

Please review the following statement:

I certify that I have not given unauthorized aid nor have I received aid in the completion of this exam.

Signature: _____

INSTRUCTIONS

Begin each problem in the space provided on the examination sheets. If additional space is required, use the white lined paper provided to you.

Work on one side of each sheet only, with only one problem on a sheet.

Each problem is worth 20 points.

Please remember that for you to obtain maximum credit for a problem, it must be clearly presented:

- The only authorized exam calculator is the TI-30IIS
- The allowable exam time for the Final Exam is 120 minutes.
- The coordinate system must be clearly identified.
- Where appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- Units must be clearly stated as part of the answer.
- You must carefully delineate vector and scalar quantities.

If the solution does not follow a logical thought process, it will be assumed in error.

When handing in the test, please make sure that all sheets are in the correct sequential order and make sure that your name is at the top of every page that you wish to have graded.

Instructor’s Name and Section:

Sections:	J Hylton 8:30-9:20AM	J Jones 9:30-10:20AM	E Nauman 11:30AM-12:20PM
	J Seipel 12:30-1:20PM	I Billionis 2:30-3:20PM	M Murphy 9:00-10:15AM
	Z Shen 4:30-5:20PM	J Jones Distance Learning	

Problem 1 _____

Problem 2 _____

Problem 3 _____

Problem 4 _____

Problem 5 _____

Total _____

PROBLEM 1 (20 points) – Prob. 1 questions are all or nothing.

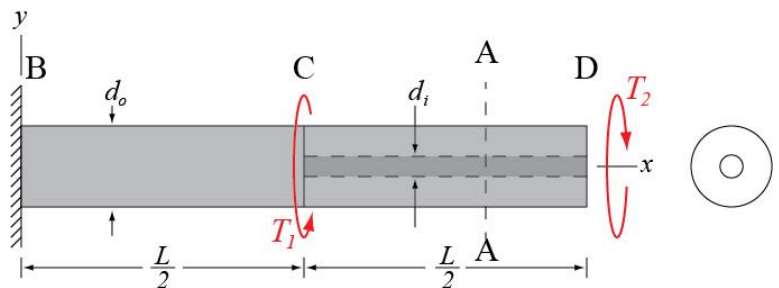
1A. Newton's laws are the basis for mechanics, including the mechanics of systems in equilibrium. In order, please state Newton's Three Laws of Motion using complete sentences and/or equations.

Newton's 1st Law (2pts):

Newton's 2nd Law (2pts):

Newton's 3rd Law (2pts):

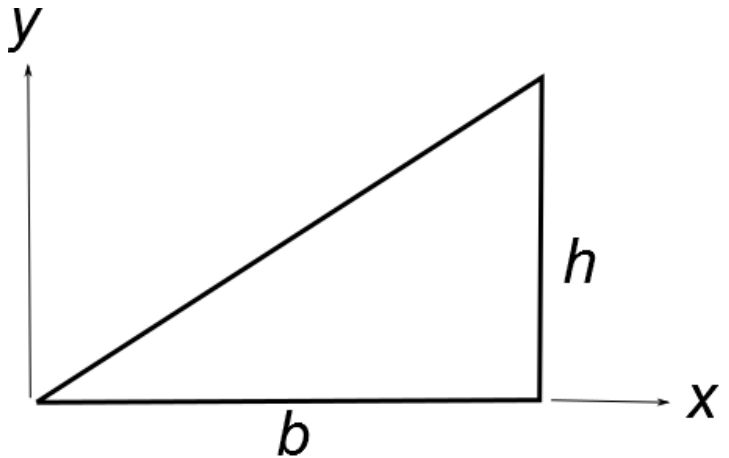
1B. Given $T_1 = 30\text{kN}\cdot\text{m}$ and $T_2 = 10\text{kN}\cdot\text{m}$, determine the maximum shear stress due to torsion in shaft BC. Determine the maximum shear stress due to torsion in shaft CD. Assume $d_o = 0.1\text{m}$, $d_i = 0.025\text{m}$, and $L = 10\text{m}$. (6 pts)



$$(\tau_{BC})_{\max} = \quad \quad \quad (3 \text{ pts})$$

$$(\tau_{CD})_{\max} = \quad \quad \quad (3 \text{ pts})$$

1C. For the triangular cross section shown below, set up but do not integrate the equation for calculating the **y coordinate of the centroid**. Be sure to clearly indicate all limits of integration as well as the function being integrated. (4 points)



