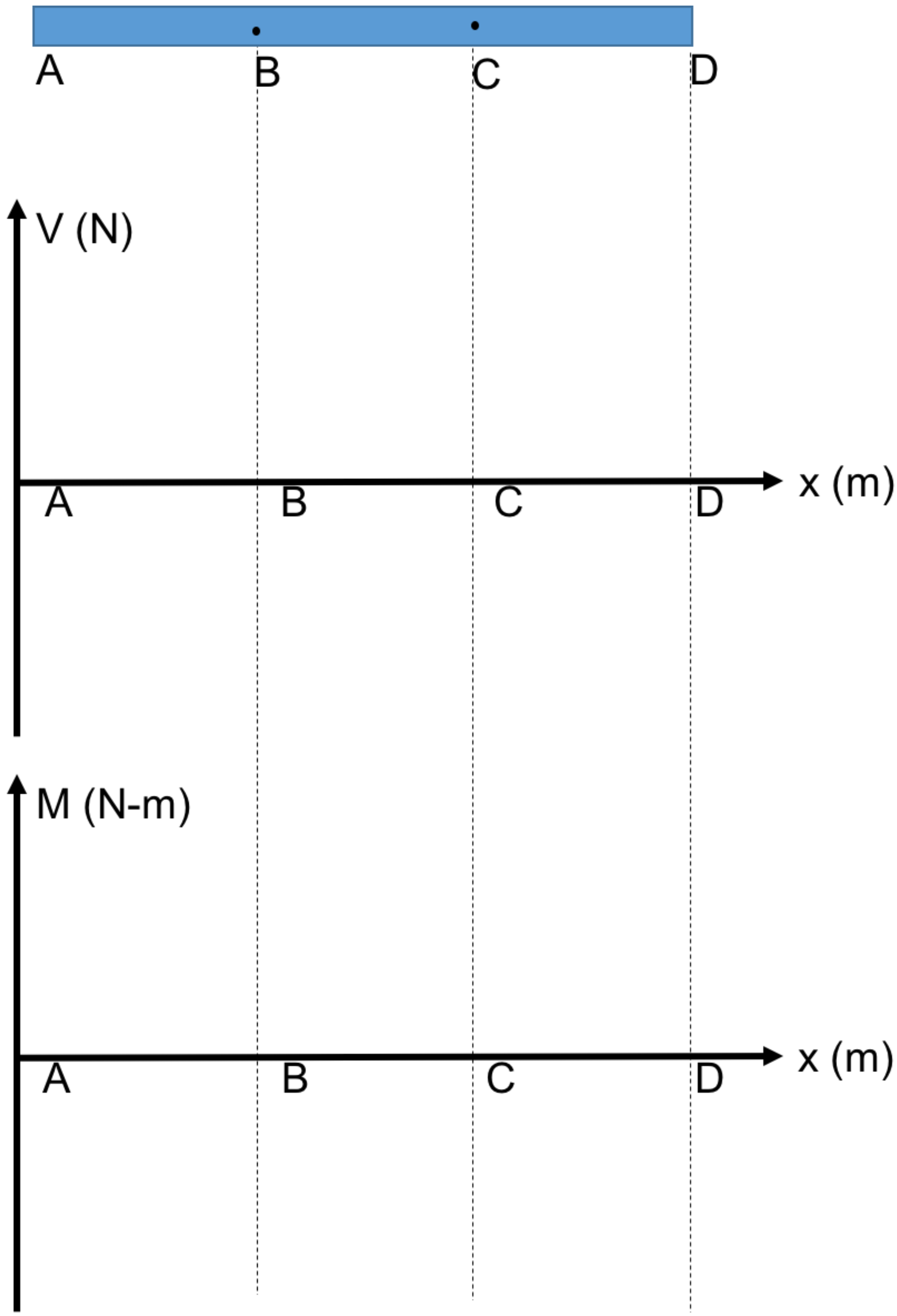
FIND:
a) Draw the **free body diagram** of the beam ABCD and **calculate the reaction forces at the supports** (4 pts).

