FRAMES AND MACHINES

Learning Objectives

1). To evaluate the *unknown reactions* at the *supports* and the *interaction forces* at the *connection points* of a rigid frame in equilibrium by solving the *equations of static equilibrium* of the *overall structure* and *each individual member*.
2). To do an *engineering estimate* of these quantities.

Definitions

Two-Force Member: a structural member that is loaded only at two pin joints along the member.

Multi-Force Member: a structural member that is loaded at more than two points along the member.

Truss: a rigid framework of straight, lightweight *two-force members* that are joined together at their ends.

Frame: an assembly of rigid members (of which at least one is a **multi – force** *member*) intended to be a stationary structure for supporting a load.

Machine: an assembly of rigid members designed to do mechanical work by transmitting a given set of input loading forces into another set of output forces.

Newton's Third Law

Newton's Third Law: For each action there is an **action** and **opposite** reaction $\left(F_{A_{Body\,1}} = -F_{A_{Body\,2}}\right)$

Frames

In frames, we are often interested not only in the reaction forces at the supports but also in the interaction forces between members and the loads carried by any *two-force members*.

Procedure:

1). Inspect structure for *two-force members*.

2). Draw FBDs of the *entire structure* and of *each member*. Be sure the interaction forces between members are equal in magnitude, opposite in direction and collinear (i.e., satisfy Newton's Third Law).

3). Count the number of unknowns and equations available for each FBD. Successively write and solve the equilibrium equating corresponding to the FBDs of interest.

Note:

1). For a structure composed of "N" members, will be "N + 1" sets of equilibrium equations and FBDs. Only "N" sets of equations are independent.

2). If all external reactions on a frame can be determined, then the internal forces between members may be determined from either member.

3). If there are more unknowns than available equations ⇒ Statistically Indeterminate. This is not always true. Sometimes by disassembling the frame, the forces can be determined using the equilibrium equations.