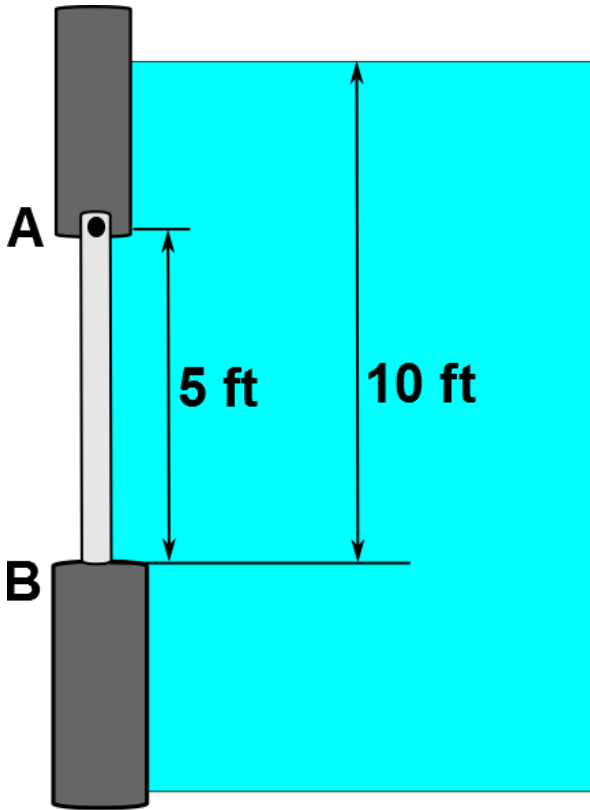
$$\bar{y} = \underline{\hspace{15cm}}$$

1D. A punching machine is capable of delivering up to 440 kips of force. The machine was designed to punch 4in diameter circles out of 2014-T6 aluminum ($E = 10.6 \times 10^3$ ksi, shear strength = 70 ksi). What is the maximum thickness of aluminum plate which the machine can punch through? (4 points)

$$\text{Maximum Thickness} = \underline{\hspace{15cm}} \text{ in}$$

PROBLEM 2 (20 points) – Prob. 2 questions are all or nothing.

2A. A submerged flood gate is 5 feet in length and 3 feet in width (in/out of the page). The bottom of the gate (point B) is 10 feet below the surface of the water. Answer the following questions about the flood gate. (4 pts)



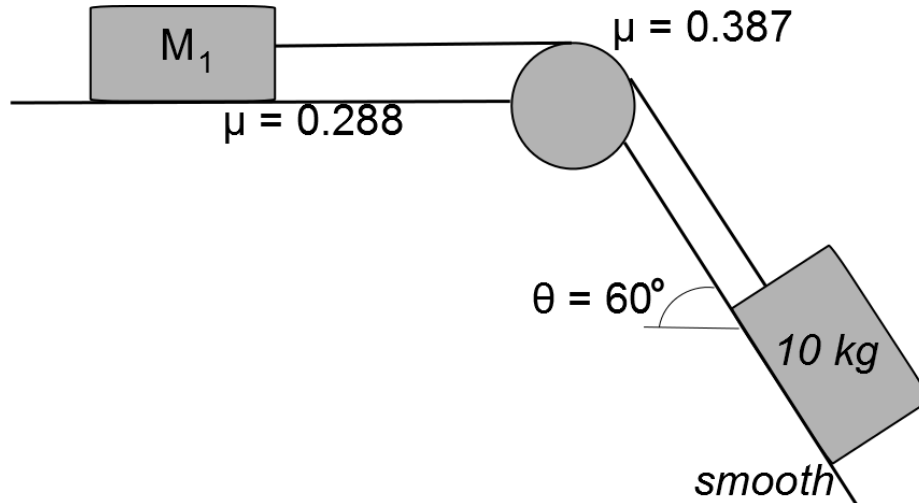
On the FBD diagram provided, draw the distributed load of the water acting on the gate.

Calculate the **pressure** at points A and B. Note that the specific gravity of water is 62.4 lbs/ft³.

