STATIC EQUILIBRIUM OF RIGID BODIES (2-D)

Learning Objectives

1). To evaluate the *unknown reactions* holding a rigid body in equilibrium by solving the *equations of static equilibrium*.
2). To recognize situations of *partial* and *improper constraint*, as well as *static indeterminacy*, on the basis of the solvability of the equations of static equilibrium.

Newton's First Law

Given *no net force*, a body at rest will remain at **rest** (and a body moving at a constant velocity will continue to do so along a straight path).

Definitions

Zero-Force Members: structural members that support no loading but aid in the stability of the truss.

Two-Force Members: structural members that are: a) subject to no applied or reaction moments, and b) are loaded only at two pin joints along the member.

Multi-Force Members: structural members that have a) applied or reaction moments, or b) are loaded at more than two points along the member.

Vector Equations

$$\overline{F_R} = \sum \overline{F} = \overline{0}$$

$$\overline{M_{R_O}} = \sum \overline{M_O} = \overline{0}$$ where O is any arbitrary point

Component Equations

There are three alternate forms of equilibrium equations for 2-D problems.

(i) Two component force equations (x and y) are one moment equation (z).

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0$$

(ii) One component force equation (x or y) and two moment equations (both about different points in the z direction).

$$\sum F_x = 0 \qquad \sum M_A = 0 \qquad \sum M_B = 0$$

(iii) Three moment equations (points A, B and C cannot be collinear).

$$\sum M_A = 0 \qquad \sum M_B = 0 \qquad \sum M_C = 0$$

Static Determinacy/Partial and Improper Constraints

Static Indeterminacy: occurs when a system has *more* constraints than is necessary to hold the system in equilibrium (i.e., the system is *overconstrained* and thus has *redundant* reactions).

Static Determinancy: occurs when a system has a *sufficient* number of constraints to prevent motion without any redundancy.

Partial Constraint: occurs when there is an *insufficient* number of reaction forces to prevent motion of the system (i.e., the system is *partially constrained*).

Improper Constraint: occurs when a system has a *sufficient* number of reaction forces but one or more are *improperly applied* so as not to prevent motion of the system (i.e., the system is *improperly constrained*).

Comments:

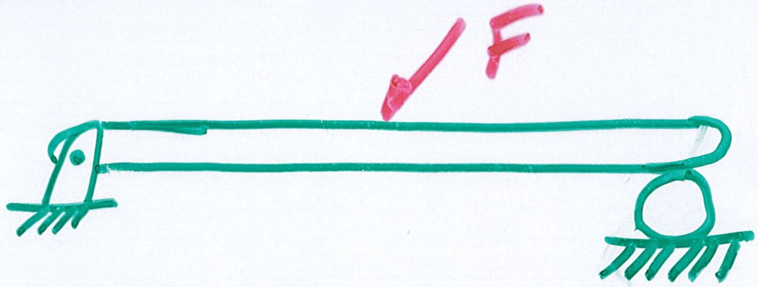
1). Equations (i) are the equilibrium eqns most commonly used.

2). NEVER attempt to use MORE THAN THREE equilibrium equations from a single planar FBD. Only three independent equations can exist for a single planar FBD.

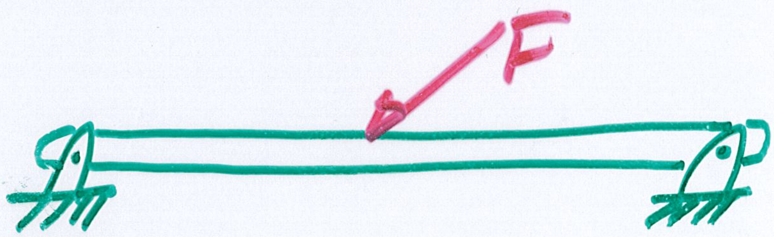
3). If you have more than three unknown forces in your three equations, then consider breaking the system or structure into smaller systems and write down equilibrium equations for each sub-structure. If this is not possible, you may have an indeterminate structure; i.e., the evaluation of member forces requires consideration of deformation of the members resulting from the loading.

4). If all forces act through a single point, then the moment equation for any point will not provide any more new information.