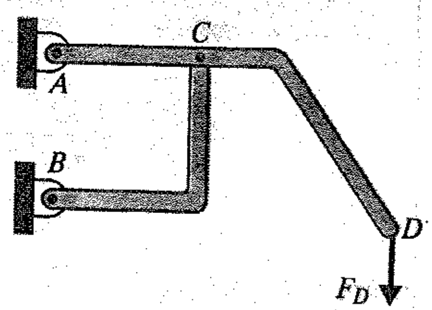


FIGURE 17

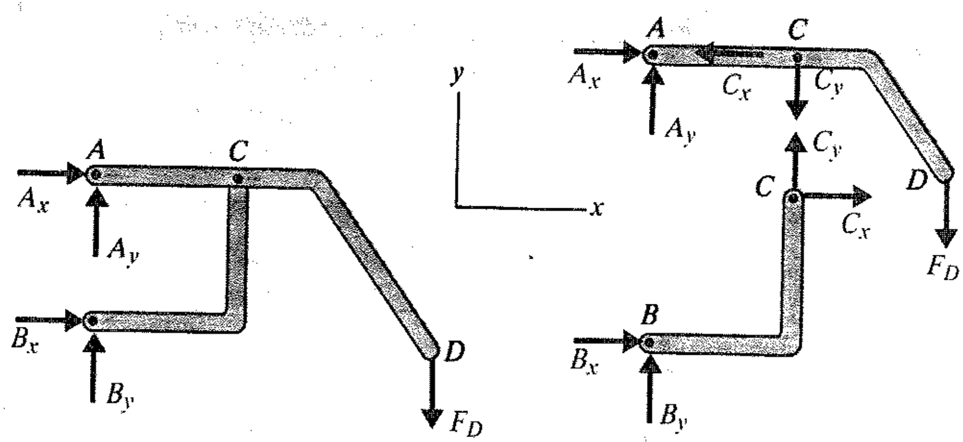


FIGURE 18

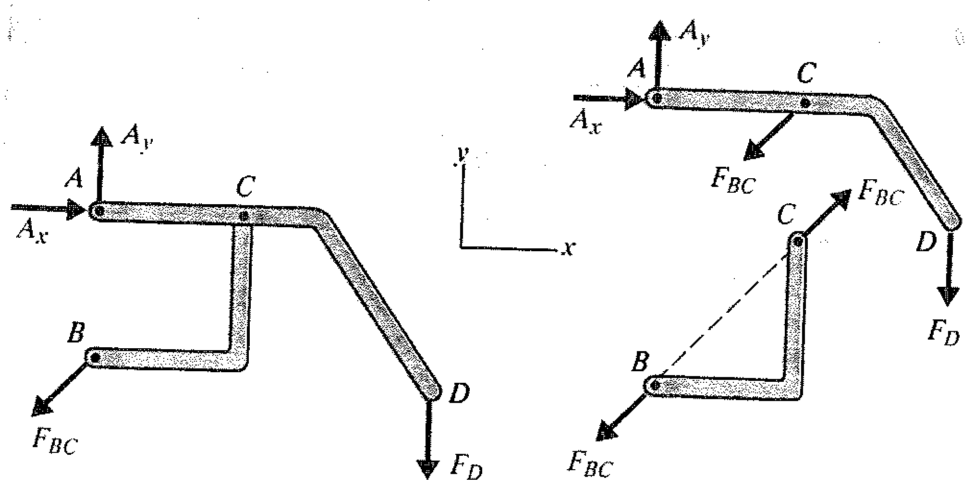


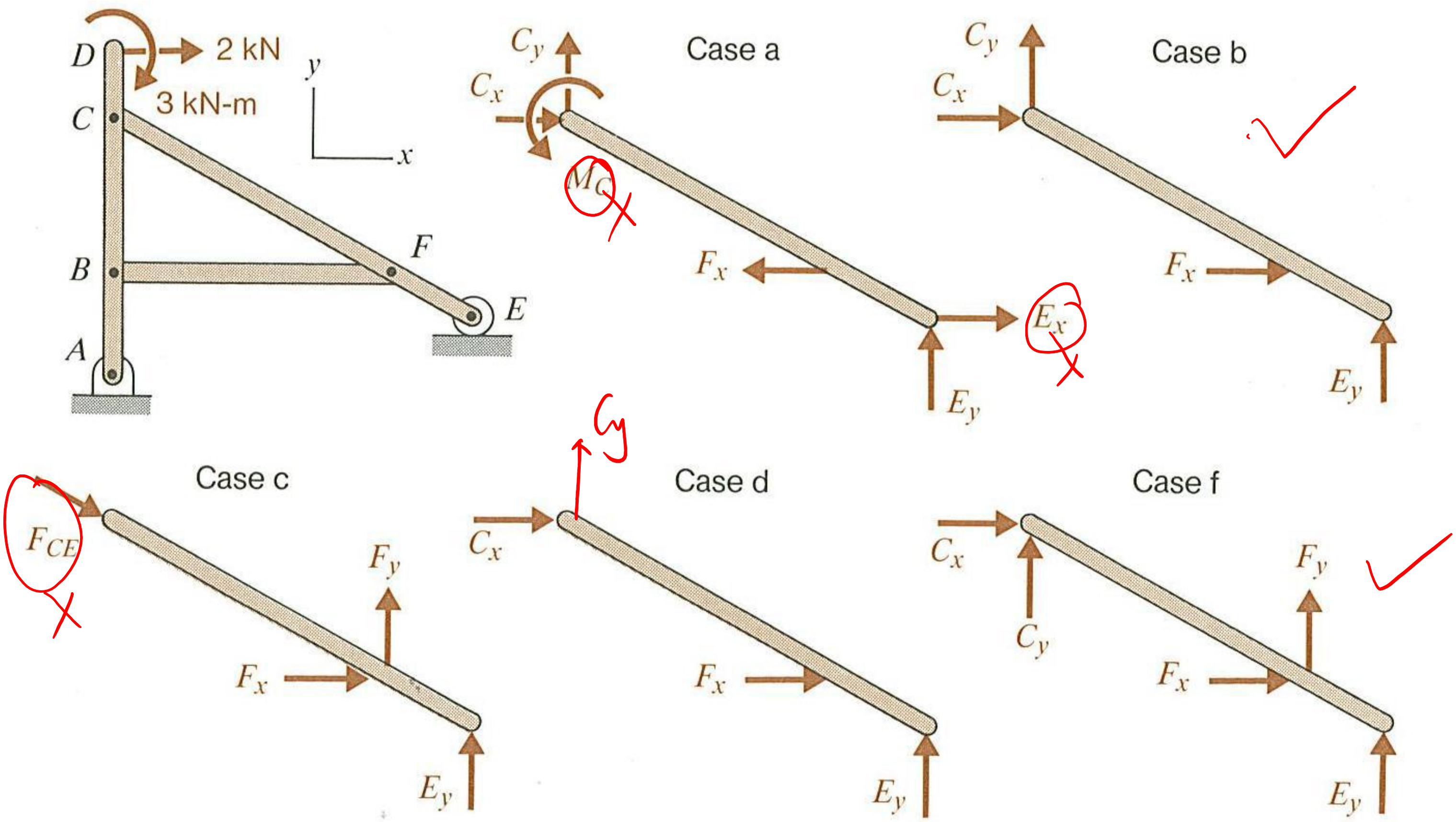
FIGURE 19

Frames and Machines

Example 3

Given: Frame ABCDEF is loaded as shown and is in static equilibrium.

Find: Five alternative free body diagrams for member CE are shown. Explain what (if anything) is erroneous in each diagram.



Frames and Machines

Example 4

Given: The frame shown is loaded with a 100kg package and is supported by pin supports at joints A and B. The frame is in static equilibrium.