Pressure at A = _____ lb/ft²

Pressure at B = _____ lb/ft²

2B. A block, M_1 , is resting on a horizontal surface with a coefficient of friction $\mu=0.288$. A second block, M_2 , is on a smooth ramp with an inclination of $\theta=60^\circ$. M_2 has a mass of 10kg. They are connected with an inextensible cable which wraps partially around a drum. The drum has a coefficient of friction $\mu=0.387$. (10 points)

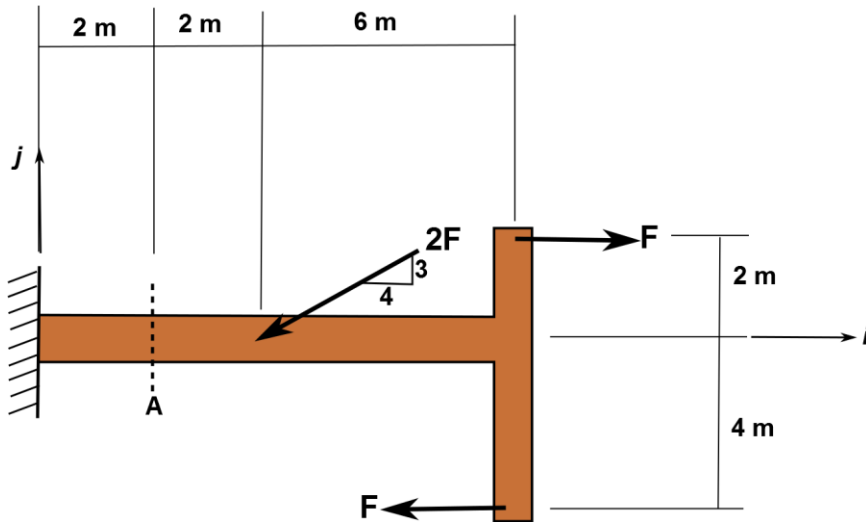


Calculate the tensions in the upper cable segment (T_1) and the lower cable segment (T_2). Then calculate the minimum mass, M_1 , to prevent motion.

$T_1 =$ _____ N $T_2 =$ _____ N

Minimum Mass $M_1 =$ _____ kg

2C. A cantilevered beam is shown below. The value of **F** is **100N**. Replace the loading shown with an equivalent force (please write in vector form) and moment (please write in vector form) at point A. Please note that this is an equivalent system problem. (6 pts)