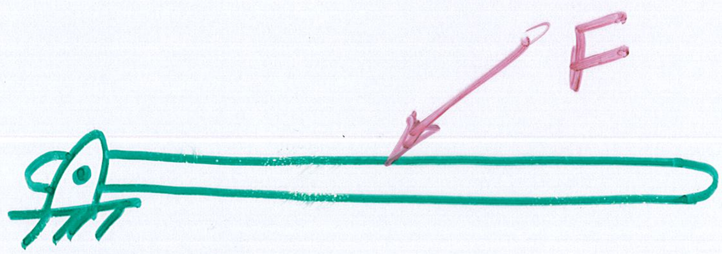
STATIC DETERMINACY



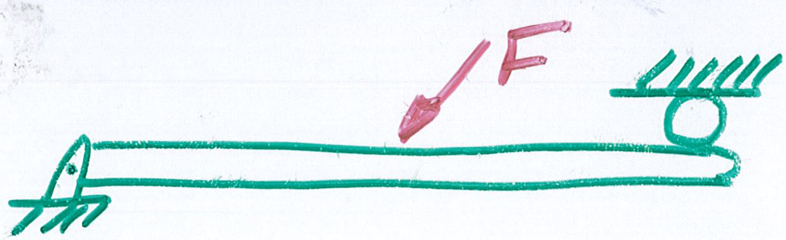
STATIC INDETERMINACY



PARTIAL CONSTRAINT



IMPROPER CONSTRAINT



2-D Static Equilibrium
Example 1

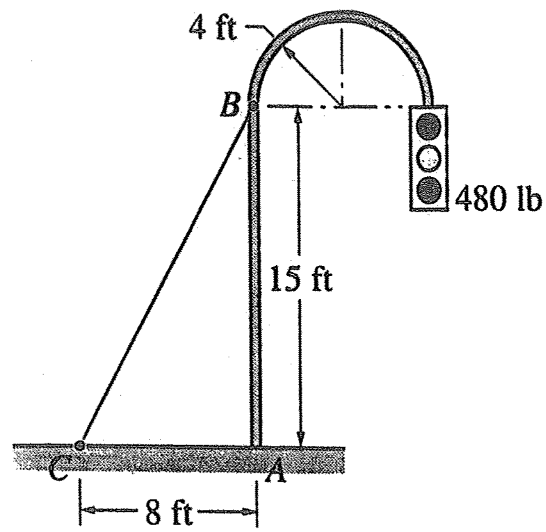
Given: Traffic light pole ABC is supported by a fixed support at A and a support cable BC. The pole is loaded with a 480 lb traffic light as shown. Assume cable AB carries a tension of 544 lbs.

Find:

a) Draw a sketch of the free body diagram of pole ABC.

b) Determine the reactions at the fixed support at A.

c) If the cable is accidently cut, what effect would this have on each of the support reactions (i.e., do they increase, decrease, or remain the same).



Solution:

2-D Static Equilibrium

Example 2

Given: A 5 kN force and a 4 kN-m couple are applied to the forked bar as shown.

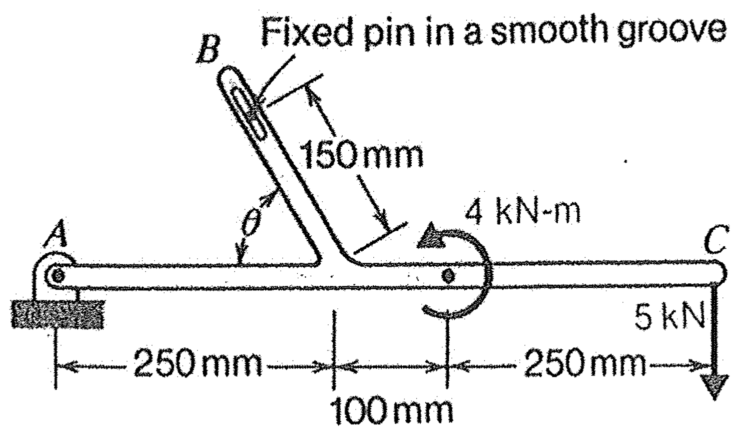
Find:

a) Draw a free body diagram of the forked bar ABC.

b) Determine the reaction forces at supports A and B when $\theta = 36.87°$.

c) If the 4 kN-m couple were applied 250 mm from pin A, qualitatively how would this effect the reactions at A and B?

d) If the 4 kN-m couple were removed from the forked bar, qualitatively how would this effect the reactions at A and B?



Solution:

2-D Static Equilibrium

Example 3

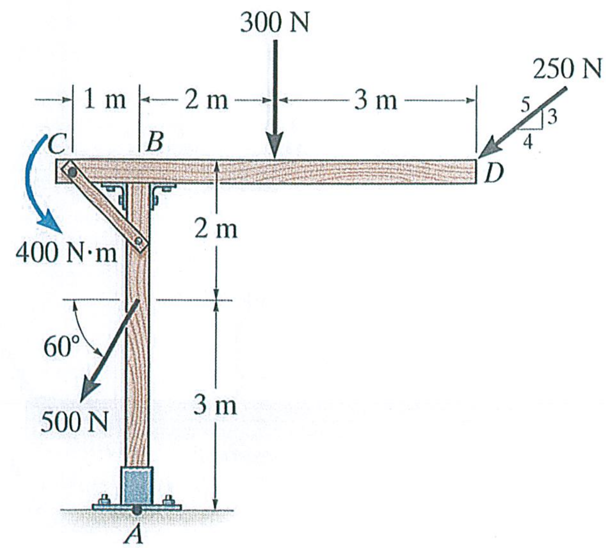
Given: Frame ABCD is loaded as shown and held in static equilibrium with a fixed support at base A.

