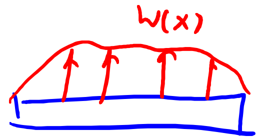
Shear-Force & Bending-Moment Diagrams
Graphical Methods



Use the relation:

$$\frac{dM}{dx} = V(x), \quad \frac{dV}{dx} = w(x)$$

Learning Objectives

1) To evaluate the *shear-force* and *bending-moment* diagrams for systems with discrete loads.

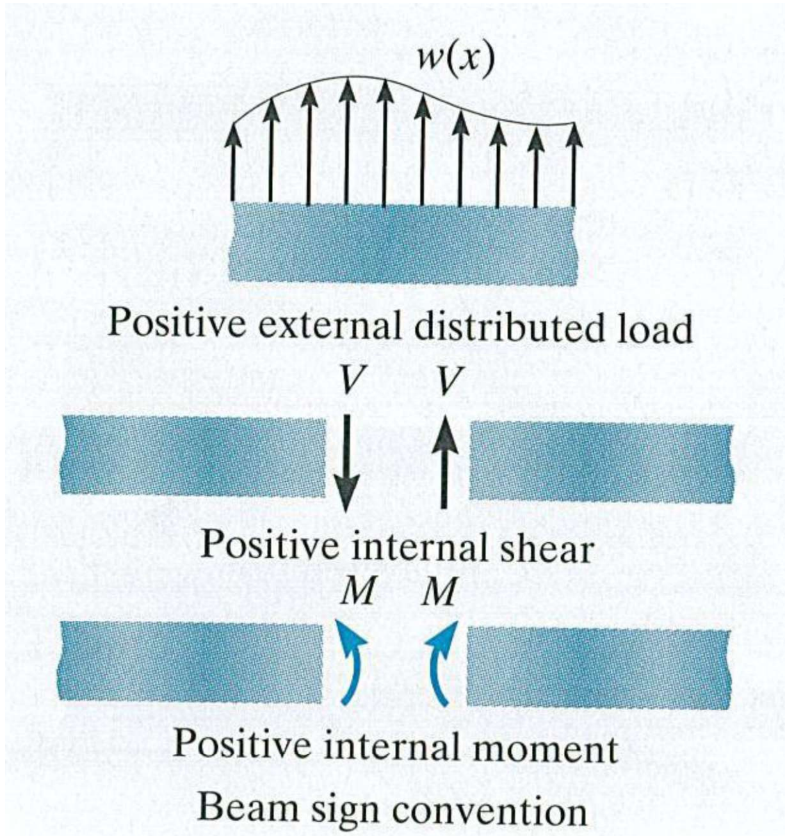
2) To do an *engineering estimate* of these quantities.

Beam Sign Convention

Distributed load -	An upward load is positive
Shear Force -	A positive internal shear force causes a clockwise rotation of beam segment. (i.e., it pushes a left-facing cross-section upward or a right-facing cross-section downward).
Bending Moment -	A positive internal moment causes compression in the top fibers of the segment (i.e., clockwise on a left-facing cross-section or counter-clockwise on a right-facing cross-section).

Procedure

1. Determine support reactions
2. Specify beam sections origin (left end) to between each discrete load (force or moment). Be sure V and M are shown acting in the positive sense.
3. Sum forces vertically to determine V
4. Sum moments at sectioned end to determine M. (This eliminates V from the moment equation).



Beam sign convention

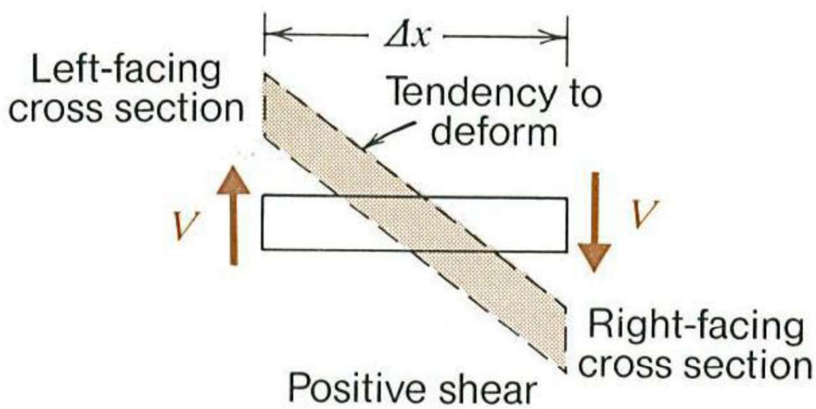


FIGURE 7a

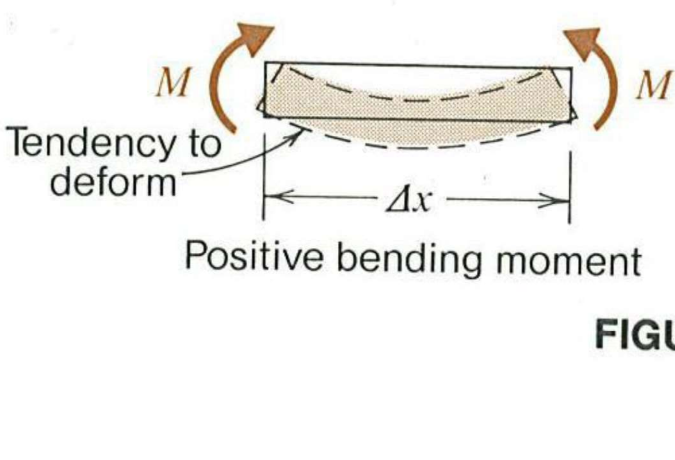
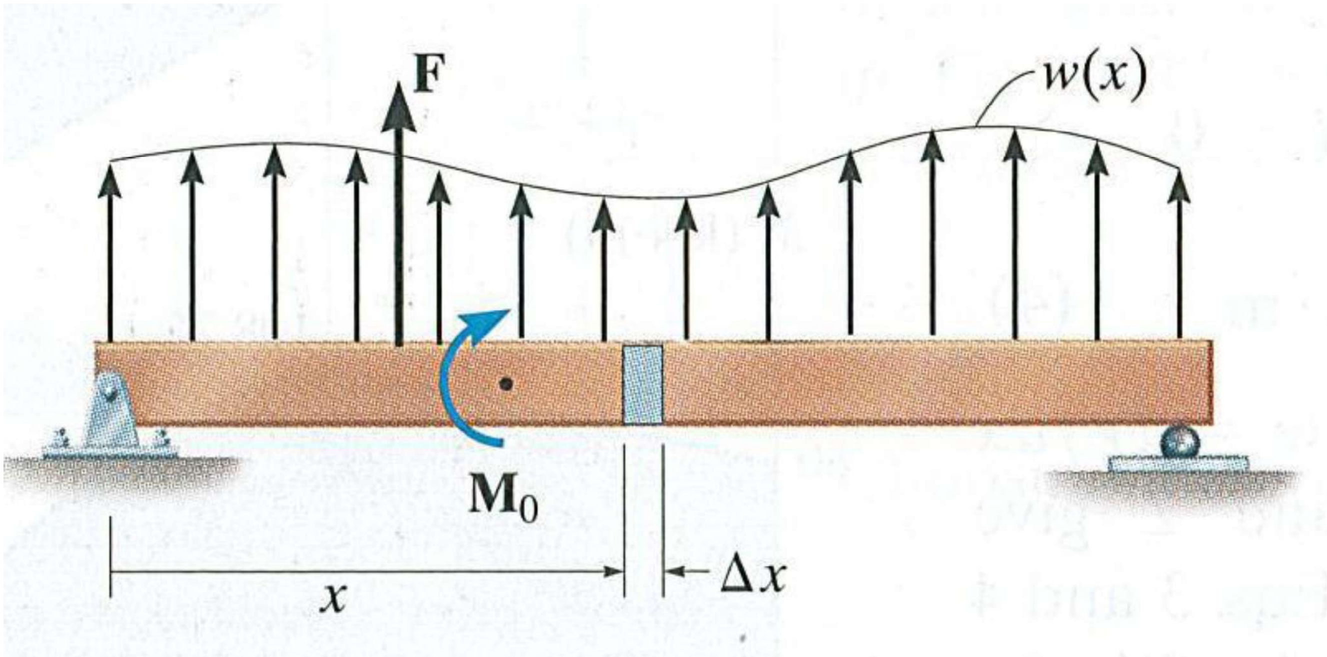
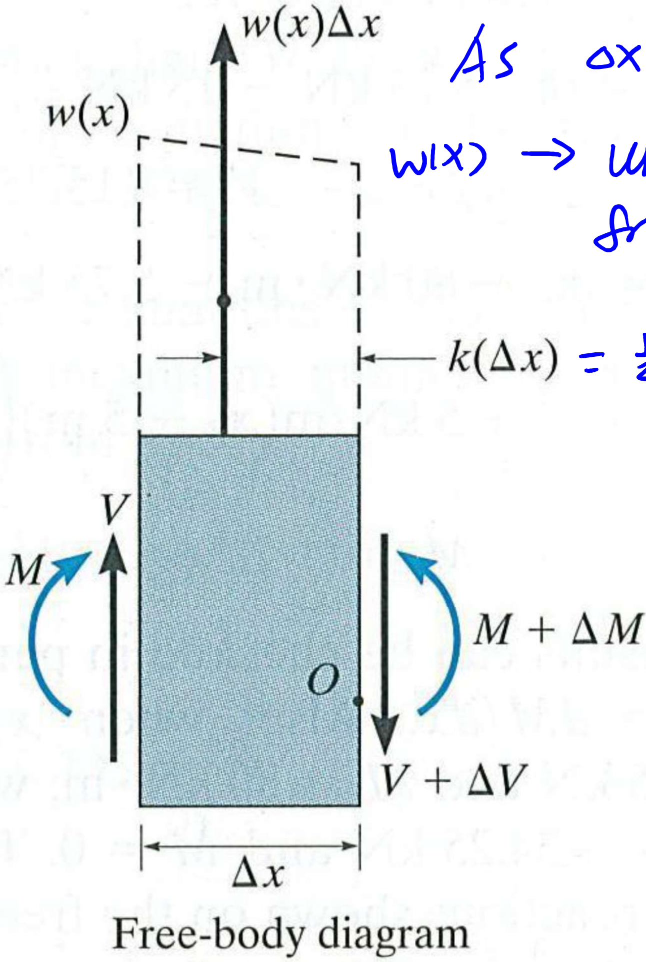