A_x = ______________________________

A_y = ______________________________ (4 pts)

M_A = ______________________________

b) Draw the **shear force and bending moment diagram** of the beam ABCD. You must **label** the **shear force** and **bending moment values** on the diagram at points **A, B, C, and D** as well as at any extreme points (e.g., location where the shear force crosses zero, maximum of the bending moment, changes in slope) to receive full credit (8 pts). You may use the graphical method.



Labeled Shear and Bending Moment diagram (above) (8 pts)

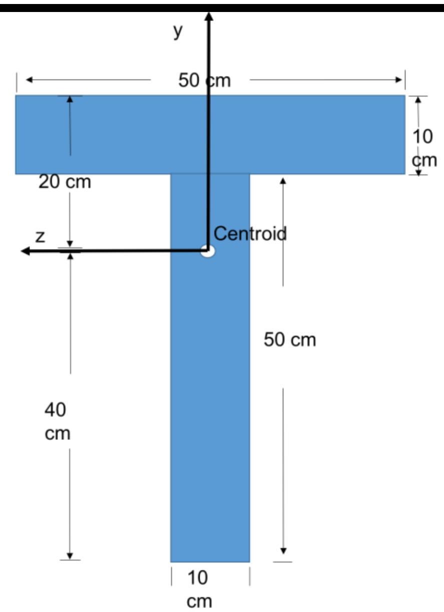
c) When is an arbitrary point **x** of the beam in **pure bending** (1 pt)? Hint: Looking for the definition.

A point **x** is in pure bending if V(**x**) is ______ and M(**x**) is _________. (1 pt)

d) At which point, if any, is the beam in **pure bending** (1 pt)?

Point: ____ (1 pt)

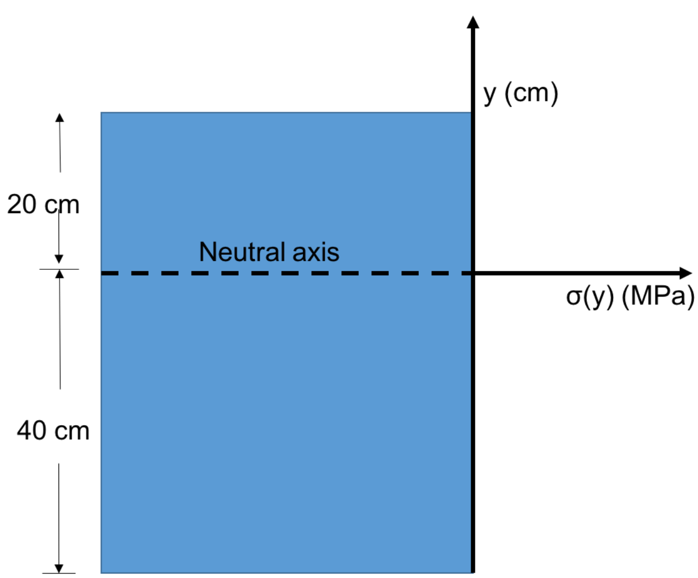
e) Calculate the **maximum bending stress of the beam (σ_{max})** and the **maximum axial strain** (ε_{max}) at the point where **maximum** pure bending occurs. For your convenience, we replicate on the right the cross section of the beam and we remind you that the second area moment about the centroid of the cross section is I = 33.33 cm^4 (3 pt). The stress must be in MPa.



σ_{max} = ____________ (4 pt)

ε_{max} = ____________

f) Suppose the the figure below is a cut of the beam at the point of pure bending. Draw the axial stress as a function of the distance y from the neutral axis of the beam (2 pt).