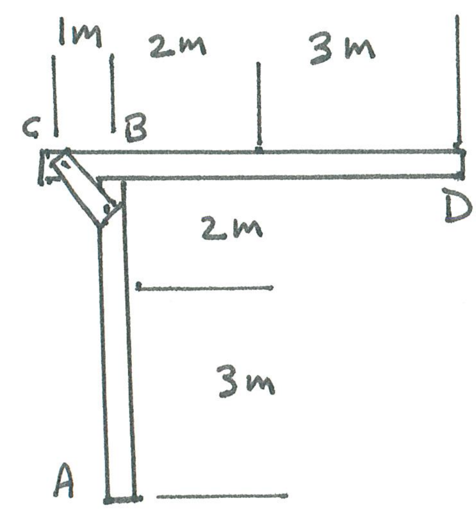
Find:

a) Draw a free body diagram of the frame.

b) Write the equations of static equilibrium.

c) Solve the equations for the reactions at base A.



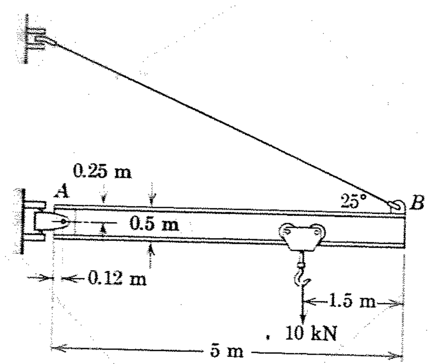


ME 270 - Basic Mechanics I - Group Quiz

Name/Group #: ______________________ Group Members: 1) ____________ 2) ____________

Date: __________ Period: __________ 3) ____________ 4) ____________

Given: A jib crane AB consists of standard 5m I-beam with a mass of 95 kg per meter of length. The end of the crane is supported using a 1 cm diameter cable attached at point B.



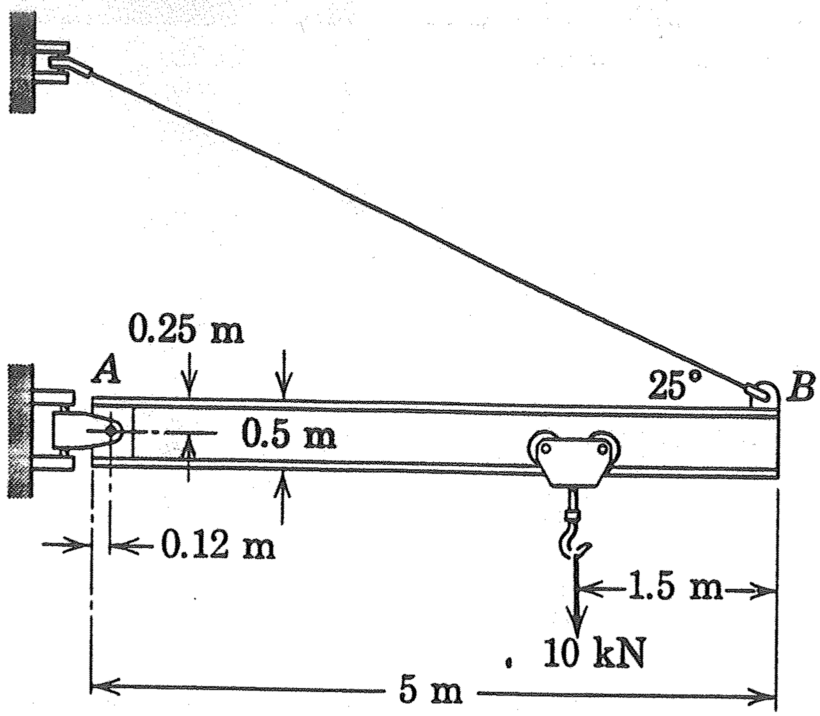
Find:

(a) Draw a free body diagram of the crane.

(b) Determine the reactions at pin A and the tension in the supporting cable. Express the reaction forces at point A in vector form.

(c) If the 10 kN load is moved further toward end B, what effect will this have on the tension in the supporting cable and the reactions at pin support A.

Solution:


0.25 m
A
25°
B
0.5 m
0.12 m
1.5 m
10 kN
5 m