FIGURE 7b





Free-body diagram of segment Δx

(b)

As $\Delta x \to 0$

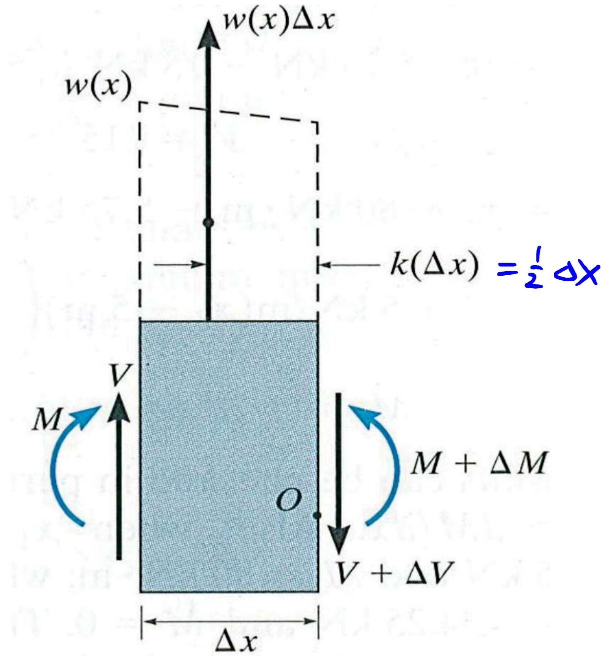
$w(x) \to$ uniform distribution from x to $x+\Delta x$

$k(\Delta x) = \frac{1}{2}\Delta x$, (i.e. $k = \frac{1}{2}$)

$\Sigma F_y = 0: \quad V + w(x)\Delta x - (V + \Delta V) = 0$

$$\Rightarrow \frac{\Delta V}{\Delta x} = w(x)$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = w(x)$$



$\Sigma M_O = 0:$

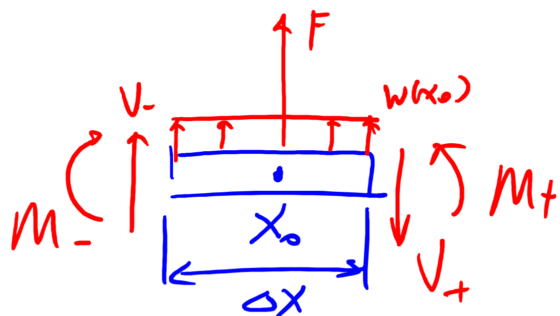
$$M + \Delta M - M - V \cdot \Delta x - (w(x)\Delta x)\left(\tfrac{1}{2}\Delta x\right) = 0$$

$$\Rightarrow \frac{\Delta M}{\Delta x} = V + \underline{\tfrac{1}{2} w(x)\Delta x}$$

$\hookrightarrow 0$ as $\Delta x \to 0$

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \frac{dM}{dx} = V$$

- Concentrated Load: F at x_0



$\Sigma F_y = 0 \quad V_- - V_+ + F + \underline{w(x_0)\Delta x} = 0$

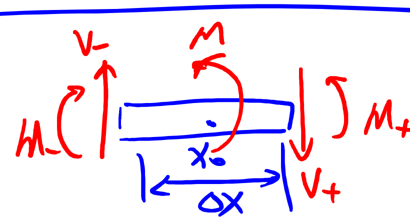
$\hookrightarrow 0$ as $\Delta x \to 0$

$\Delta x \to 0 \Rightarrow V_+ - V_- = F$

$\Sigma M_{x_0} = 0: \quad -M_- + M_+ - V_-\left(\tfrac{1}{2}\Delta x\right) - V_+\left(\tfrac{1}{2}\Delta x\right) = 0$

$\Delta x \to 0 \Rightarrow M_- = M_+$

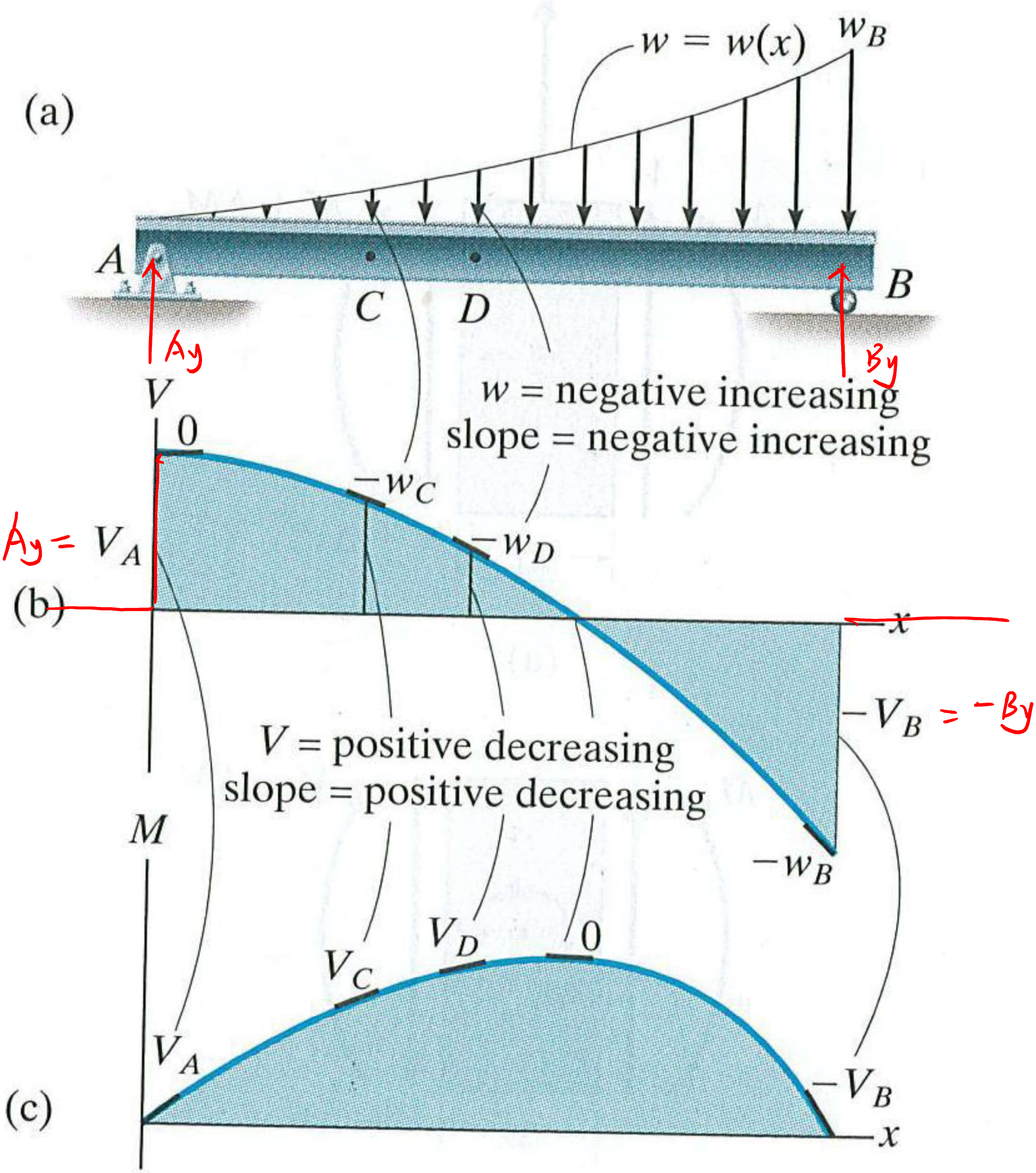
- Concentrated Moment: M at x_0



$\Sigma F_y = 0: \quad V_- = V_+$

$\Sigma M_{x_0} = 0: \quad M_+ - M_- + M - V_-\left(\tfrac{1}{2}\Delta x\right) - V_+\left(\tfrac{1}{2}\Delta x\right) = 0$