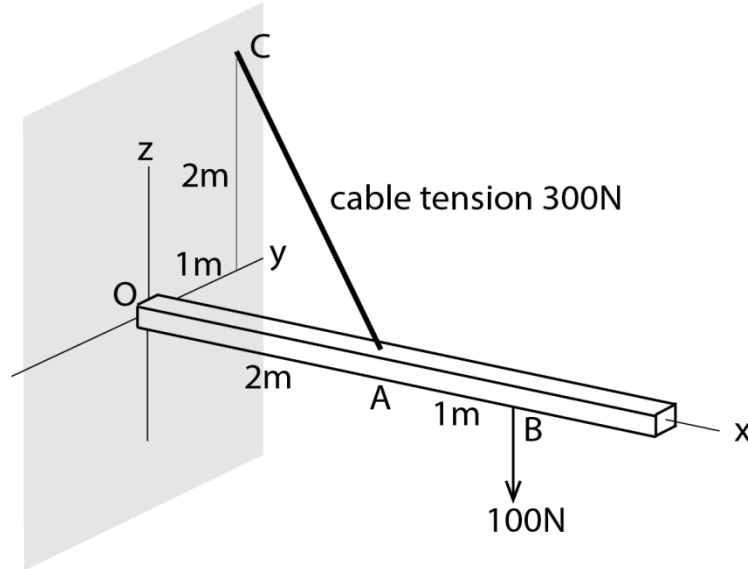


Equivalent Force = { _____ \hat{i} + _____ \hat{j} + _____ \hat{k} } N

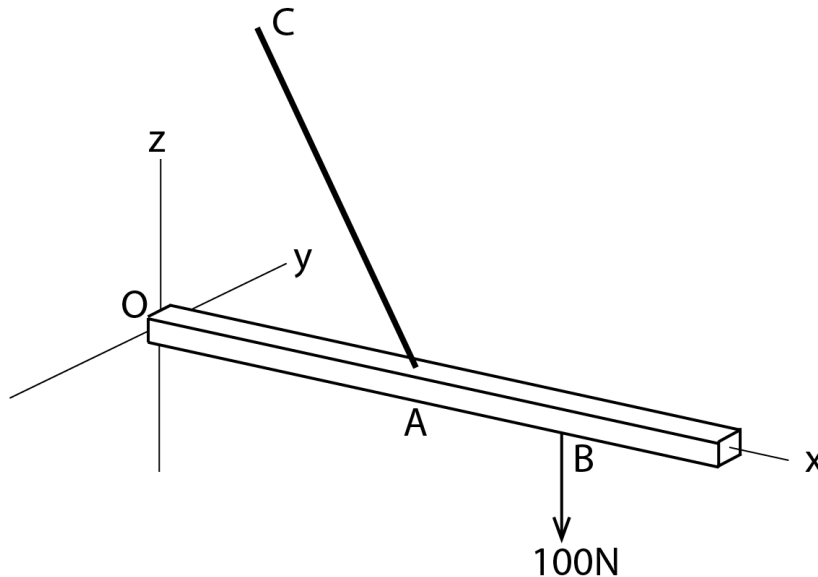
Equivalent Moment = { _____ \hat{i} + _____ \hat{j} + _____ \hat{k} } N-m

PROBLEM 3 (20 points)

A massless beam OAB shown in the figure is rigidly connected to the wall (indicated as gray area). It also connects to cable AC with 300N in tension at point A, the other end of the cable C is located on the wall. In addition, a vertical downward force of 100N is applied at point B of the beam. The distance between O and A is 2m, and 1m between A and B.



3A. Using the figure below, complete the free body diagram of the beam (4pts)



3B. The tension in cable AC is given as 300N. Rewrite this tension in vector form as a magnitude times a unit vector. (5pts)

$$\vec{F}_{AC} = \text{_____} \{ \text{_____} \hat{i} + \text{_____} \hat{j} + \text{_____} \hat{k} \} \text{ N}$$

3C. Calculate the reaction **forces** at connecting point O. Express your answers in standard vector format. (5pts)

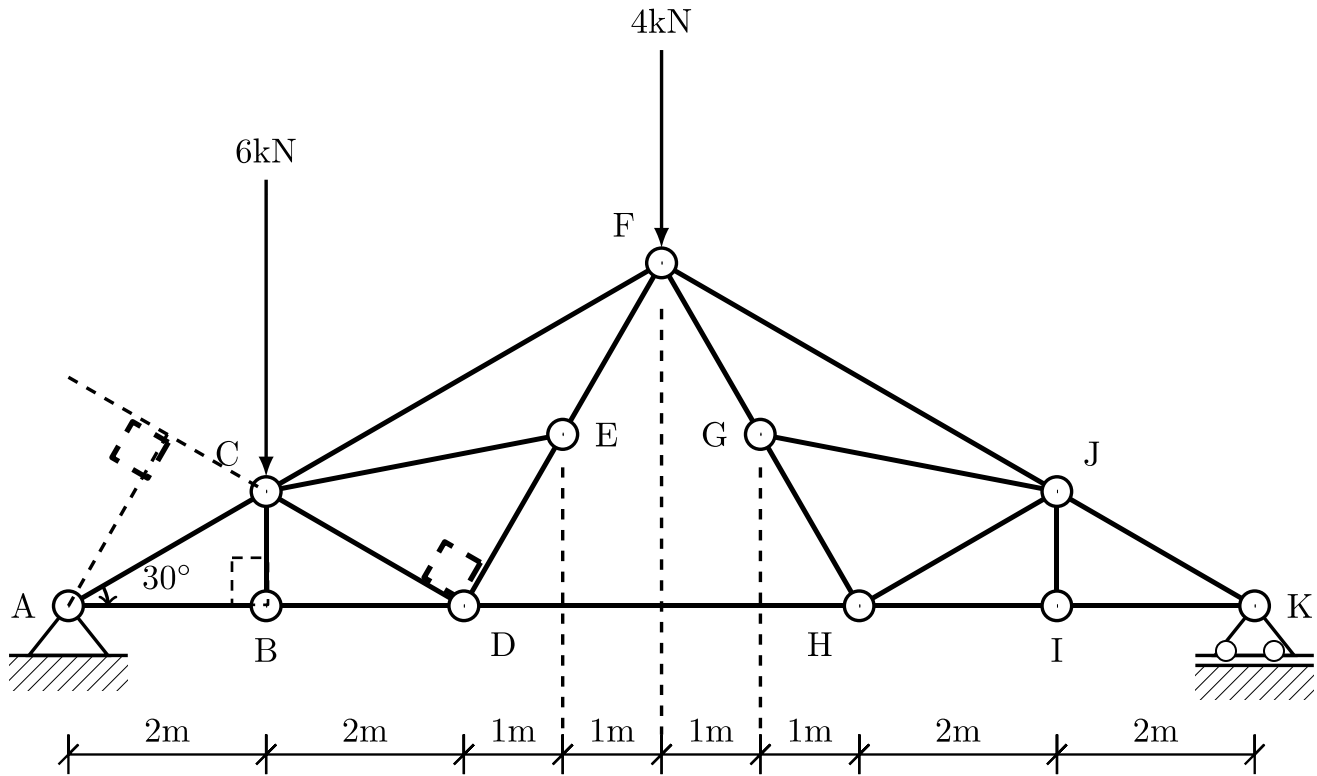
$$\vec{F}_O = \{ \text{_____} \hat{i} + \text{_____} \hat{j} + \text{_____} \hat{k} \} \text{ N}$$

