DOT PRODUCT (Scalar Product)

Learning Objectives

1). To use the dot product to determine:

i) the *projection of a vector* in another direction,

ii) the *angle* (θ) between two vectors and

2). To do an *engineering estimate* of these quantities.

Definitions

$$\bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A} \;=\; \text{scalar quantity} = |\bar{A}|\;|\bar{B}|\;\cos\theta$$

In words,

$\bar{A} \cdot \bar{B} =$ the magnitude $|\bar{A}|$ times the magnitude of the projection of $\bar{B}$ on $\bar{A}$ (i.e., $|\bar{B}|\cos\theta$).

Projection of a Vector

$$\left|\bar{A}_u\right| = \bar{A} \bullet \bar{u} \qquad\qquad \bar{A}_u = \left(\bar{A} \bullet \bar{u}\right)\bar{u}$$

Angle (θ) Between Two Vectors

$$\theta = \cos^{-1}\left(\frac{\bar{A} \bullet \bar{B}}{|\bar{A}||\bar{B}|}\right)$$

Mechanics

$$\overline{A}\cdot\overline{B} = (A_x\overline{i} + A_y\overline{j} + A_z\overline{k}) \cdot (B_x\overline{i} + B_y\overline{j} + B_z\overline{k})$$

$$= A_xB_x + A_y B_y + A_z B_z$$

Recall,

$$\overline{i}\cdot\overline{i} = \overline{j}\cdot\overline{j} = \overline{k}\cdot\overline{k} = 1$$

$$\overline{i}\cdot\overline{j} = \overline{i}\cdot\overline{k} = \overline{j}\cdot\overline{i} = \overline{j}\cdot\overline{k} = \overline{k}\cdot\overline{i} = \overline{k}\cdot\overline{j} = 0$$

Basic Properties

1. $\left.\begin{array}{l} p(\overline{A}\cdot\overline{B}) = (p\overline{A})\cdot\overline{B} = \overline{A}\cdot(p\overline{B}) \\ \overline{A}\cdot(\overline{B}+\overline{C}) = \overline{A}\cdot\overline{B} + \overline{A}\cdot\overline{C} \end{array}\right\}$ similar to multiplication

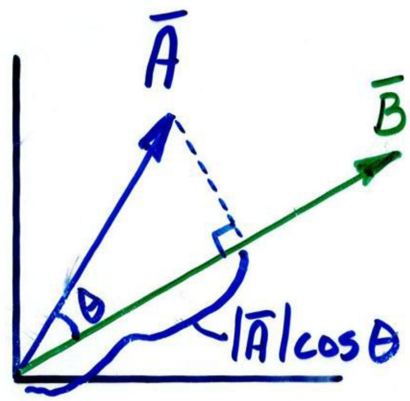
2. If $\overline{A}\cdot\overline{B} > 0$, then the projection of $\overline{B}$ is in the direction of $\overline{A}$ (i.e., $0 \le \theta \le 90°$)

If $\overline{A}\cdot\overline{B} < 0$, then the project of $\overline{B}$ is in the opposite direction of $\overline{A}$ (i.e., $90° \le \theta \le 180°$)

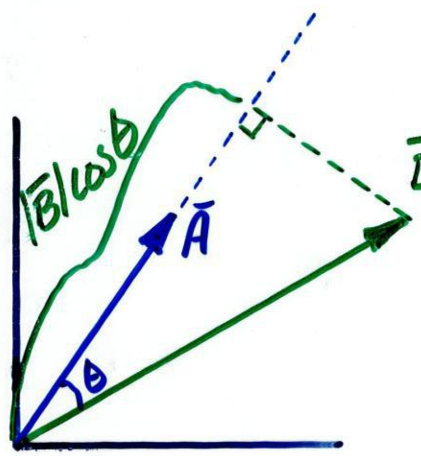
3. If $\overline{A}\cdot\overline{B} = 0 \Rightarrow \theta = 90^\circ$ and $\overline{A}$ is perpendicular to $\overline{B}$

   If $\overline{A}\cdot\overline{B} = |\overline{A}||\overline{B}| \Rightarrow \theta = 0^\circ$ and $\overline{A}$ is parallel to $\overline{B}$