$\Delta x \to 0 \Rightarrow M_+ - M_- = -M$ ☆



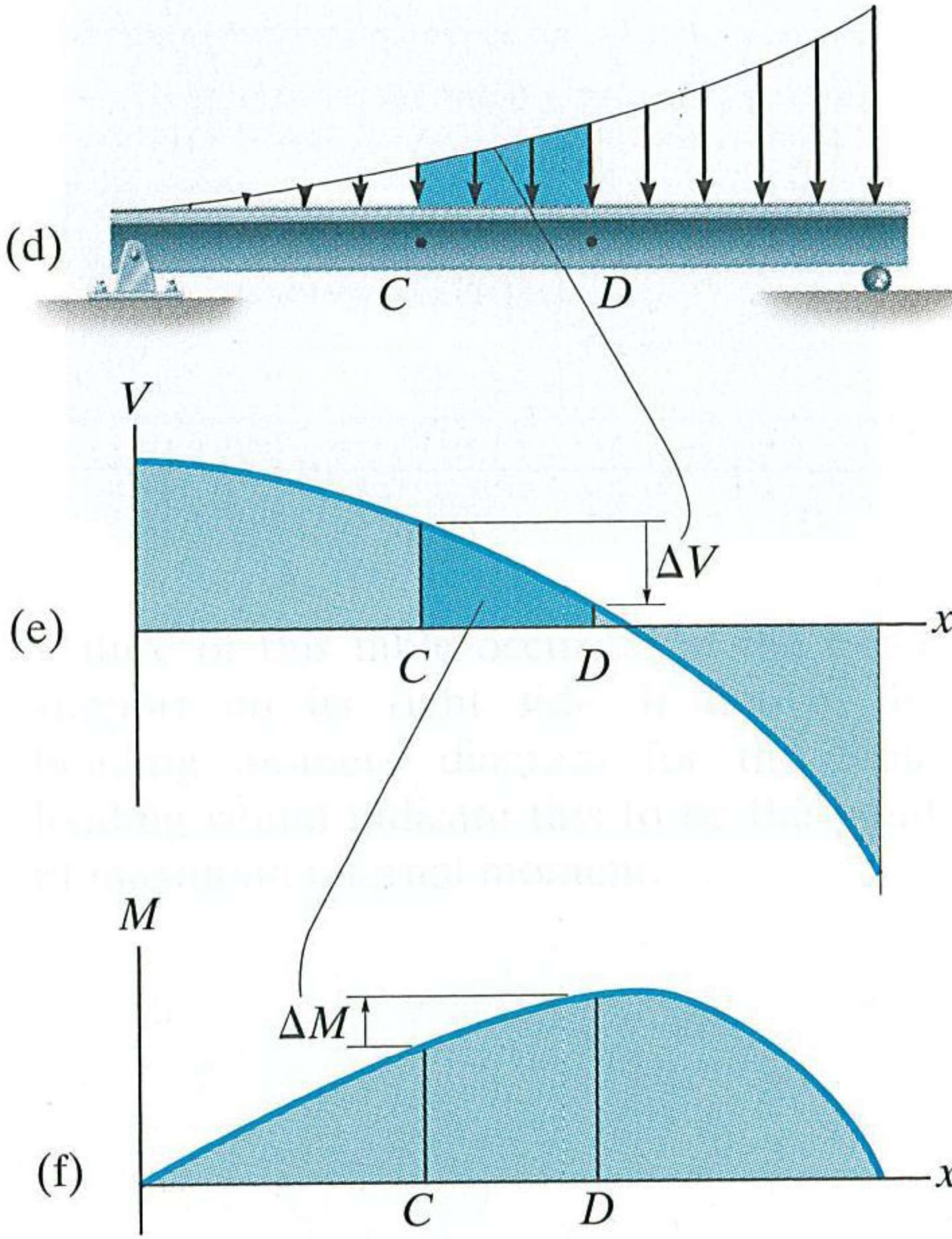
(a)
w = w(x)
w_B
A
C
D
B
A_y
B_y
w = negative increasing
slope = negative increasing
V
0
−w_C
−w_D
A_y = V_A
(b)
x
−V_B = −B_y
V = positive decreasing
slope = positive decreasing
−w_B
M
V_D
0
V_C
V_A
(c)
−V_B
x

$$\Delta V = \int w(x)\, dx$$

change in shear = area under distributed loading

$$\Delta M = \int V(x)\, dx$$

change in moment = area under shear diagram

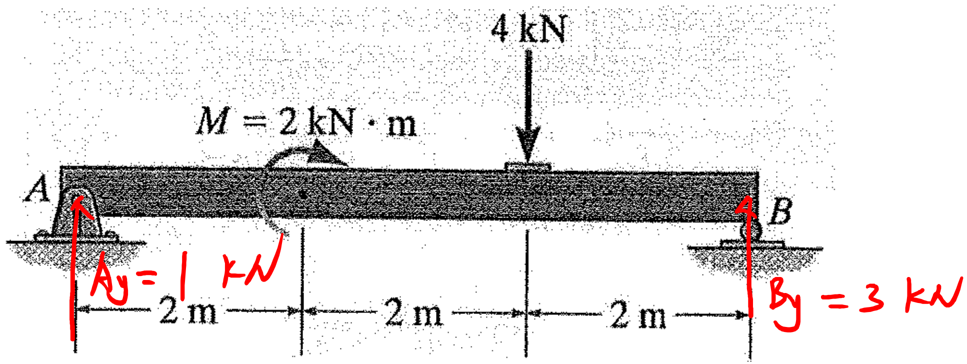


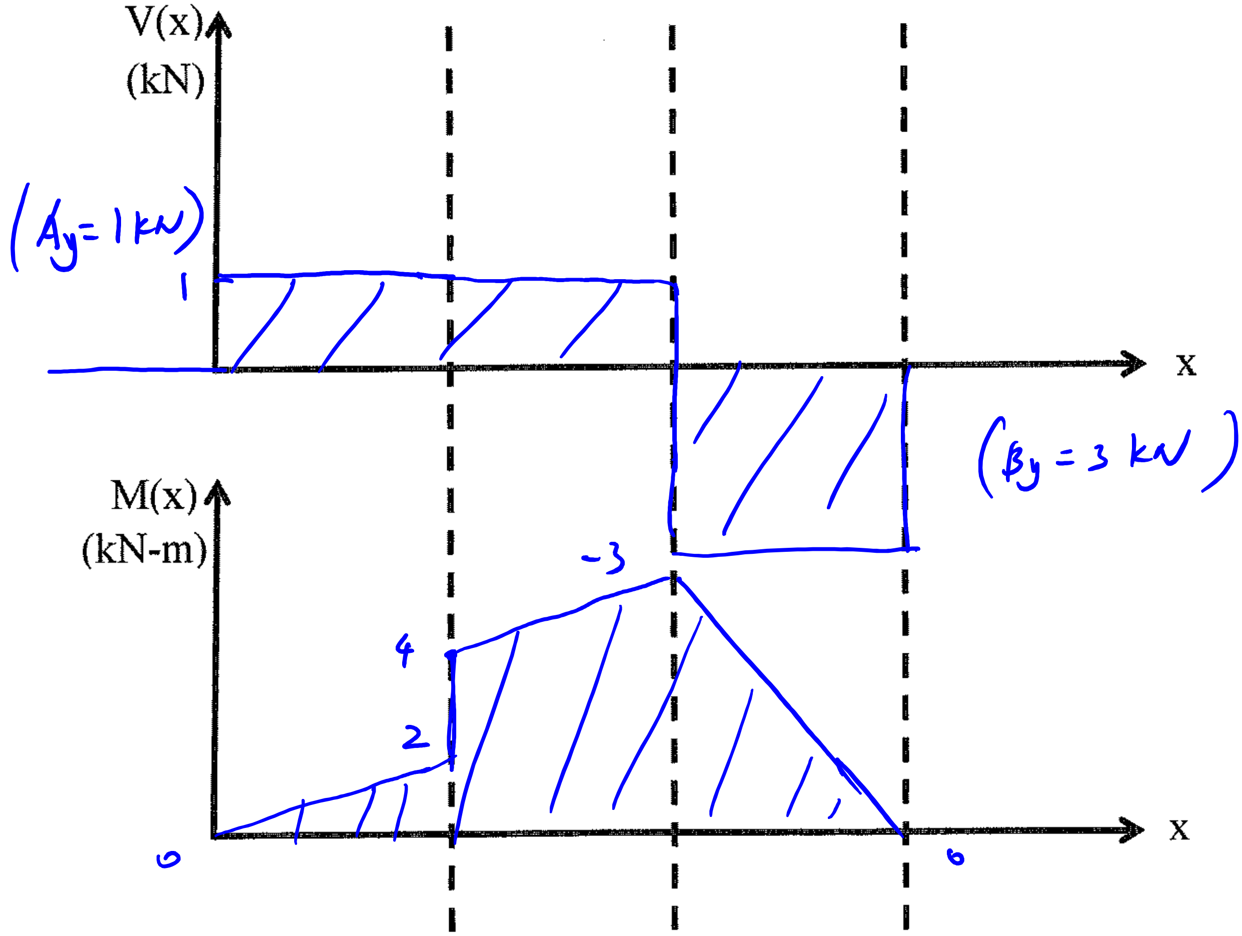
(d)
C
D
V
ΔV
(e)
x
C
D
M
ΔM
(f)
x
C
D

Shear-Force & Bending-Moment Diagrams
Graphical Methods
Example 1

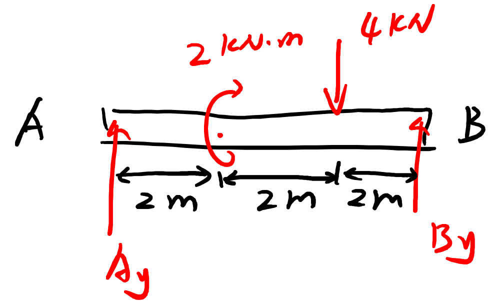
Given: A simply-supported beam is loaded with a 2 kN-m couple and a 4 kN load as shown.