standard vector format. (5pts)

$$\vec{M}_O = \{ \text{_____ } \hat{i} + \text{_____ } \hat{j} + \text{_____ } \hat{k} \} \text{ N-m}$$

PROBLEM 4 (20 points)

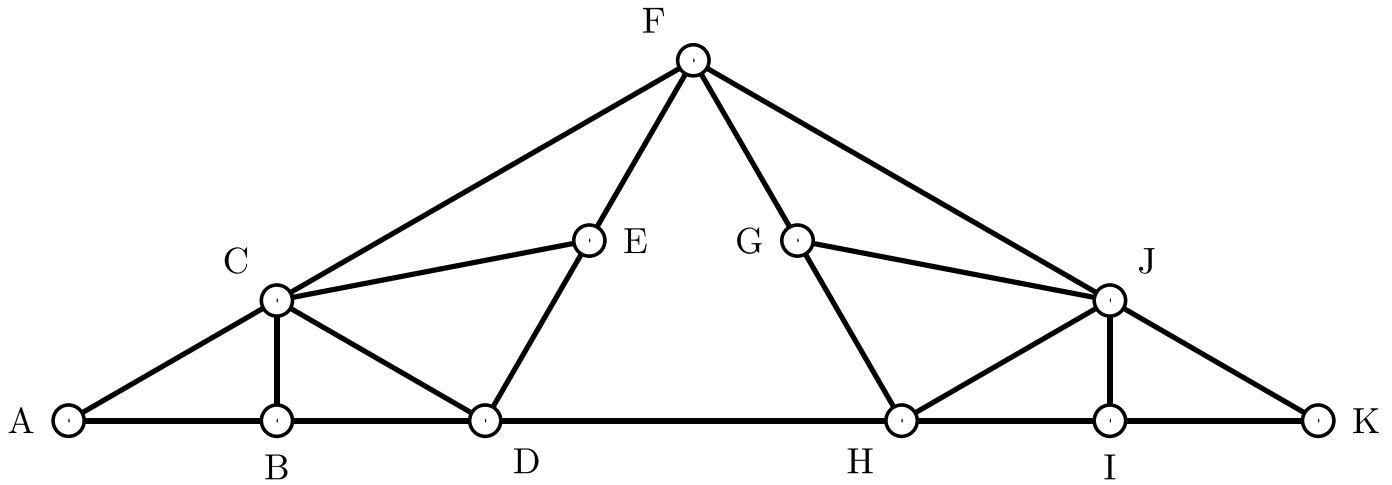
The truss below is in static equilibrium and supports loads at joints C, and F. The support at K is a roller.



4A. Identify all zero-force members (no explanation is required). (4 pts)

Zero force members: _____

4B. Draw the free body diagram of the truss on the figure provided below. (4pts).