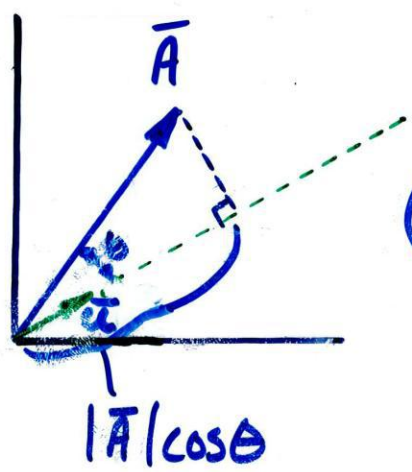
4. $\overline{A}\cdot\overline{A} = |\overline{A}|^2$



$$\bar{A}\cdot\bar{B} = \left[|\bar{A}|\cos\theta\right]|\bar{B}|$$

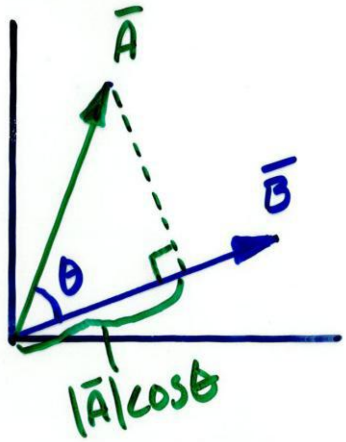


$$\bar{B}\cdot\bar{A} = \left[|\bar{B}|\cos\theta\right]|\bar{A}|$$



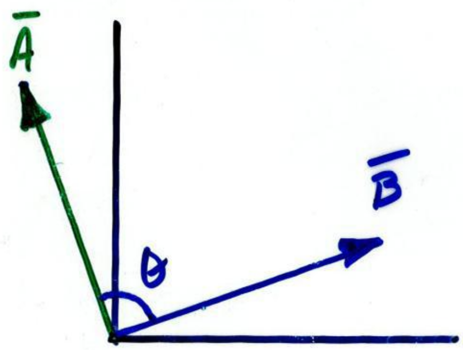
$$\bar{A}\cdot\bar{u} = \left[|\bar{A}|\cos\theta\right]|\bar{u}| = A_u$$

$$(\bar{A}\cdot\bar{u})\bar{u} = \bar{A}_u = A_u\,\bar{u}$$



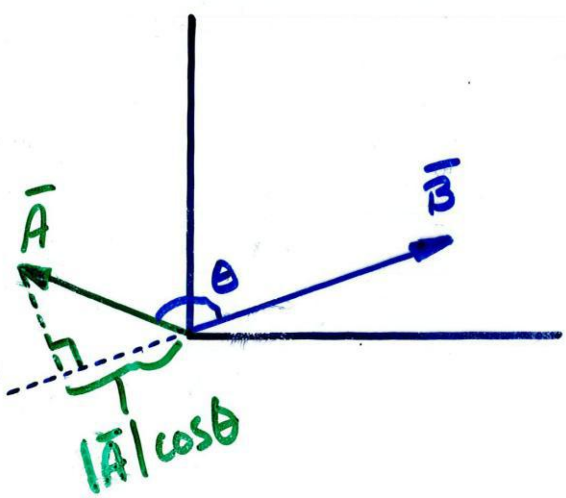
$0^\circ \leq \theta < 90^\circ$

$\bar{A} \cdot \bar{B} = (+)$



$\theta = 90^\circ$

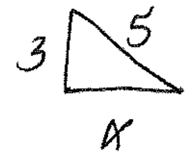
$\bar{A} \cdot \bar{B} = 0$



$90^\circ < \theta \leq 180^\circ$

$\bar{A} \cdot \bar{B} = (-)$

Basic Trigonometric Identities



cos 0° = 1   sin 0° = 0

cos 30° = $\frac{\sqrt{3}}{2} = 0.866$ sin 30° = $\frac{1}{2} = 0.5$

cos 36.87° = 0.8   sin 36.87° = 0.6

cos 45° = $\frac{\sqrt{2}}{2} = 0.707$ sin 45° = $\frac{\sqrt{2}}{2} = 0.707$

cos 53.13° = 0.6   sin 53.13° = 0.8

cos 60° = $\frac{1}{2} = 0.5$ sin 60° = $\frac{\sqrt{3}}{2} = 0.866$

cos 90° = 0   sin 90° = 1

Dot Product

Example 1

Given: The tension in cable AC (T_{AC}) is 1,000 lbs.