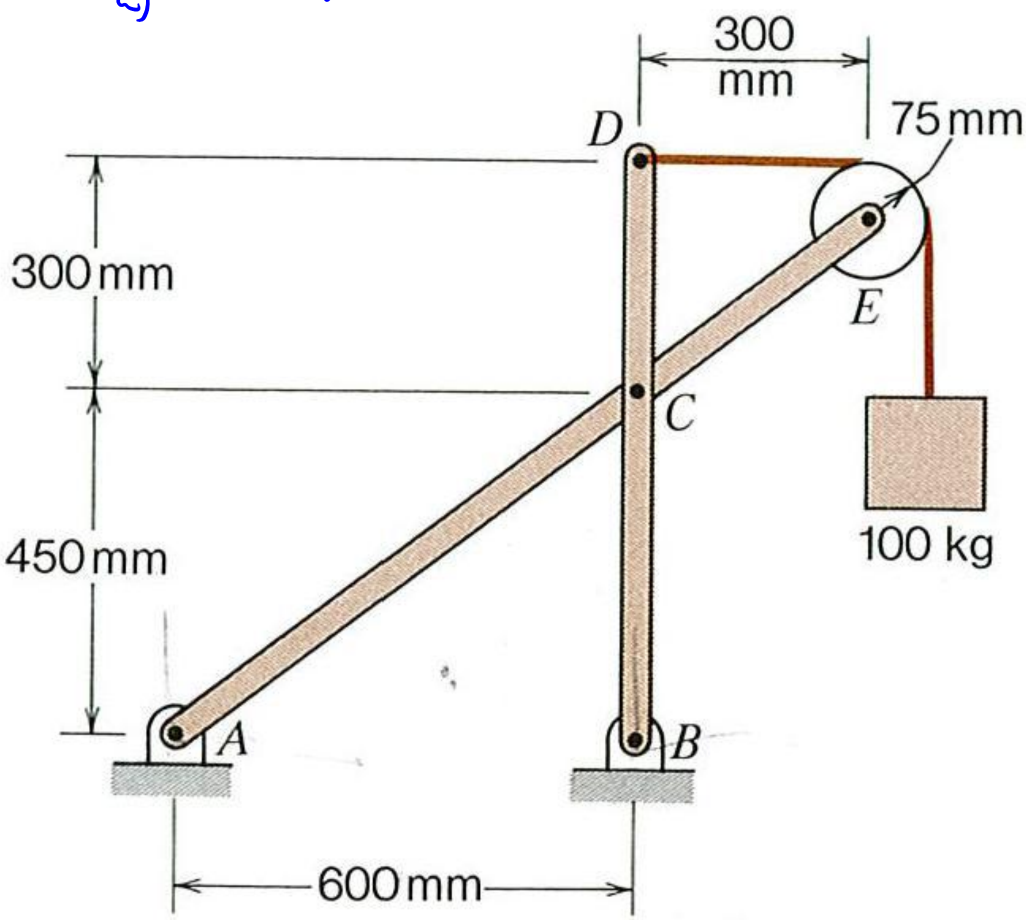
Find:

a) Identify any two-force members in the frame.

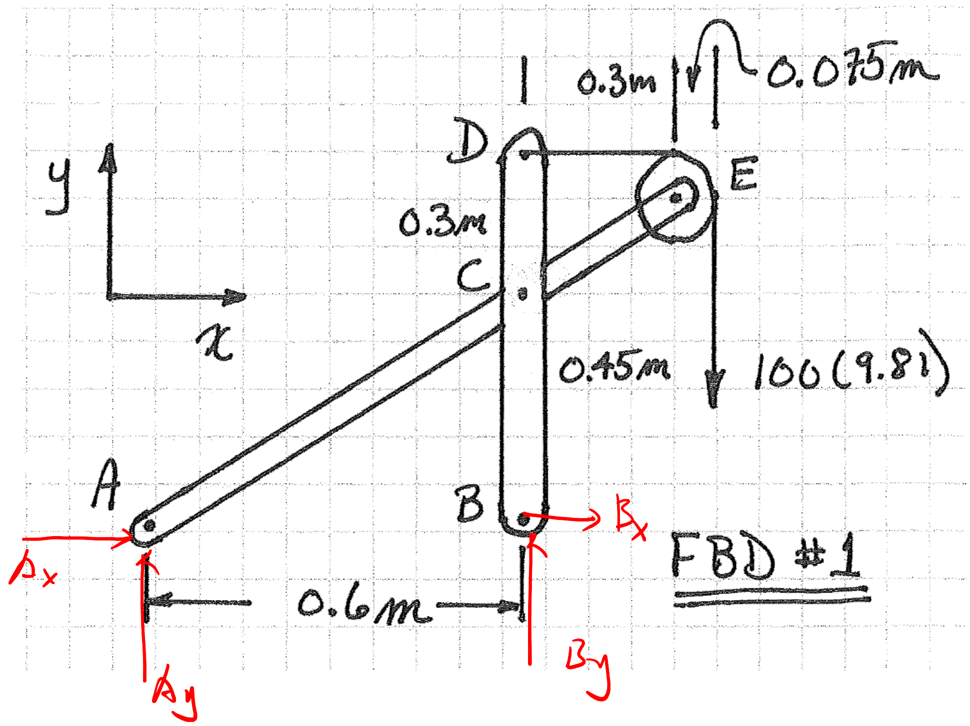
b) Draw the overall free body diagram and the individual free body diagrams of members ACE and BCD, and pulley E.