4C. Compute the reaction forces on supports A and K. (4pts)

$A_x =$ _____

$A_y =$ _____

$K_y =$ _____

4D. Determine the forces in members CD and EF. Write your answers in the box below. Indicate if the member is in tension or compression by circling the appropriate option. Show your work including the necessary free body diagrams. (8pts)

$$T_{EF} = \underline{\hspace{10em}} \quad (4 \text{ pts})$$

Member EF is in:

compression

tension

$$T_{CD} = \underline{\hspace{10em}} \quad (4 \text{ pts})$$

Member CD is in:

compression

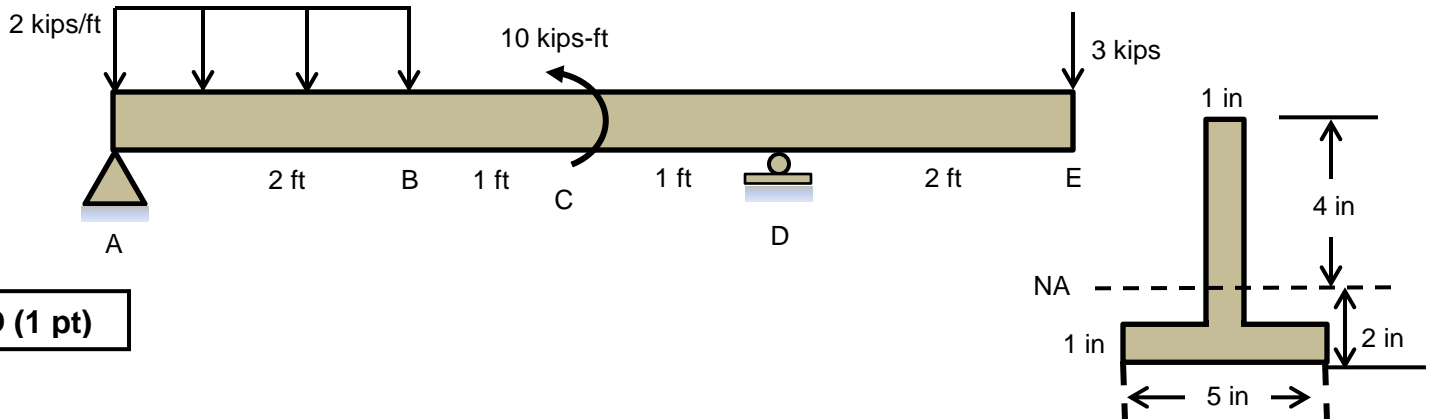
tension

PROBLEM 5 (20 points)

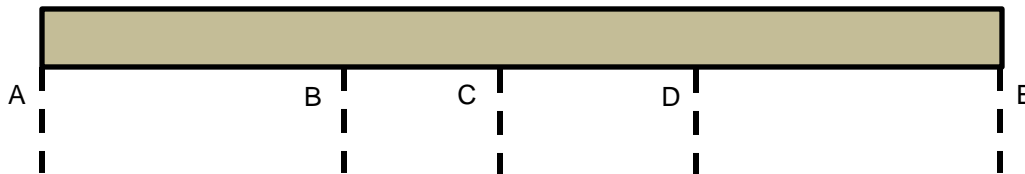
Given: Beam ABCDE is loaded as shown and is held in static equilibrium by a pin support at A and a roller support at D. The beam cross-section is an “inverted T-shape” and has the dimensions shown with a second moment of inertia of $I_x = 33.3 \text{ in}^4$. (Note – NA refers to the neutral axis.)

Find:

5a) Sketch a free-body diagram of the beam and determine the magnitudes of the reactions at A and D. (5 pts)