Find: Using graphical methods, draw the shear-force and bending-moment diagrams.





Reaction:



$$\Sigma M_A = 0: \quad -2 - 4(4) + B_y(6) = 0$$

$$\Rightarrow B_y = 3 \text{ kN}$$

$$\Sigma F_y = 0: \quad A_y + B_y - 4 = 0$$

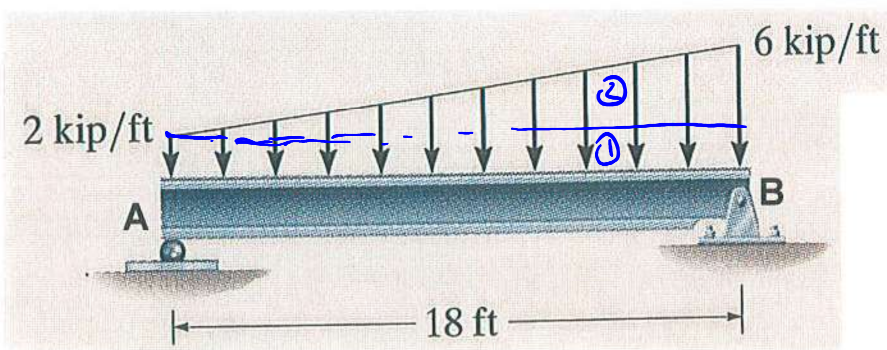
$$\Rightarrow A_y = 1 \text{ kN}$$

Shear-Force & Bending-Moment Diagrams
Graphical Methods
Example 2

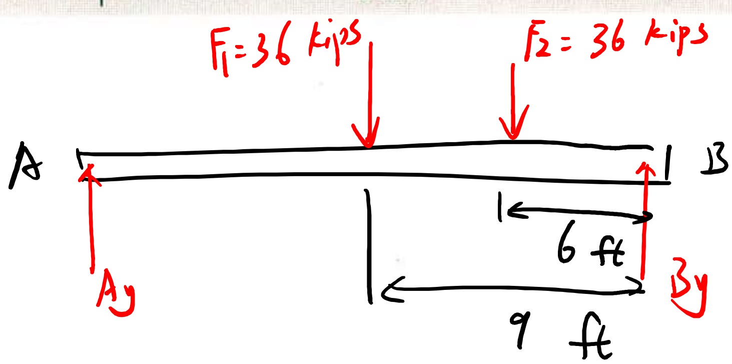
Given: A simply supported beam is supporting a trapezoidal loading as shown and is in static equilibrium.

Find:

a) Determine the reactions at supports A and B.
b) Write an algebraic expression for the trapezoidal loading shown $(w(x))$.
c) Using the relationship $\frac{dV}{dx} = w(x)$, determine an expression for the shear force as a function of x $(\text{i.e., } V(x))$.
d) Using the relationship $\frac{dM}{dx} = V(x)$, determine an expression for the bending moment as a function of x $(\text{i.e., } M(x))$.



a) Reaction:



$\Sigma M_A = 0: \quad -F_1 \cdot (9) - F_2 (12) + B_y (18) = 0$

$\Rightarrow B_y = 42 \text{ kips}$

$\Sigma F_y = 0: \quad A_y + B_y - F_1 - F_2 = 0$

$\Rightarrow A_y = 30 \text{ kips}$

b). $w(x) = -2 - \frac{2}{9}x$ kip/ft

c) $\frac{dV}{dx} = w(x) = -\frac{2}{9}x - 2$

$V(x) - \underbrace{V(0)}_{A_y = 30} = \int_0^x w(s)ds = \int_0^x \left(-\frac{2}{9}s - 2\right)ds$

$= \left[-\frac{1}{9}s^2 - 2s\right]\Big|_0^x = -\frac{1}{9}x^2 - 2x$

$\Rightarrow V(x) = -\frac{x^2}{9} - 2x + 30$ kips

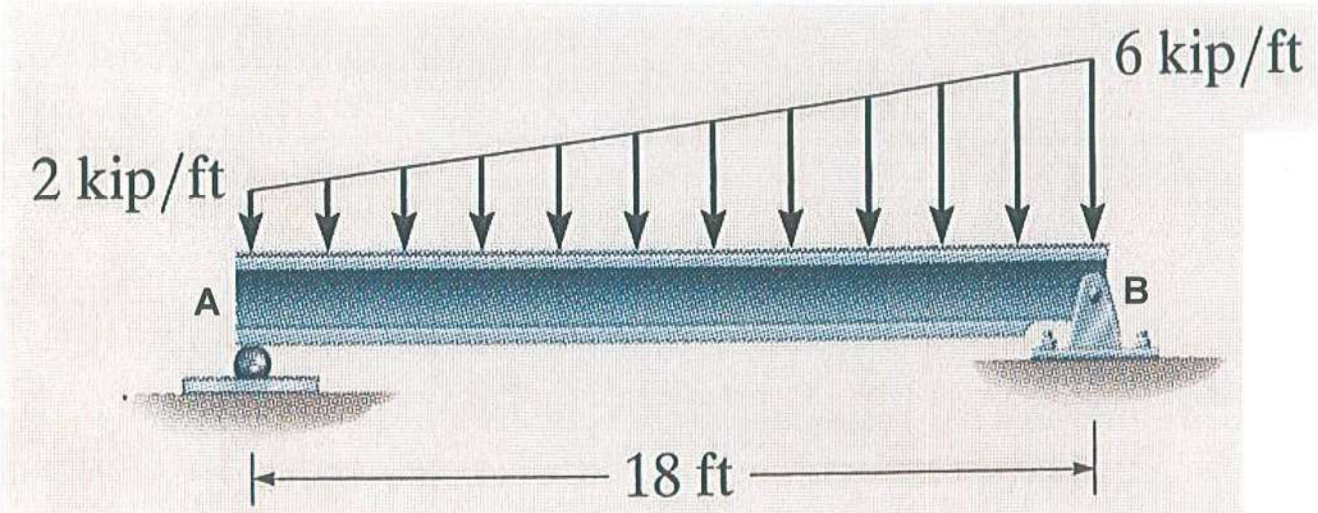
d) $\frac{dM}{dx} = V(x) = -\frac{x^2}{9} - 2x + 30$

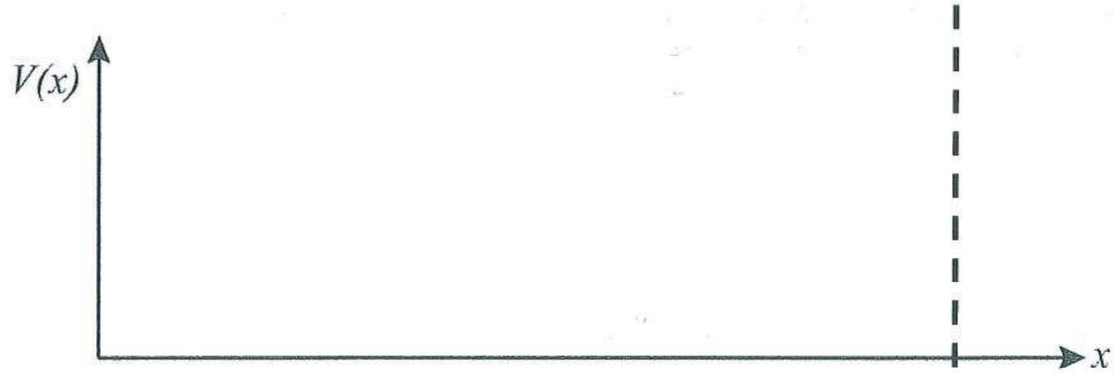
$M(x) - \underbrace{M(0)}_{0} = \int_0^x V(s)\,ds = \int_0^x \left(-\frac{s^2}{9} - 2s + 30\right)ds$

