Equivalent system Prob. V.S. Static Equilibrium Prob.

Equivalent system Prob. $\downarrow$

Apply a force and a moment at a point to make the <u>same</u> effect as the system

$$\begin{cases} \bar{F}_{eq} = \Sigma \bar{F} \\ \bar{M}_{eq} = \Sigma \bar{r} \times \bar{F} + \Sigma \bar{C} \end{cases}$$

Static Equilibrium Prob. $\downarrow$

Apply forces and moments to <u>balance</u> the effect of the system

$$\begin{cases} \Sigma \bar{F} = 0 \\ \Sigma \bar{M} = 0 \end{cases}$$

- A Static Equilibrium system is equivalent to a sys with <u>zero</u> force and <u>zero</u> moment

FREE BODY DIAGRAMS (FBDs)

Learning Objectives

1). To inspect the supports of a rigid body in order to determine the nature of the reactions, and to use that information to draw a *free body diagram* (FBD).

Force/Moment Classifications

External Forces/Moments: applied forces/moments which are typically known or prescribed (e.g., forces/moments due to cables springs, gravity, etc.).

Reaction Forces/Moments: constraining forces/moments at supports intended to prevent motion (usually nonexistent unless system is externally loaded).

Free Body Diagram (FBD)

Free Body Diagram (*FBD*): a graphical sketch of the system showing a coordinate system, all external/reaction forces and moments, and key geometric dimensions.

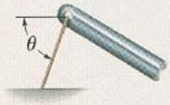
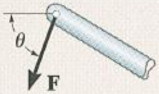
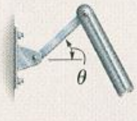
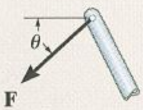
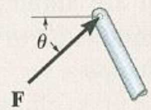
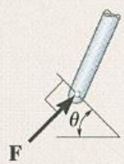
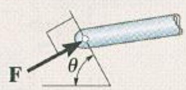
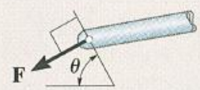
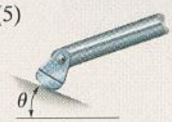
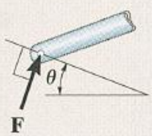
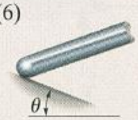
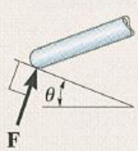
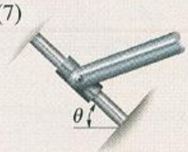
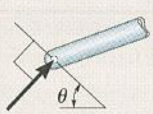
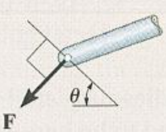
Benefits:

1). Provides a *coordinate system* to establish a solution methodology.

2). Provides a *graphical display* of all forces/moments acting on the rigid body.

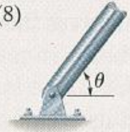
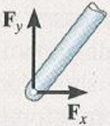
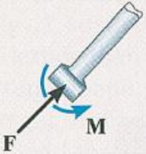
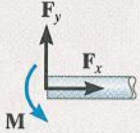
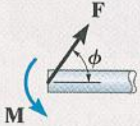
3). Provides a record of *geometric dimensions* needed for establishing moments of the forces.

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

5

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge	 or 	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support	 or 	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

STATIC EQUILIBRIUM OF RIGID BODIES (2-D)

Learning Objectives

1). To evaluate the *unknown reactions* holding a rigid body in equilibrium by solving the *equations of static equilibrium*.
2). To recognize situations of *partial* and *improper constraint*, as well as *static indeterminacy*, on the basis of the solvability of the equations of static equilibrium.

Newton's First Law

Given *no net force*, a body at rest will remain at **rest** (and a body moving at a constant velocity will continue to do so along a straight path).

Definitions

Zero-Force Members: structural members that support no loading but aid in the stability of the truss.

Two-Force Members: structural members that are: a) subject to no applied or reaction moments, and b) are loaded only at two pin joints along the member.

Multi-Force Members: structural members that have a) applied or reaction moments, or b) are loaded at more than two points along the member.