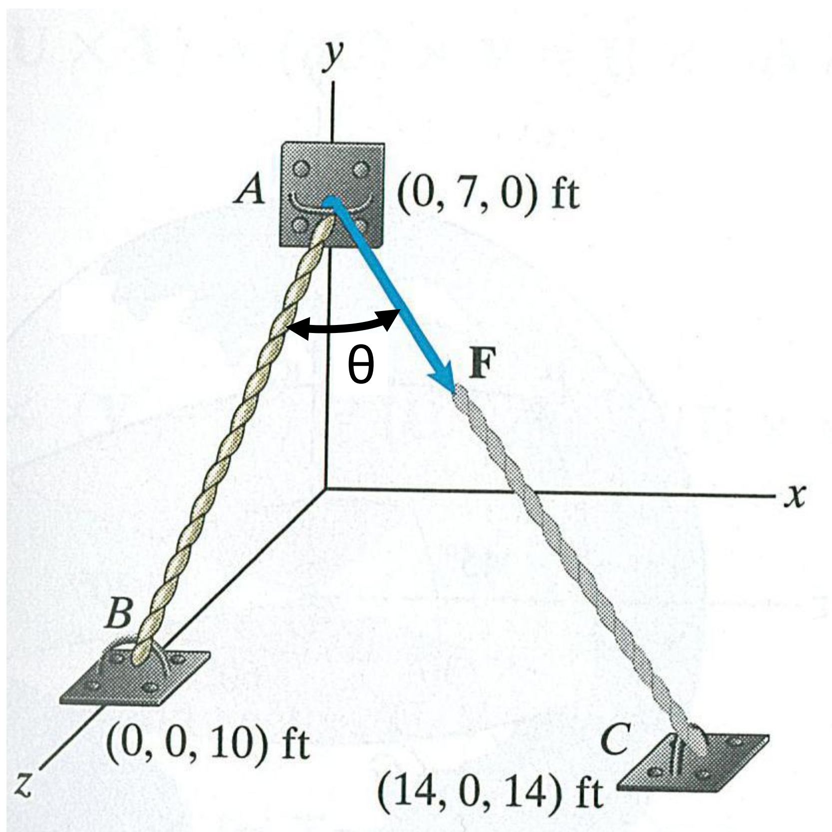
Find: a) Estimate the angle θ between cables AB and AC.

b) Calculate the angle θ between cables AB and AC.

c) Estimate the magnitude of the projection of force $\overline{F}_{AC}$ in the direction of cable AB.

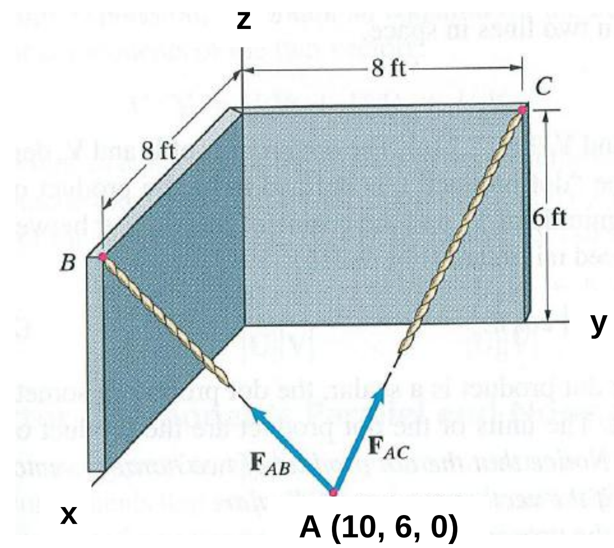
d) Determine the unit vector $\overline{U}_{AB}$ and force vector $\overline{F}_{AC}$.

e) Calculate the magnitude and vector projection of force $\overline{F}_{AC}$ in the direction of cable AB.



Dot Product

Example 2



Given: Cables AB and AC exert forces on point A of 200 lbs and 100 lbs, respectively, in the directions shown.

Find: a) <u>Estimate</u> the angle $\angle BAC$.

b) Using the dot product, compute the angle $\angle BAC$.
Does it matter which AB and BC vectors I use (position, unit force vectors)?

c) <u>Estimate</u> the magnitude of the projection of F_{AB} in the direction of AC.

d) <u>Calculate</u> the magnitude of the vector projection of F_{AB} in the direction of AC and then express the projection in vector form.

e) Is it possible to have a projection magnitude greater than the magnitude of the original vector?