$= \left[-\frac{s^3}{27} - s^2 + 30s\right]\Big|_0^x$

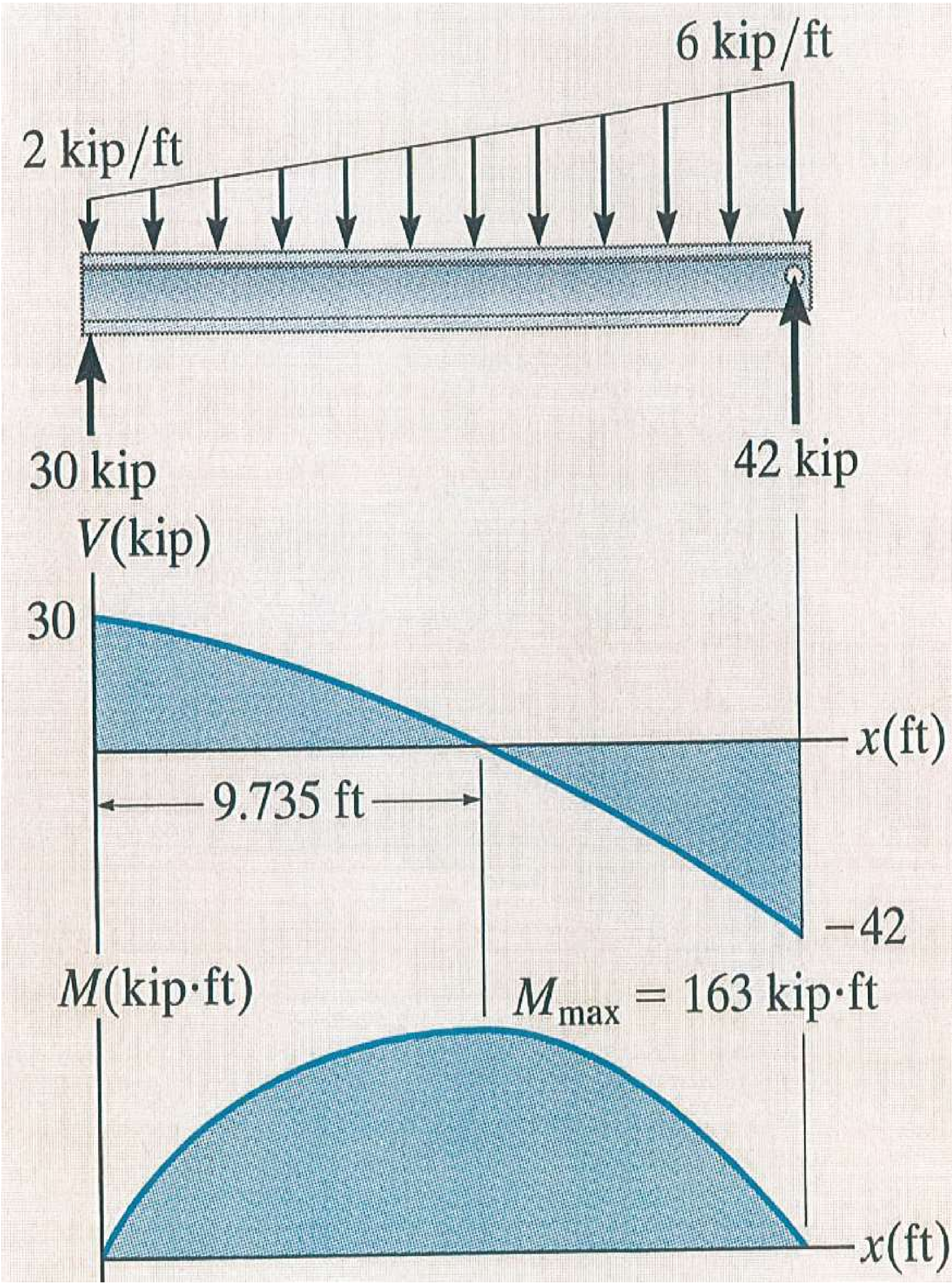
$= -\frac{x^3}{27} - x^2 + 30x$

$M(x) = -\frac{x^3}{27} - x^2 + 30x$ kip·ft.


6 kip/ft
2 kip/ft
A
B
18 ft


V(x)
x


M(x)
x

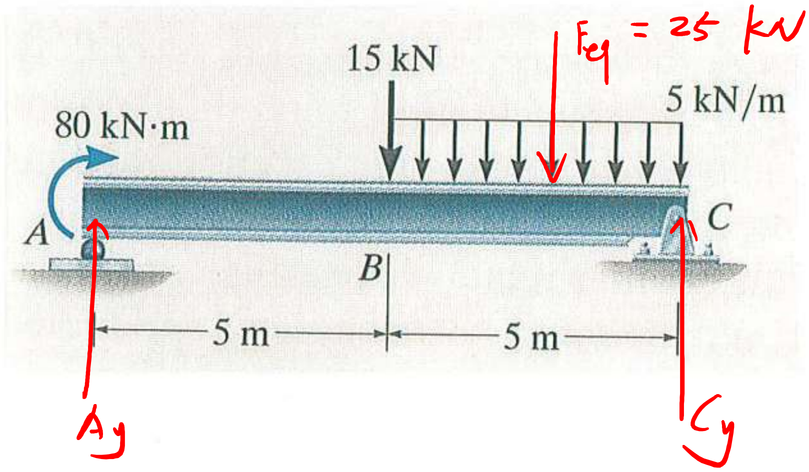

6 kip/ft
2 kip/ft
30 kip
42 kip
V(kip)
30
x(ft)
9.735 ft
−42
M(kip·ft)
M_max = 163 kip·ft
x(ft)

Shear-Force and Bending-Moment Diagrams

Example 3

Given: A simply-supported beam is loaded as shown.

Find: Using graphical methods, draw the shear-force and bending-moment diagrams on the axes provided below.