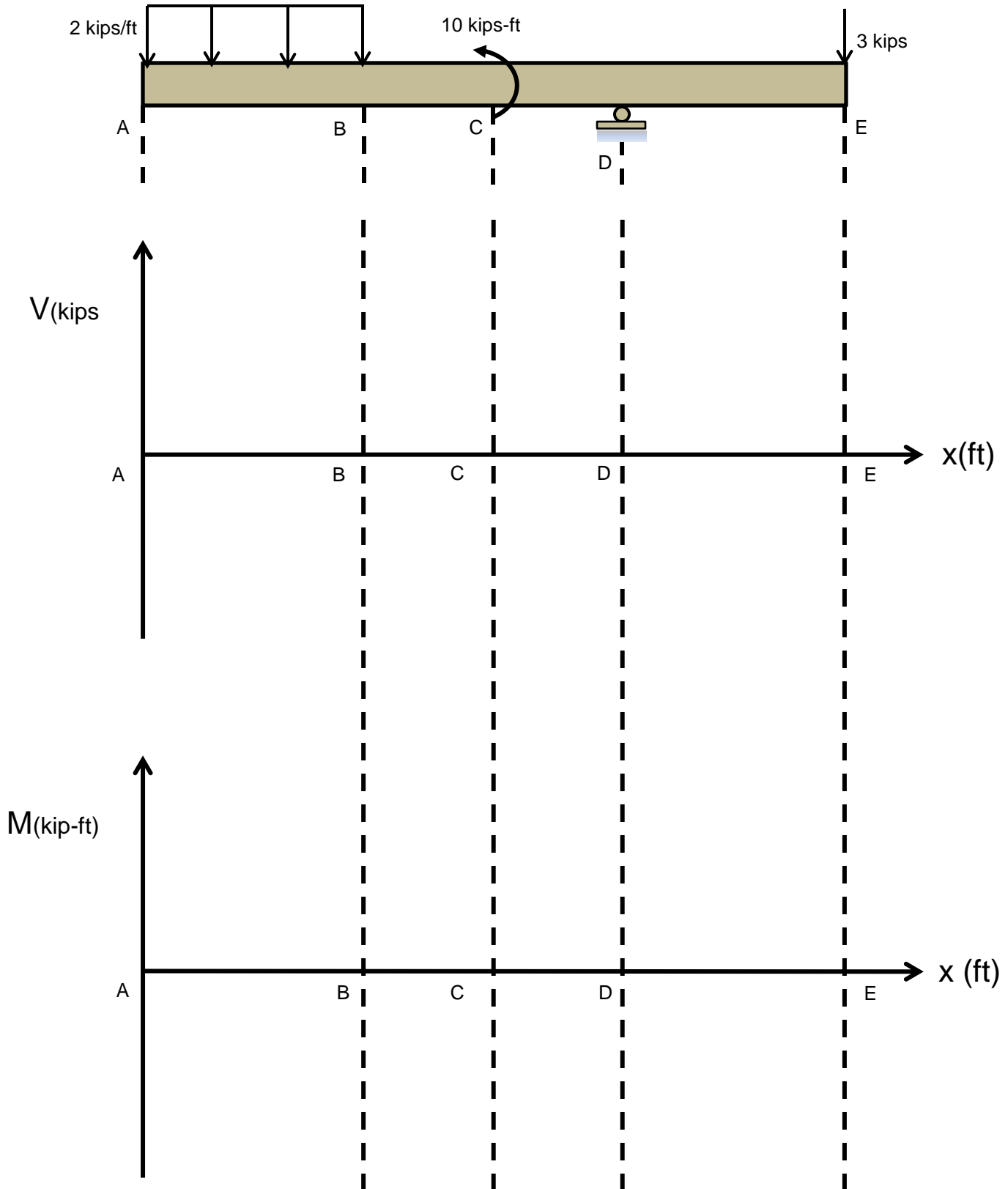


FBD (1 pt)



$A_y =$	(2pts)
$D_y =$	(2pts)

5b) On the axes provided, sketch the shear-force and bending-moment diagrams of the beam. Please specify the magnitudes of the shear and moment at each of the transition points. (8 pts)



5c) In which segment(s) of the beam does pure bending occur? (1 pts)

AB	BC	CD	DE	None	(Circle all that apply)
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5d) In the segment of the beam where pure bending exists, determine the magnitudes of the maximum tensile bending stress and the maximum compressive bending stress. (6 pts)

$\left (\sigma_{\max})_T \right =$	(3pts)
$\left (\sigma_{\max})_C \right =$	(3pts)

Normal Stress and Strain

$$\sigma_x = \frac{F_n}{A}$$

$$\sigma_x(y) = \frac{-My}{I}$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{\Delta L}{L}$$