c) Determine the forces at pin C on member BCD.

a) No two-force members



b)



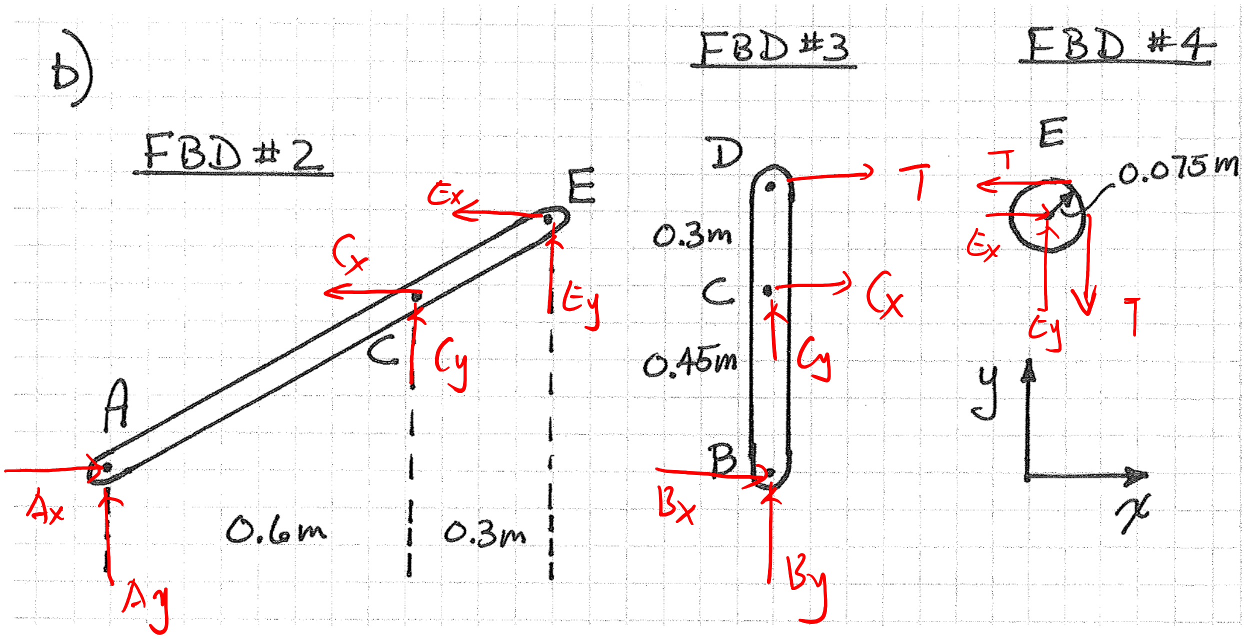
$\Sigma M_A = 0$: $\quad B_y(0.6) - 100(9.81)(0.975) = 0$

$\Rightarrow B_y = 1.59$ kN

$\Sigma F_x = 0$: $\quad A_x + B_x = 0$ $\quad \Rightarrow A_x = -B_x$

$\Sigma F_y = 0$: $\quad A_y + B_y - 100(9.81) = 0$

$\Rightarrow A_y = -0.613$ kN



c) From FBD #3 :

$$\Sigma M_B = 0 : \quad -C_x\,(0.045) - T\,(0.75) = 0$$

$$\Rightarrow C_x = -1.64 \text{ kN}$$

$$\Sigma F_y = 0 : \quad C_y + B_y = 0 \quad \Rightarrow \quad C_y = -1.59 \text{ kN}$$

$$(\bar{C})_{\text{on } BCD} = -1.64\,\bar{i} - 1.59\,\bar{j} \text{ kN}$$

$$(\bar{C})_{\text{on } ACE} = 1.64\,\bar{i} + 1.59\,\bar{j} \text{ kN}$$

Frames and Machines

Example 5

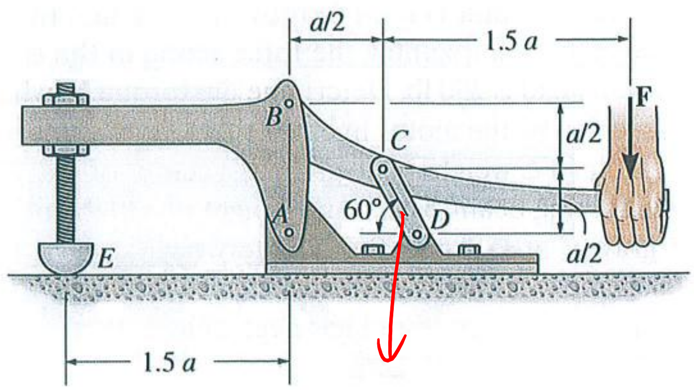
Given: A toggle clamp is subjected to a force "F" at the handle.