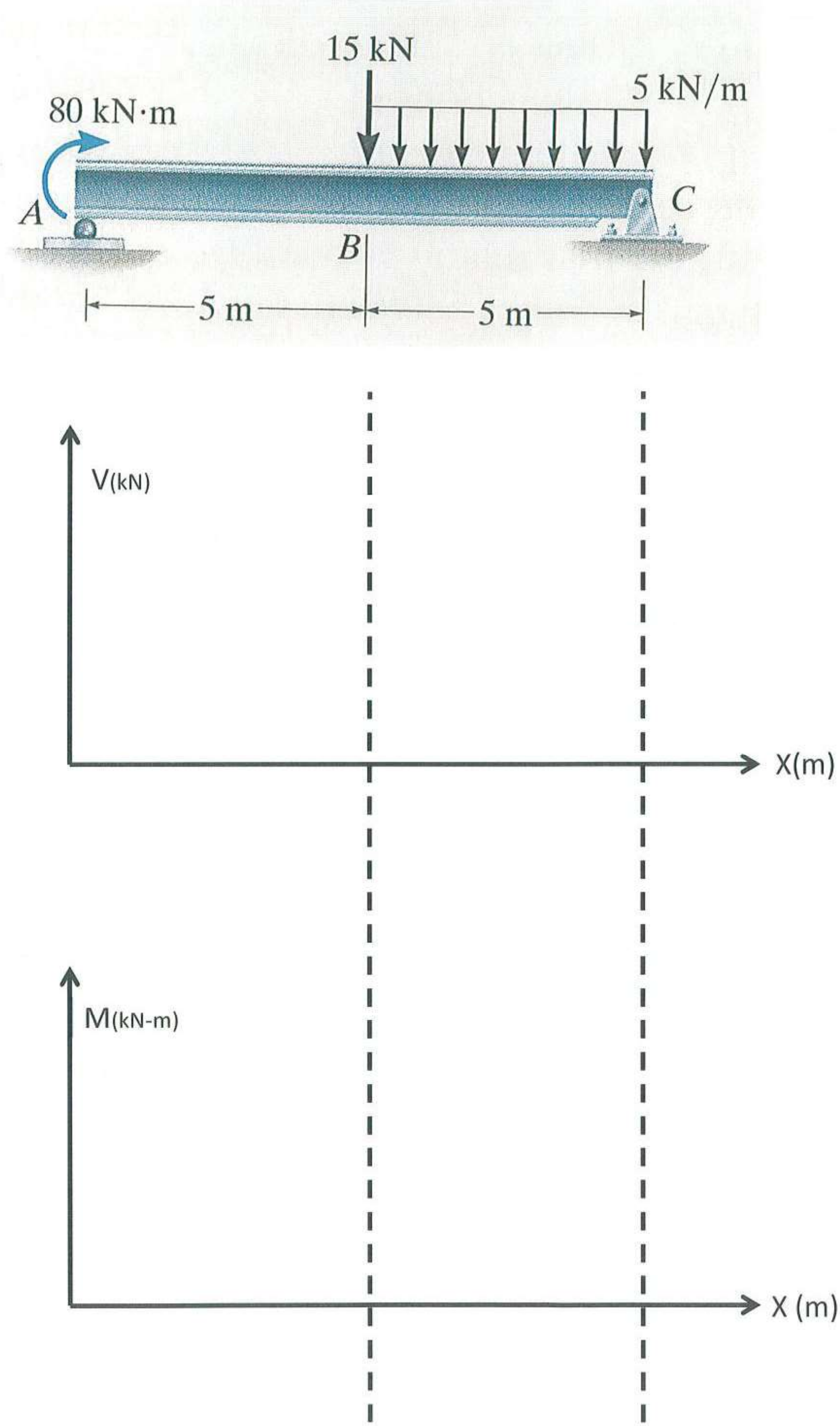


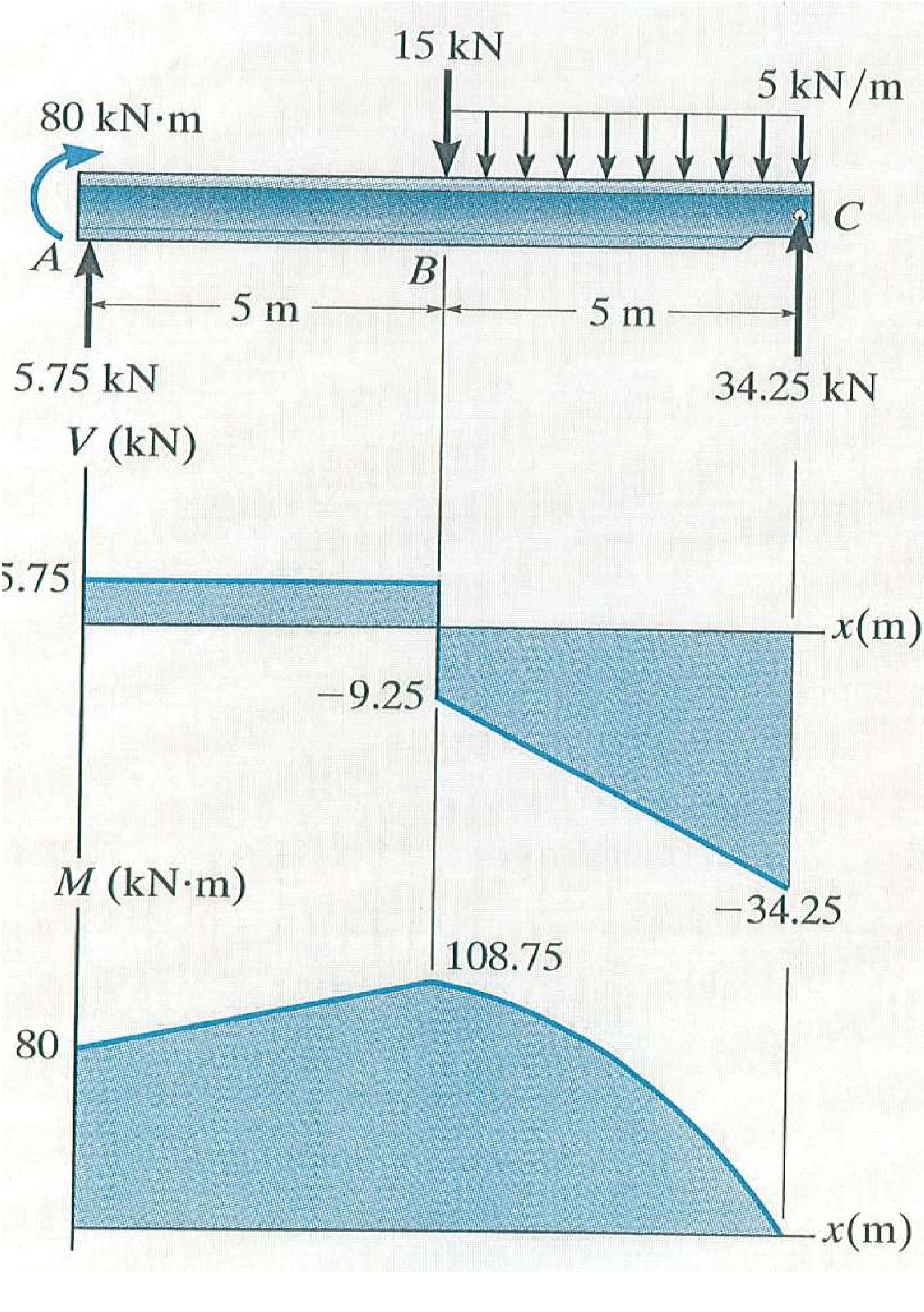
$$\Sigma M_C = 0: \quad -80 - A_y(10) + 15(5) + 25(2.5) = 0$$

$$\Rightarrow A_y = 5.75 \text{ kN}$$

$$\Sigma F_y = 0: \quad A_y - 15 - 25 + C_y = 0$$

$$\Rightarrow C_y = 34.25 \text{ kN}$$


15 kN
5 kN/m
80 kN·m
A
B
C
5 m
5 m
V(kN)
X(m)
M(kN-m)
X (m)

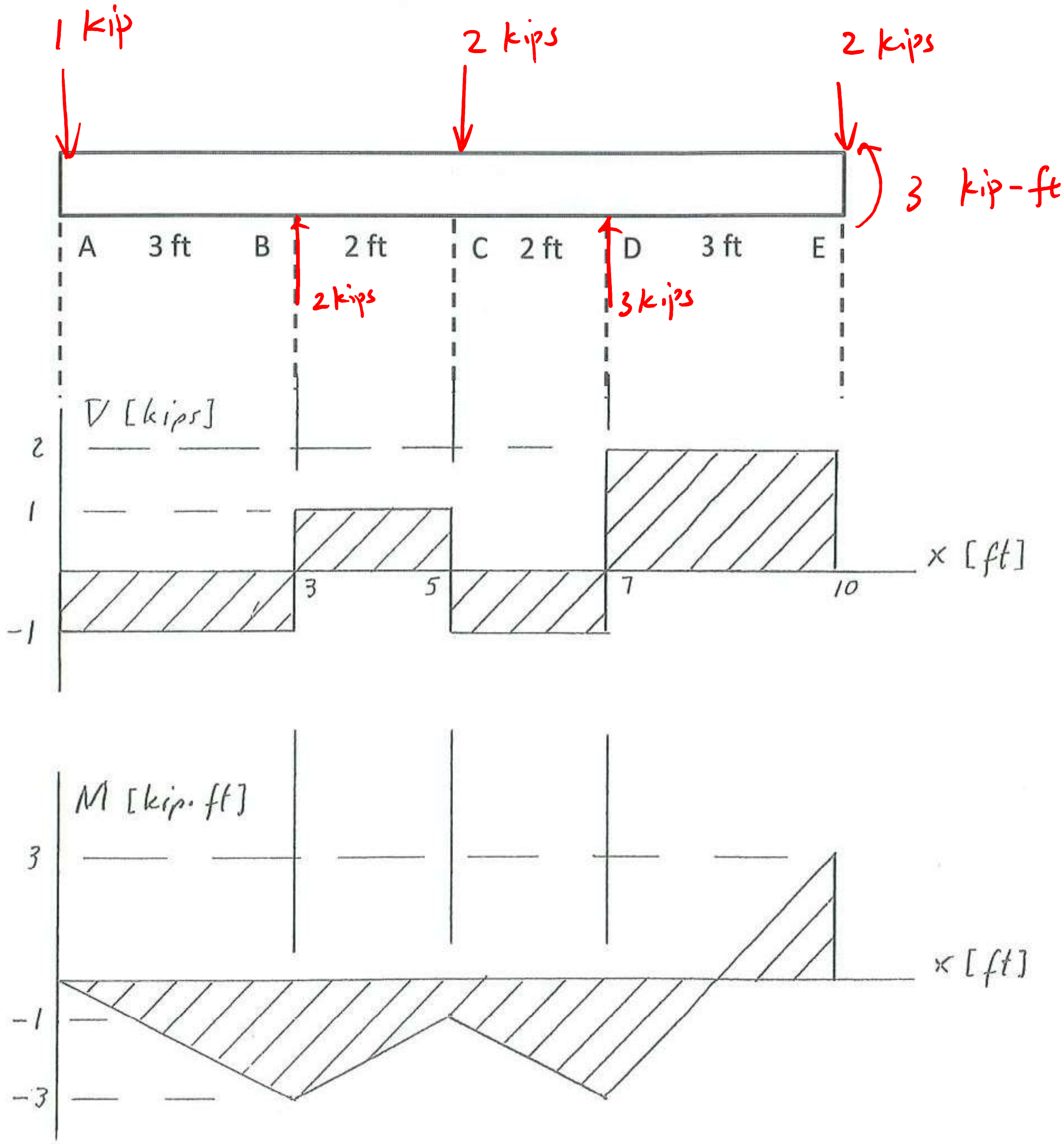


15 kN
5 kN/m
80 kN·m
A
B
C
5 m
5 m
5.75 kN
34.25 kN
V (kN)
5.75
x(m)
−9.25
−34.25
M (kN·m)
108.75
80
x(m)

Shear-Force and Bending-Moment Diagrams
Example 4

Given: The shear-force and bending-moment diagrams for a loaded beam are provided below.

Find: On the beam provided, sketch a valid set of loads that would result in the shear-force and bending-moment diagrams shown.