Vector Equations

$$\overline{F_R} = \sum \overline{F} = \overline{0}$$

$$\overline{M_{R_O}} = \sum \overline{M_O} = \overline{0}$$ where O is any arbitrary point

Component Equations

There are three alternate forms of equilibrium equations for 2-D problems.

(i) Two component force equations (x and y) are one moment equation (z).

$$\boxed{\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_A = 0}$$

(ii) One component force equation (x or y) and two moment equations (both about different points in the z direction).

$$\boxed{\sum F_x = 0 \qquad \sum M_A = 0 \qquad \sum M_B = 0}$$

(iii) Three moment equations (points A, B and C cannot be collinear).

$$\boxed{\sum M_A = 0 \qquad \sum M_B = 0 \qquad \sum M_C = 0}$$

Static Determinacy/Partial and Improper Constraints

Static Indeterminacy: occurs when a system has *more* constraints than is necessary to hold the system in equilibrium (i.e., the system is *overconstrained* and thus has *redundant* reactions).

Static Determinancy: occurs when a system has a *sufficient* number of constraints to prevent motion without any redundancy.

Partial Constraint: occurs when there is an *insufficient* number of reaction forces to prevent motion of the system (i.e., the system is *partially constrained*).

Improper Constraint: occurs when a system has a *sufficient* number of reaction forces but one or more are *improperly applied* so as not to prevent motion of the system (i.e., the system is *improperly constrained*).

Comments:

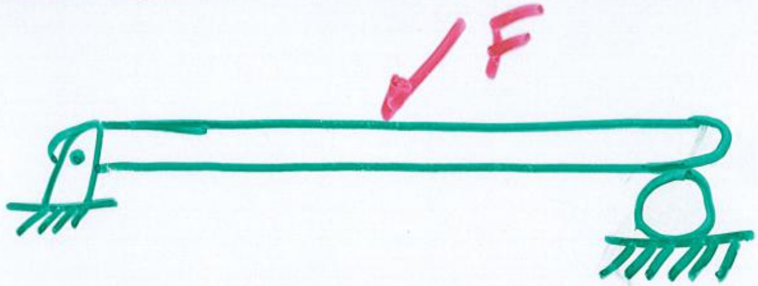
1). Equations (i) are the equilibrium eqns most commonly used.

2). NEVER attempt to use MORE THAN THREE equilibrium equations from a single planar FBD. Only three independent equations can exist for a single planar FBD.

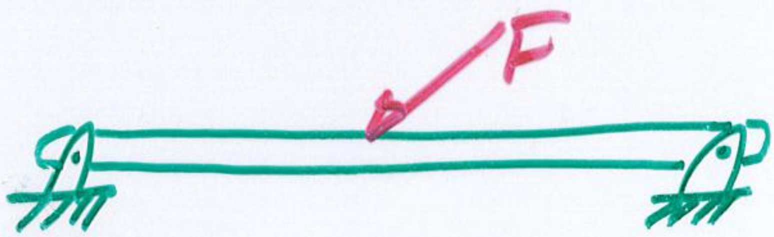
3). If you have more than three unknown forces in your three equations, then consider breaking the system or structure into smaller systems and write down equilibrium equations for each sub-structure. If this is not possible, you may have an indeterminate structure; i.e., the evaluation of member forces requires consideration of deformation of the members resulting from the loading.

4). If all forces act through a single point, then the moment equation for any point will not provide any more new information.