Find:

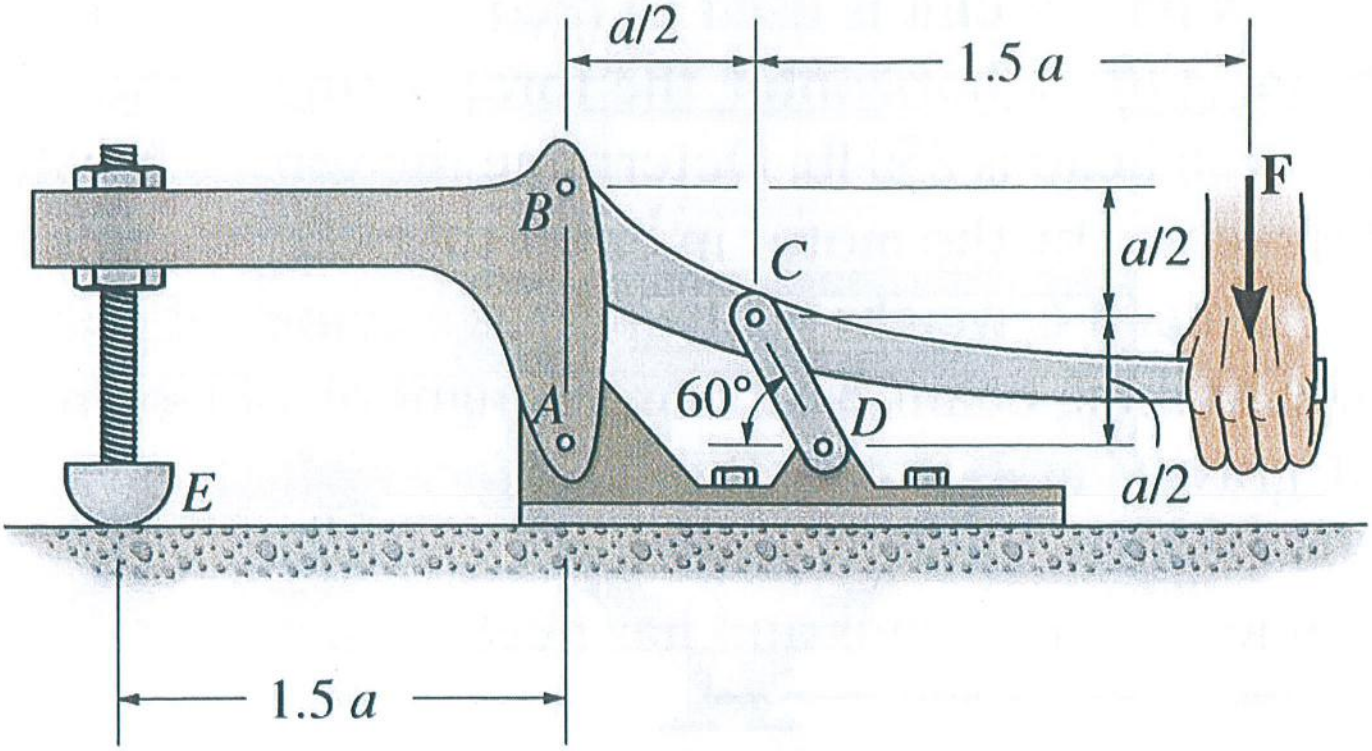
a) Determine the loads at joint C and pin B on member BC. Express these loads in vector form.

b) Determine the vertical clamping force acting at E as a function of the applied force "F".

c) If the applied force is doubled, what happens to the clamping force?