ME 270 – Basic Mechanics I – Group Quiz

Your Name: ______________________ Group Members: 1) ____________________________

Date: ____________ Period: ________ 2) ____________________________

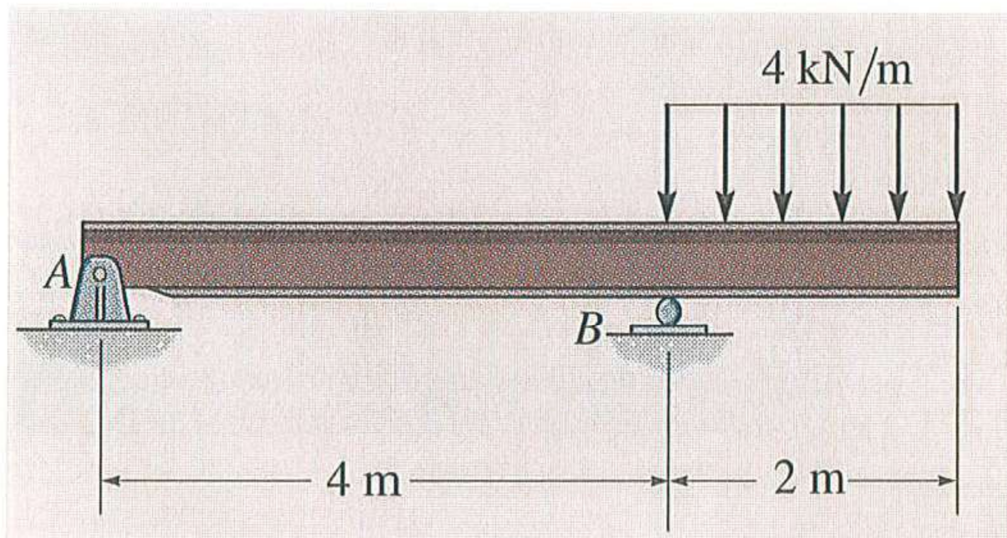
3) ____________________________

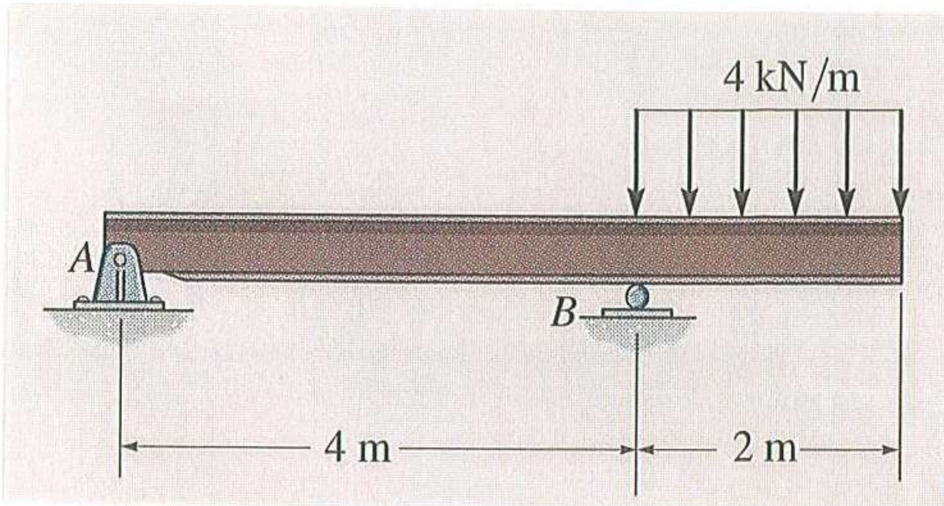
4) ____________________________

Given: Cantilever beam AB is loaded as shown.

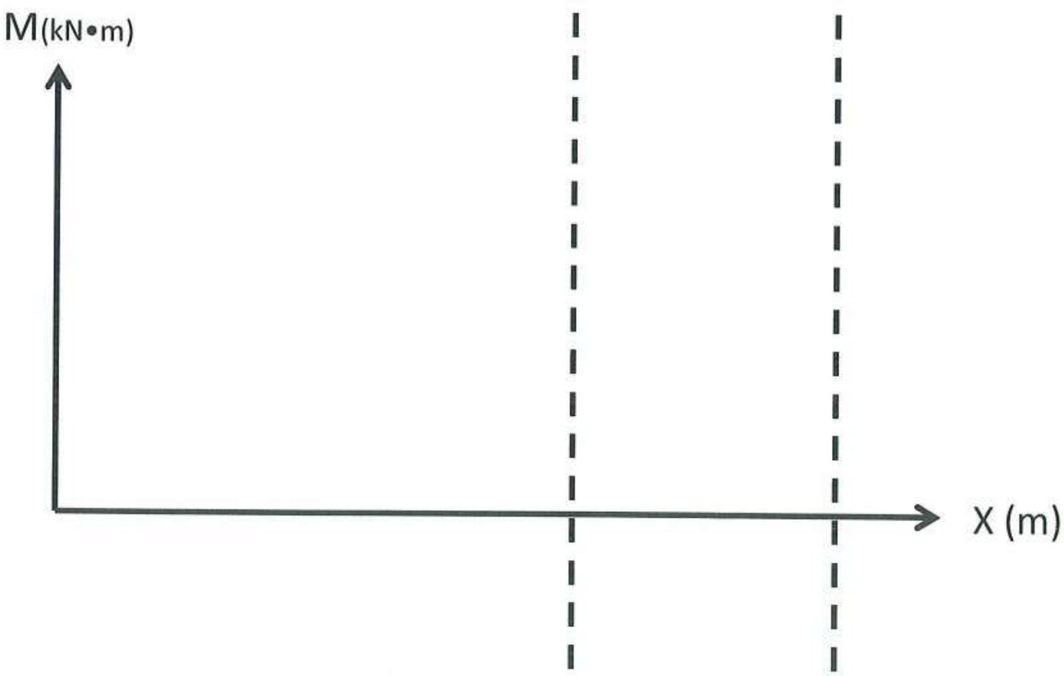
Find: Using the graphical method, sketch the shear-force and bending moment diagrams.

Solution:




4 kN/m
A
B
4 m
2 m


V(kN)
X(m)


M(kN•m)
X (m)