$$\epsilon_y = \epsilon_z = -\nu \epsilon_x$$

$$\epsilon_x(y) = \frac{-y}{\rho}$$

$$FS = \frac{\sigma_{fail}}{\sigma_{allow}}$$

Shear Stress and Strain

$$\tau = \frac{V}{A}$$

$$\tau(\rho) = \frac{T\rho}{J}$$

$$\tau = G\gamma$$

$$G = \frac{E}{2(1 + \nu)}$$

$$\gamma = \frac{\delta_s}{L_s} = \frac{\pi}{2} - \theta$$

Second Area Moment

$$I = \int_A y^2 dA$$

$$I = \frac{1}{12}bh^3 \quad \text{Rectangle}$$

$$I = \frac{\pi}{4}r^4 \quad \text{Circle}$$

$$I_B = I_O + Ad_{OB}^2$$

Polar Area Moment

$$J = \frac{\pi}{2}(r_o^4 - r_i^4) \quad \text{Tube}$$

Shear Force and Bending Moment

$$V(x) = V(0) + \int_0^x p(\epsilon) d\epsilon$$

$$M(x) = M(0) + \int_0^x V(\epsilon) d\epsilon$$

Buoyancy

$$F_B = \rho g V$$

Fluid Statics

$$p = \rho gh$$

$$F_{eq} = p_{avg}(Lw)$$

Belt Friction

$$\frac{T_L}{T_S} = e^{\mu\beta}$$

Distributed Loads

$$F_{eq} = \int_0^L w(x) dx$$

$$\bar{x}F_{eq} = \int_0^L x w(x) dx$$

Centroids

$$\bar{x} = \frac{\int x_c dA}{\int dA}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA}$$

$$\bar{x} = \frac{\sum x_{ci} A_i}{\sum A_i} \quad \bar{y} = \frac{\sum y_{ci} A_i}{\sum A_i}$$

$$\text{In 3D, } \bar{x} = \frac{\sum x_{ci} V_i}{\sum V_i}$$

Centers of Mass

$$\tilde{x} = \frac{\int x_{cm} \rho dA}{\int \rho dA} \quad \tilde{y} = \frac{\int y_{cm} \rho dA}{\int \rho dA}$$

$$\tilde{x} = \frac{\sum x_{cmi} \rho_i A_i}{\sum \rho_i A_i}$$

$$\tilde{y} = \frac{\sum y_{cmi} \rho_i A_i}{\sum \rho_i A_i}$$

ME 270 Final Exam Solutions for Fall 2014

1a. See Newton (1687). Reasonable variations accepted.

$$1b. (\tau_{BC})_{\max} = 101.9 \text{ MPa} \qquad (\tau_{CD})_{\max} = 50.99 \text{ MPa}$$

$$1c. \bar{y} = \int_0^b \int_0^{hx/b} y \, dy \, dx / \left(\int_0^b \int_0^{hx/b} dy \, dx \right)$$

1d. Maximum thickness = 0.50 in.

$$2a. \text{FBD} \qquad \text{Pressure at A} = 312 \text{ lb/ft}^2 \qquad \text{Pressure at B} = 624 \text{ lb/ft}^2$$

$$2b. T_1 = 56.67 \text{ N} \qquad T_2 = 85 \text{ N} \qquad \text{Minimum Mass } M_1 = 20 \text{ kg}$$

$$2c. \text{Equivalent Force} = \{-160\hat{i} + -120\hat{j} + 0\hat{k}\} \text{ N}$$

$$\text{Equivalent Moment} = \{0\hat{i} + 0\hat{j} + -840\hat{k}\} \text{ N-m}$$

3a. FBD

$$3b. = 300 \left\{ -\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k} \right\} \text{ N}$$

$$3c. = \{200\hat{i} + -100\hat{j} + -100\hat{k}\} \text{ N}$$

$$3d. = \{0\hat{i} + 100\hat{j} + -200\hat{k}\} \text{ Nm}$$

4a. Zero-Force Members are: CB, CE, GJ, JI, HJ, GH, and GF

4b. FBD

$$4c. A_x = 0 \text{ kN} \qquad A_y = 7 \text{ kN} \qquad K_y = 3 \text{ kN}$$

$$4d. T_{EF} = 3.46 \text{ kN} \qquad \text{Member EF is in: tension}$$

$$T_{CD} = -6 \text{ kN} \qquad \text{Member CD is in compression}$$

$$5a. \text{FBD} \qquad A_y = 4 \text{ kips} = 4,000 \text{ lbs} \qquad D_y = 3 \text{ kips} = 3,000 \text{ lbs}$$

5b. Shear-force and bending-moment diagram

5c. Segments in pure bending: BC and CD

$$5d. |(\sigma_{\max})_T| = 8650 \text{ psi} = 8.650 \text{ ksi} \qquad |(\sigma_{\max})_C| = 5770 \text{ psi} = 5.770 \text{ ksi}$$