Two-force member



a/2
1.5 a
F
B
C
a/2
60°
A
D
E
a/2
1.5 a

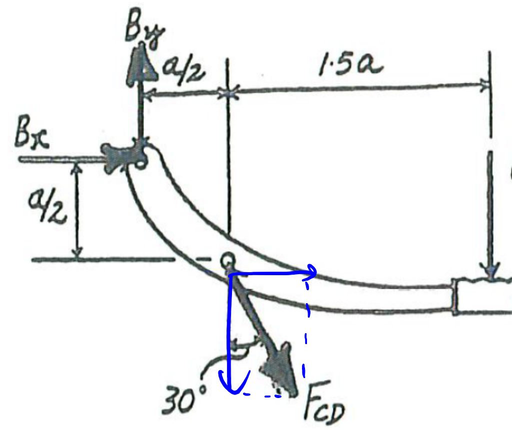
From FBD #1

$\Sigma M_B = 0$:

$- F_{CD} \cos 30° \left(\frac{a}{2}\right) + F_{CD} \sin 30° \left(\frac{a}{2}\right)$

$- F(2a) = 0$

$\Rightarrow F_{CD} = -10.93 F$, (compression)



FBD #1

$\Sigma F_y = 0$,

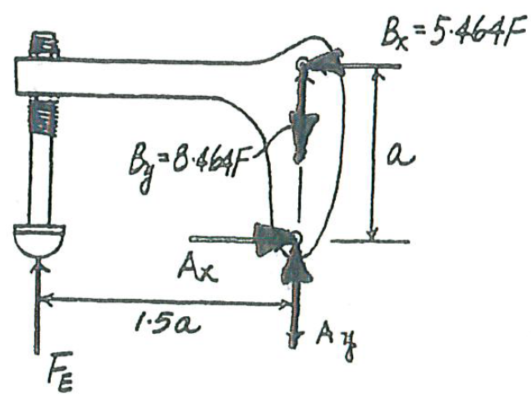
$-F_{CD} \cos 30° - F + B_y = 0$

$\Rightarrow B_y = -8.464 F$

$\Sigma F_x = 0$:

$B_x + F_{CD} \sin 30° = 0$

$\Rightarrow B_x = 5.464 F$



FBD #2

From FBD #2.

$\Sigma M_A = 0$: $B_x(a) - F_E (1.5a) = 0$

$\Rightarrow F_E = 3.64 F$

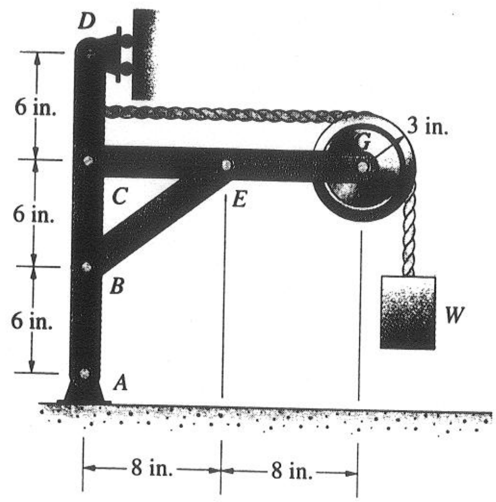
Frames and Machines
Group Quiz 2

Group #: ____________ Group Members: 1) ______________________________
(Present Only)
Date: ______________ Period: _________ 2) ______________________________

3) ______________________________

4) ______________________________

Given: A frame is composed of lightweight rigid members ABCD, BE, and CEG. The frame is supported by a pin reaction at A and a roller at D. The frame is loaded with a suspended weight of W = 40 lb.