ME 270 – Basic Mechanics I – Group Quiz

Your Name: ______________________ Group Members: 1) ____________________________

Date: ____________ Period: ________ 2) ____________________________

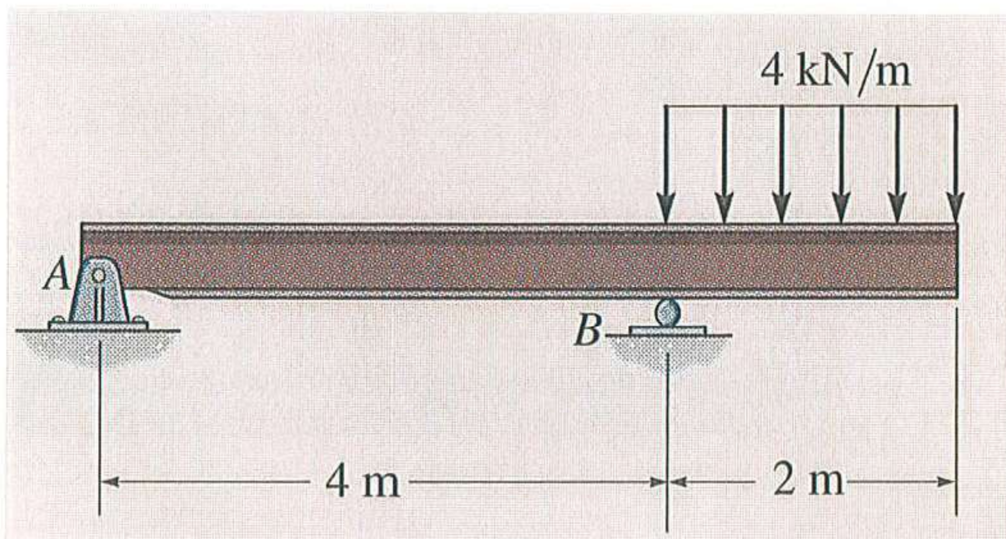
3) ____________________________

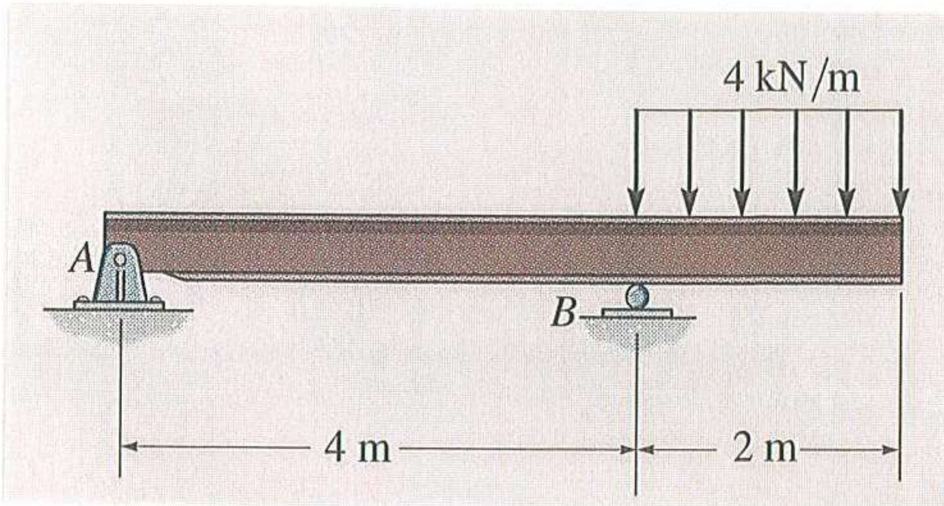
4) ____________________________

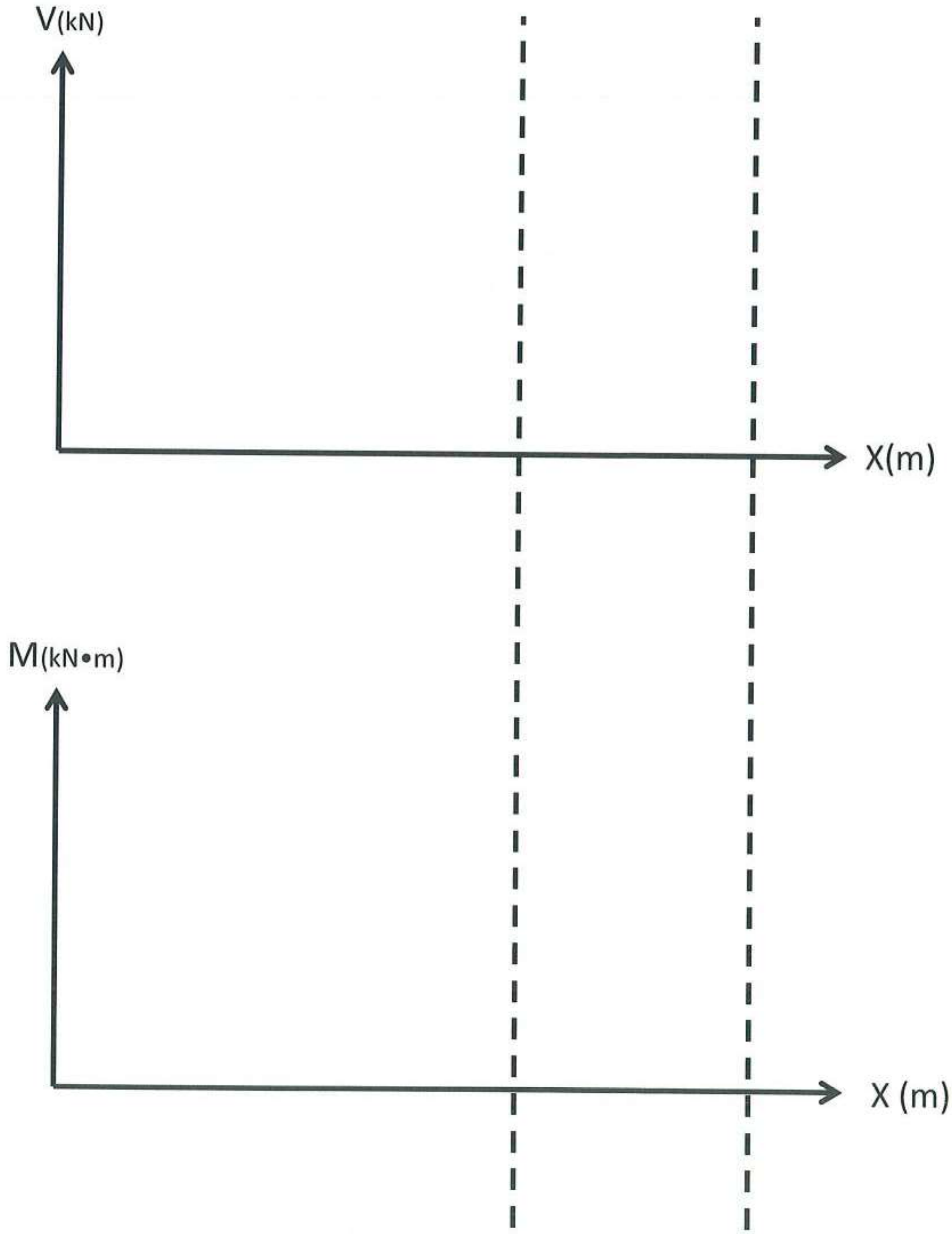
Given: Cantilever beam AB is loaded as shown.

Find: Using the graphical method, sketch the shear-force and bending moment diagrams.

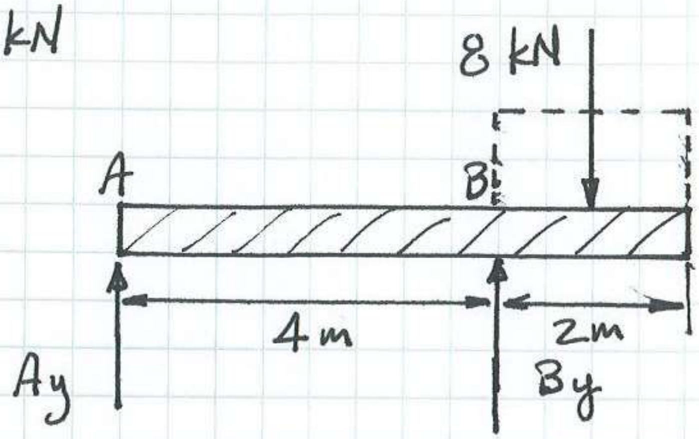
Solution:




4 kN/m
A
B
4 m
2 m


V(kN)
X(m)
M(kN•m)
X (m)

$F_{eq} = \left(4\ \frac{kN}{m}\right)(2\ m) = 8\ kN$

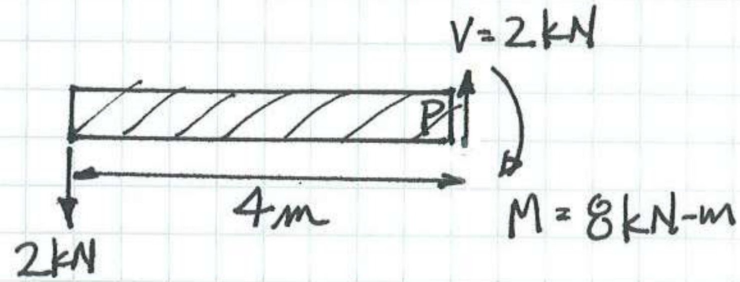


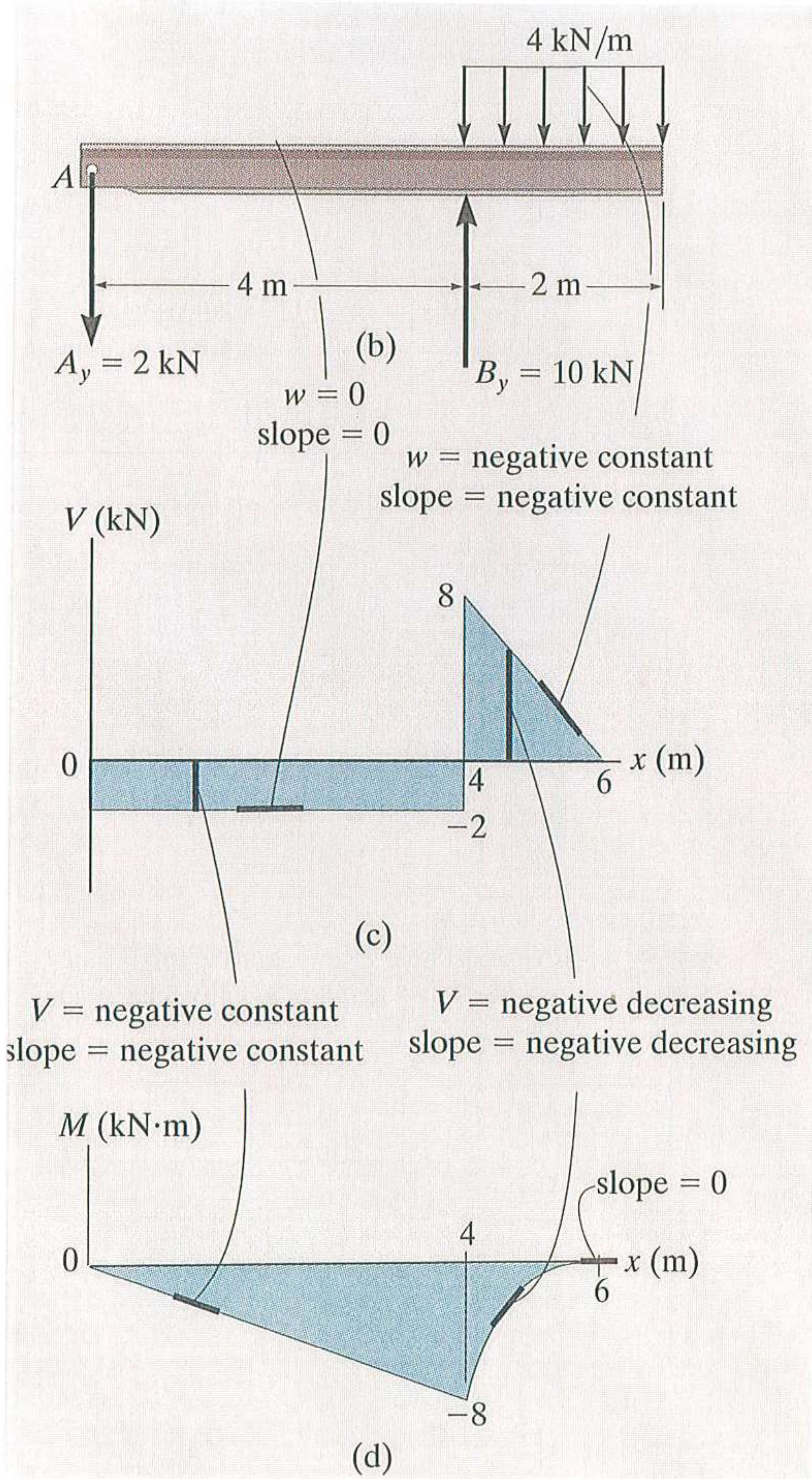
$\Sigma M_A = 0 = B_y(4) - 8(5) \Rightarrow \boxed{B_y = 10\,kN}$

$\Sigma F_y = 0 = A_y + B_y - 8 \Rightarrow \boxed{A_y = -2\,kN}$

$M|_{x=4} = M|_{x=0} + \Delta M = 0 + (-2\,kN)(4\,m)$

$\boxed{M|_{x=4} = -8\ kN\text{-}m}$




4 kN/m
A
4 m
2 m
(b)
$A_y = 2$ kN
$B_y = 10$ kN
$w = 0$
slope = 0
w = negative constant
slope = negative constant
V (kN)
8
0
4
6
x (m)
−2
(c)
V = negative constant
slope = negative constant
V = negative decreasing
slope = negative decreasing
M (kN·m)
slope = 0
4
0
x (m)
6
−8
(d)