ME 270 Final Exam Equation Sheet

Normal Stress and Strain

$$\sigma_x = \frac{F_n}{A}$$

$$\sigma_x(y) = \frac{-My}{I}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{\Delta L}{L}$$

$$\varepsilon_y = \varepsilon_z = -\vartheta\varepsilon_x$$

$$\varepsilon_x(y) = \frac{-y}{\rho}$$

$$FS = \frac{\sigma_{fail}}{\sigma_{allow}}$$

Shear Stress and Strain

$$\tau = \frac{V}{A}$$

$$\tau(\rho) = \frac{T\rho}{J}$$

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\vartheta)}$$

$$\gamma = \frac{\delta_s}{L_s} = \frac{\pi}{2} - \theta$$

For a rectangular cross-section,

$$\tau(y) = \frac{6V}{Ah^2}\left(\frac{h^2}{4} - y^2\right)$$

$$\tau_{max} = \frac{3V}{2A}$$

Second Area Moment

$$I = \int_A y^2 dA$$

$$I = \frac{1}{12}bh^3 \quad \text{Rectangle}$$

$$I = \frac{\pi}{4}r^4 \quad \text{Circle}$$

$$I_B = I_O + Ad_{OB}{}^2$$

Polar Area Moment

$$J = \frac{\pi}{2}(r_o^4 - r_i^4) \quad \text{Tube}$$

Shear Force and Bending Moment

$$V(x) = V(0) + \int_0^x p(\epsilon)d\epsilon$$

$$M(x) = M(0) + \int_0^x V(\epsilon)d\epsilon$$

Buoyancy

$$F_B = \rho g V$$

Fluid Statics

$$p = \rho g h$$

$$F_{eq} = p_{avg}(Lw)$$

Belt Friction

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

Distributed Loads

$$F_{eq} = \int_0^L w(x)dx$$

$$\bar{x}F_{eq} = \int_0^L x\, w(x)dx$$

Centroids

$$\bar{x} = \frac{\int x_c dA}{\int dA}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA}$$

$$\bar{x} = \frac{\sum_i x_{ci}A_i}{\sum_i A_i}$$

$$\bar{y} = \frac{\sum_i y_{ci}A_i}{\sum_i A_i}$$

In 3D, $\bar{x} = \frac{\sum_i x_{ci}V_i}{\sum_i V_i}$

Centers of Mass

$$\tilde{x} = \frac{\int x_{cm}\rho dA}{\int \rho dA}$$

$$\tilde{y} = \frac{\int y_{cm}\rho dA}{\int \rho dA}$$

$$\tilde{x} = \frac{\sum_i x_{cmi}\rho_i A_i}{\sum_i \rho_i A_i}$$

$$\tilde{y} = \frac{\sum_i y_{cmi}\rho_i A_i}{\sum_i \rho_i A_i}$$

Final Answers

1a) $F_{OB} = 120N$ $F_{OA} = 75N$

1b) Zero-Force Members = F_{bf}, F_{bh}, F_{ch}

1c) $3.69\ kg < m_z < 2708\ kg$

1d) F = 545 kN

2a) FBD 2b) $\bar{T}_{BD} = -0.8\ T_D\ \bar{j} + 0.6\ T_D\ \bar{k}$ $\bar{T}_{CE} = 0.8\ T_E\ \bar{i} - 0.6\ T_E\ \bar{j}$

2c) $T_{BD} = 1000N$ $T_{CE} = 1000N$

2d) $\bar{O} = -800\bar{i} + 1400\bar{j}\ N$

3a) CD, BE 3b) FBDs 3c) BE = 25 kN 3d) $\mathbf{T}_B = 15.9\times10^6$ Pa

3e) b = 0.11 m

4a) Beam Loading 4b) $\mathbf{T}_B = 70MN/m^2$ $d_i = 139$ mm

4c) $y_c = 1.75$ in $I_G = 22.4\ In^4$ 4d) $I_x = 307.2\ in^4$ $I_H = 81\ in^4$

5a) $A_x = 0N$ $A_y = 2N$ $M_A = 20$ N-m

5b) Shear-Force and Bending-Moment Diagrams

5c) V (x) = 0N M(x) ≠ 0N-m

5d) x = 3m 5e) $\sigma_{max} = 30.0$ MPa $\epsilon_{max} = 0.00015$

5f) Sketch of axial stress distribution.