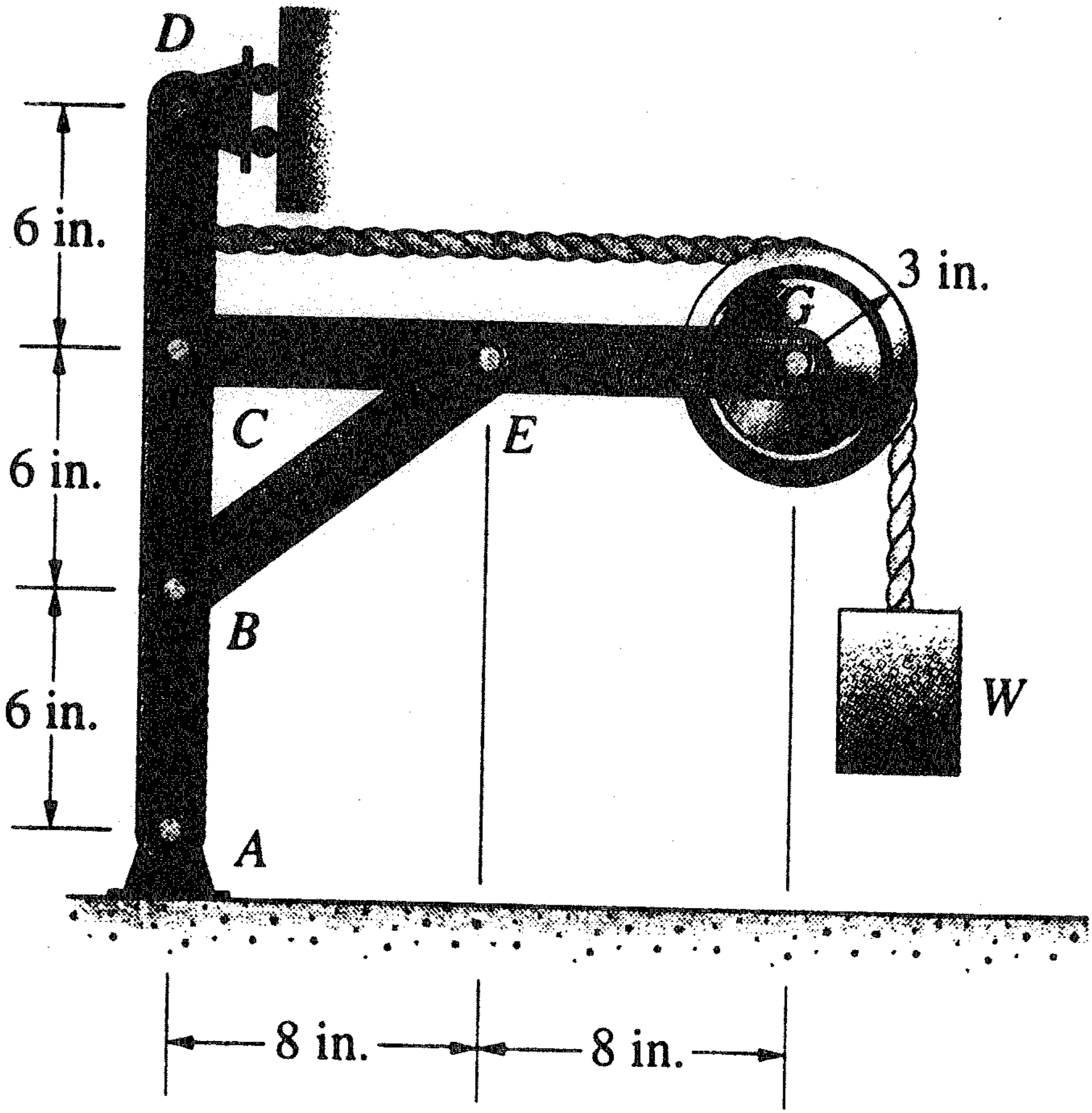
Find:

a) Draw a free body diagram of the overall frame and of individual members ABCD, CEG and pulley G.

b) Determine the reaction at the pin support at A and the roller at D.

c) Determine the forces acting on member CEG. Represent each of the forces in vector form.

Solution:

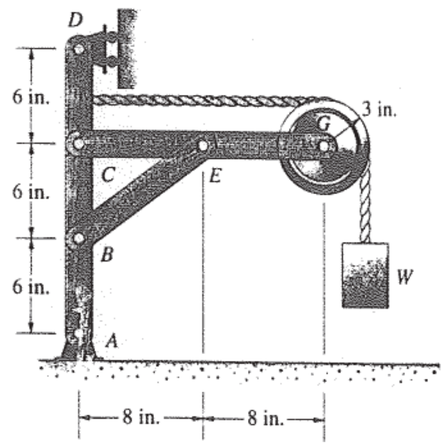

D
6 in.
3 in.
G
C
E
6 in.
B
W
6 in.
A
8 in.
8 in.

ME 270 - Basic Mechanics I - Group Quiz

Name/Group #: ____________ Group Members: 1) ____________ 2) ____________

Date: ____________ Period: ____________ 3) ____________ 4) ____________

Given: A frame is composed of lightweight rigid members ABCD, BE and CEG. The frame is supported by a pin reaction at A and a roller at D. The frame is loaded with a suspended weight of W = 40 lb.



Find:

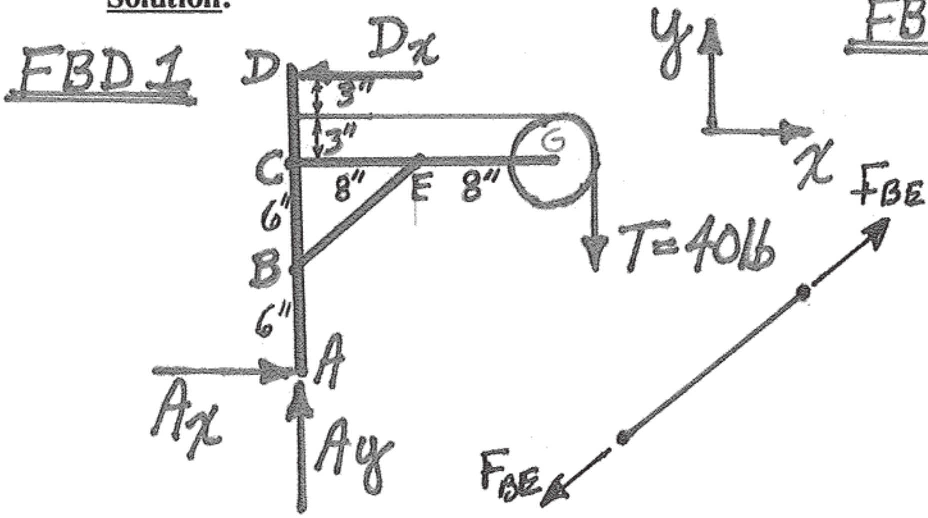
(a) Draw a free body diagram of the overall frame and of individual members ABCD, CEG and pulley G.