Dot Product
Group Quiz

Group #: ____________ Group Members: 1) ______________________________
(Present Only)

Date: ______________ Period: _________ 2) ______________________________

3) ______________________________

4) ______________________________

Given: A mast is restrained by three cables as shown. The tension in cable CE is 2kN.

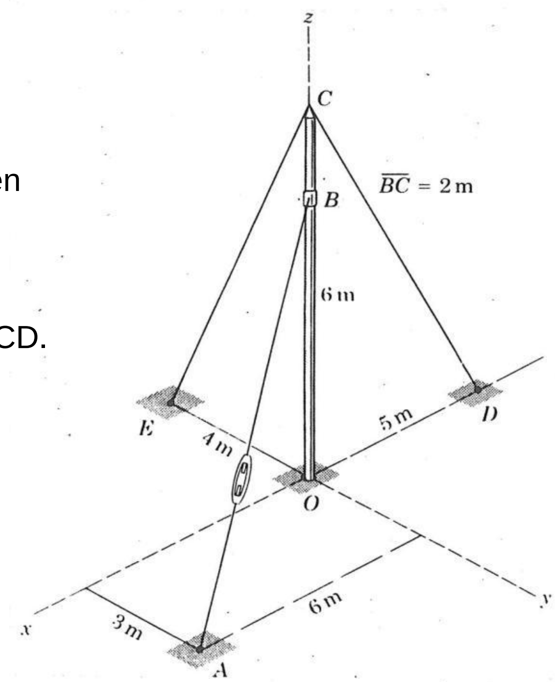
Find:

a) Estimate the angle between cables CE and CD.

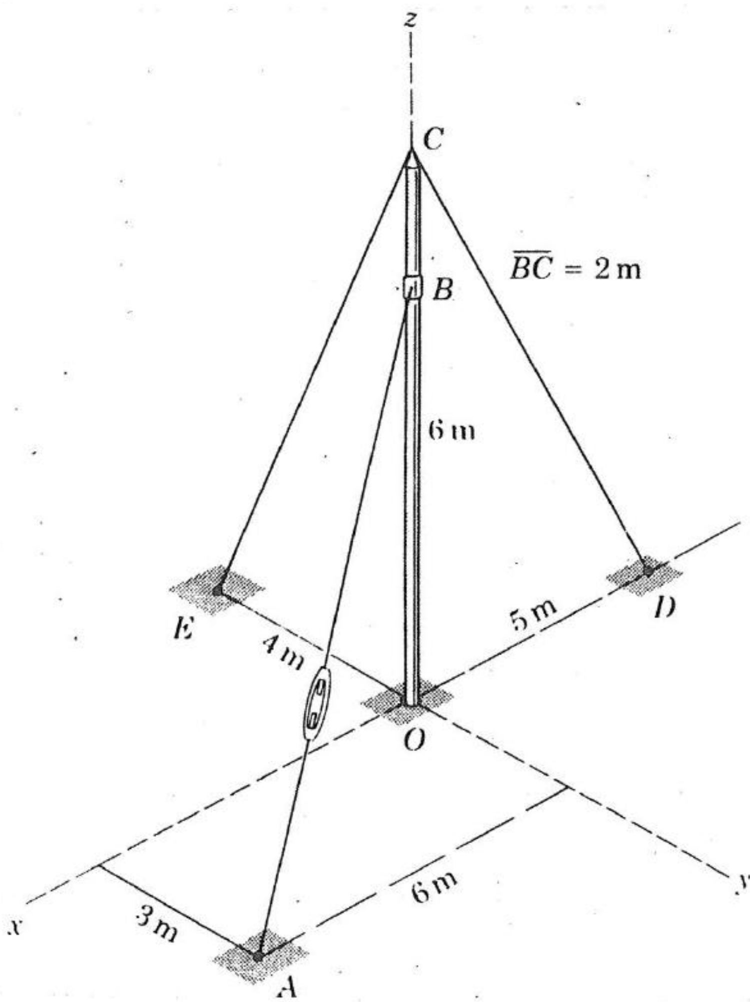
b) Use the dot product to determine the angle between cables CE and CD.

c) Write force vector $\overline{F}_{CE}$.

d) Determine the projection of $\overline{F}_{CE}$ in the direction of CD. Express the projection as a vector.



Solution:


z
C
$\overline{BC}$ = 2 m
B
6 m
E
4 m
5 m
D
O
x
3 m
6 m
y
A