“CENTROID” AND “CENTER OF MASS” BY COMPOSITE PARTS

Learning Objectives

1). To evaluate the volume, mass, centroid and center of mass of a composite body.
2). To do an engineering estimate of the volume, mass, centroid and center mass of a composite body.

Definitions

Centroid: geometric center of a line, area or volume

Center of Mass: gravitational center of a line, area or volume.

The centroid and center of mass coincide when the density is uniform throughout the part.
Centroid by Composite Parts

a). Line

\[
L = \sum_{i=1}^{n} L_i \quad \text{and} \quad \bar{L} = \frac{\sum_{i=1}^{n} L_i \cdot (x_c)_i}{\sum_{i=1}^{n} L_i} \quad \bar{L} = \frac{\sum_{i=1}^{n} L_i \cdot (y_c)_i}{\sum_{i=1}^{n} L_i}
\]

b). Area

\[
A = \sum_{i=1}^{n} A_i \quad \text{and} \quad \bar{A} = \frac{\sum_{i=1}^{n} A_i \cdot (x_c)_i}{\sum_{i=1}^{n} A_i} \quad \bar{A} = \frac{\sum_{i=1}^{n} A_i \cdot (y_c)_i}{\sum_{i=1}^{n} A_i}
\]

c). Volume

\[
V = \sum_{i=1}^{n} V_i \quad \text{and} \quad \bar{V} = \frac{\sum_{i=1}^{n} V_i \cdot (x_c)_i}{\sum_{i=1}^{n} V_i} \quad \bar{V} = \frac{\sum_{i=1}^{n} V_i \cdot (y_c)_i}{\sum_{i=1}^{n} V_i} \quad \bar{V} = \frac{\sum_{i=1}^{n} V_i \cdot (z_c)_i}{\sum_{i=1}^{n} V_i}
\]

where,

\[
\bar{x}, \quad \bar{y}, \quad \bar{z} = \text{centroid of line, area, or volume}
\]

\[
(x_c)_i, \quad (y_c)_i, \quad (z_c)_i = \text{centroid of individual parts.}
\]
Center of Mass by Composite Parts

\[ m = \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} \rho_i V_i \]

\[ m \bar{x} = \sum_{i=1}^{n} (x_{ci})_i m_i = \sum_{i=1}^{n} (x_{ci})_i (\rho_i V_i) \]

\[ m \bar{y} = \sum_{i=1}^{n} (y_{ci})_i m_i = \sum_{i=1}^{n} (y_{ci})_i (\rho_i V_i) \]

\[ m \bar{z} = \sum_{i=1}^{n} (z_{ci})_i m_i = \sum_{i=1}^{n} (z_{ci})_i (\rho_i V_i) \]

where

\[ \bar{x}, \bar{y}, \bar{z} \] = center of mass of the composite body.

\[ (x_{ci})_i, (y_{ci})_i, (z_{ci})_i \] = center of mass of individual parts.

Note:
- Tabulated values of centroid and center of mass of several standard shapes can be found on the back inside cover of the textbook.
Given: A trapezoided block has a 75 mm radius hole cut through the block as shown.

Find:

a) Estimate the area and the \( x \) and \( y \) centroid of the block.

b) Calculate the \( x \) and \( y \) centroid of the block using the method of composite parts.

Estimate:

\[
\bar{x} \approx \quad \quad \quad \bar{y} \approx
\]

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area (m(^2))</th>
<th>( \bar{x} ) (m)</th>
<th>( \bar{y} ) (m)</th>
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</thead>
<tbody>
<tr>
<td>Rectangle</td>
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<td>Triangle</td>
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<td>Circle</td>
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Centroids and Center of Mass By Composite Parts

Example 2

**Given:** The trapezoid plate shown has two cutouts, a rectangular cutoff and a semi-circular cutout.

Note – the $y$-centroid of a semicircle is $\frac{4r}{3\pi}$.

**Find:** Using the method of composite parts,

a) Estimate and then calculate the shaded area.

b) Estimate and then calculate the $x$-centroid, and

c) Estimate and then calculate the $y$-centroid.
Centroids and Center of Mass By Composite Parts

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<td>2) Triangle</td>
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<td>3) Semicircle</td>
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<td>4) Sml. Rectangle</td>
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</tbody>
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Centers of Mass of Centroids: By Composite Parts
Group Quiz 1

Given: A toy block is shaped as shown.

Find:
   a) Prepare a table of the areas and centroids of each of the basic shapes which make up the composite block shown. Be sure to use a common origin. Estimate the location of the centroid.
   b) Determine the centroid \((\bar{x}, \bar{y})\) of the composite block.
   c) Can the block stand upright as shown without tipping over?
   d) If the quarter circle cut out were in the upper right hand corner of the rectangular section instead of the lower right corner, estimate the effect this would have on the centroid \((\bar{x}, \bar{y})\).

Solution:
\[ I_x = \frac{1}{16} \pi r^4 \]

\[ I_y = \frac{1}{16} \pi r^4 \]

Quarter circle area

\[ A = \frac{1}{4} \pi r^2 \]

\[ \frac{4r}{3\pi} \]
Fig. P5-56