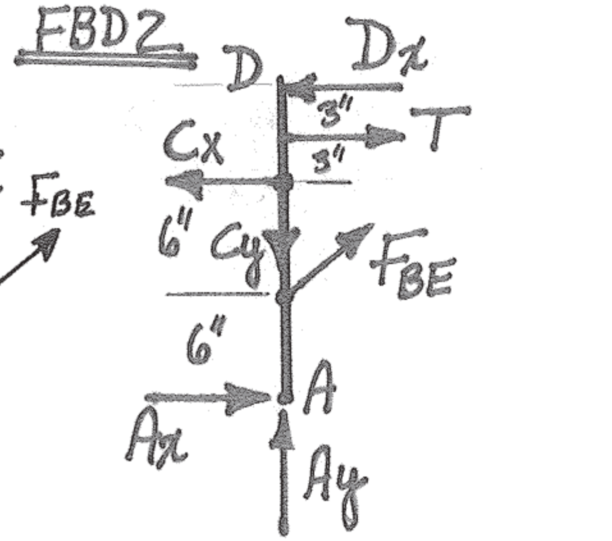
(b) Determine the reactions at the pin support at A and the roller at D.

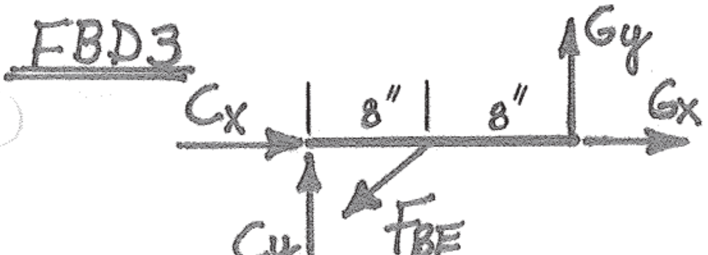
(c) Determine the forces acting on member CEG. Represent each of the forces in vector form.

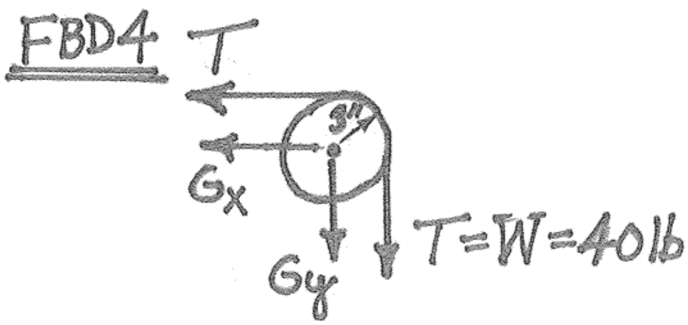
(a) NOTE MEMBER "BE" IS A TWO-FORCE MEMBER

Solution:









FROM FBD ①

(b) $\sum M_A = 0 = -40(19) + (D_x)(18) \Rightarrow \boxed{D_x = 42.2\,lb}$

$\sum F_x = 0 = -D_x + A_x \Rightarrow \boxed{A_x = D_x = 42.2\,lb}$

$\sum F_y = 0 = A_y - 40 \Rightarrow \boxed{A_y = 40\,lb}$

(c) FROM FBD ④

$\sum F_x = -G_x - 40 \Rightarrow G_x = -40\,lb$

$\sum F_y = -G_y - 40 \Rightarrow G_y = -40\,lb$

$\therefore$ On MEMBER CEG $\boxed{\vec{G} = -40\vec{i} - 40\vec{j}\ lb}$

FROM FBD ③

$\sum M_C = -\frac{6}{10} F_{BE}(8) + G_y(16) \Rightarrow \boxed{F_{BE} = -133\,lb = 133\,lb\ (C)}$

$\sum F_x = 0 = C_x - \frac{8}{10}(F_{BE}) + G_x$ $\quad \boxed{\therefore \vec{F}_{BE} = 106\vec{i} + 79.8\vec{j}\ lb}$ on member CEG

$\therefore C_x = +\frac{8}{10}(-133) + 40 = -66.4\,lb$

$\sum F_y = 0 = C_y - \frac{6}{10}(F_{BE}) + G_y$

$\therefore C_y = +\frac{6}{10}(-133) + 40 = -39.8\,lb$

$\boxed{\therefore \vec{C} = -66.4\vec{i} - 39.8\vec{j}\ lb}$ on member CEG