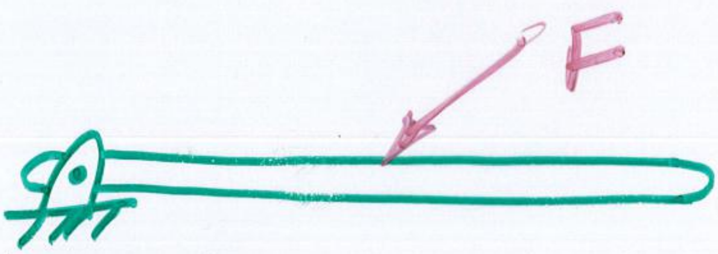
STATIC DETERMINACY



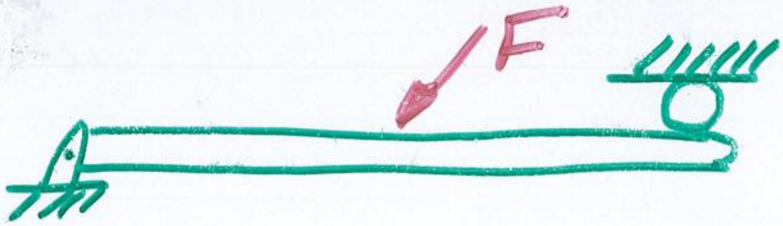
STATIC INDETERMINACY



PARTIAL CONSTRAINT



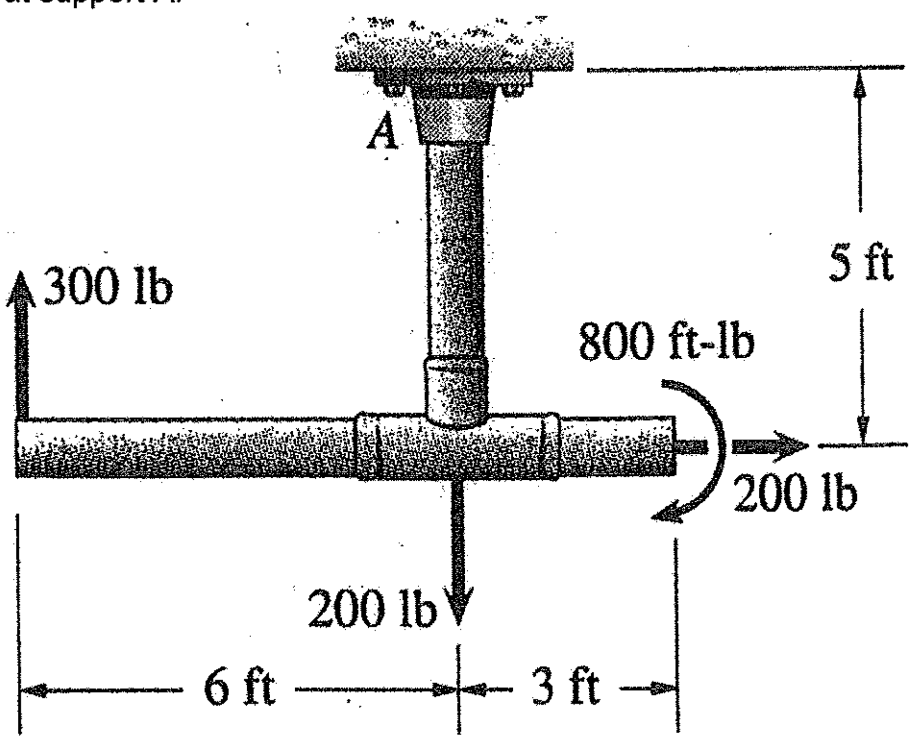
IMPROPER CONSTRAINT



Static Equilibrium of a Rigid Body 2-D
Example 1

Given: Frame loaded as shown.

Find: Determine the reactions at support A.



Solution:

2-D Static Equilibrium

Example 2

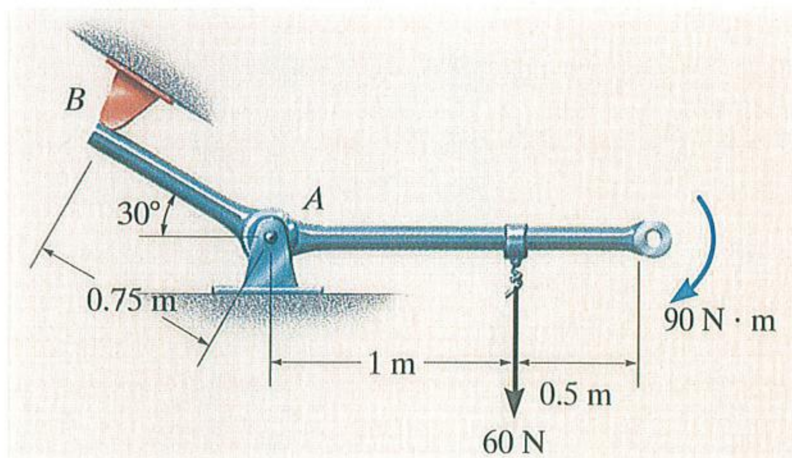
Given: The angled bar is loaded with a 60 N force and a 90 N-m couple as shown and is held in static equilibrium by a pin support at A and a smooth support at B.

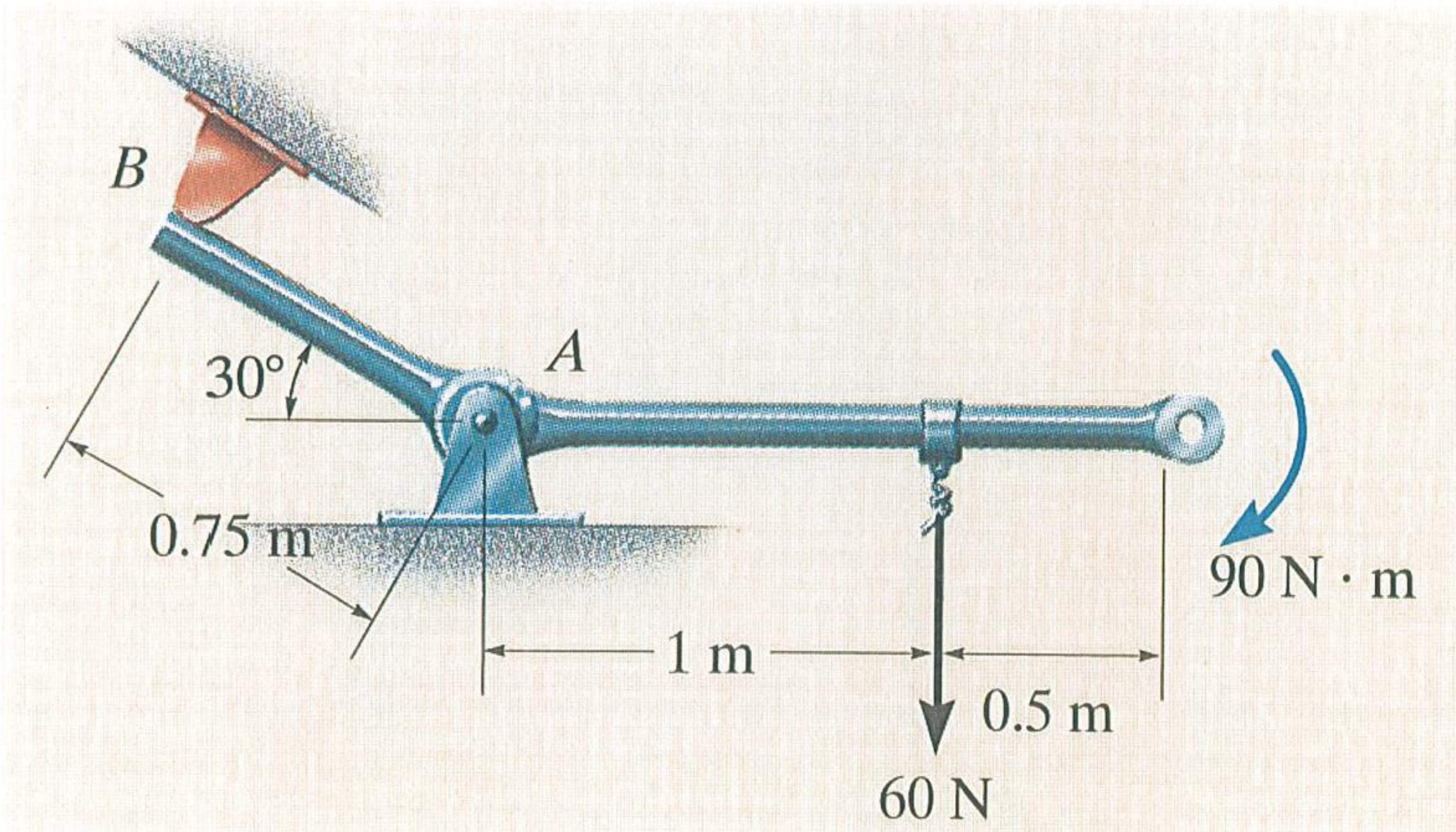
Find:

a) Draw a free body diagram of the frame.

b) Write the equations of static equilibrium.

c) Solve the equations for the reactions at base A.



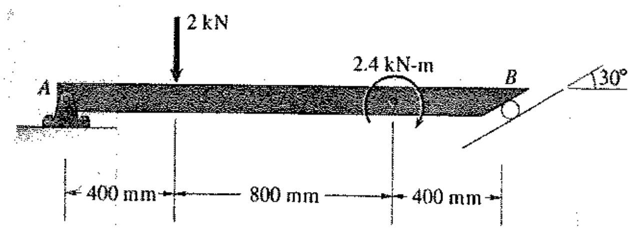

B
30°
A
0.75 m
1 m
0.5 m
60 N
90 N · m

ME 270 - Basic Mechanics I - Group Quiz

Name/Group #: ______________________ Group Members: 1) ____________ 2) ____________

Date: __________ Period: __________ 3) ____________ 4) ____________

Given: Beam AB is loaded with a 2 kN force and a 2.4 kN-m couple as shown.



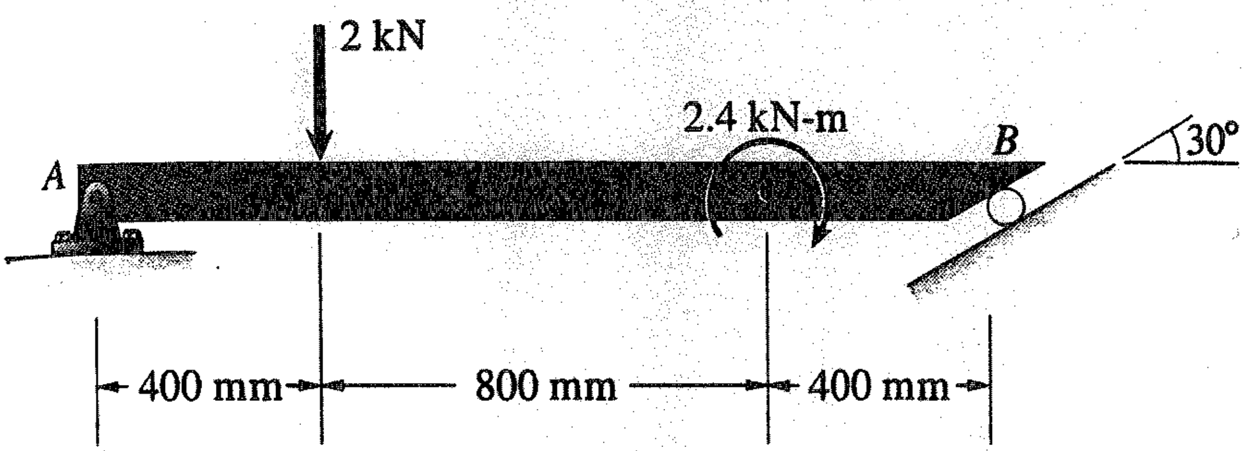
Find:

(a) Draw a free body diagram of beam AB.

(b) Determine the reactions at the supports.

(c) If the reaction at roller B was found to be negative, what would this imply physically about the system.

Solution:


2 kN
2.4 kN-m
A
B
30°
400 mm
800 mm
400 mm