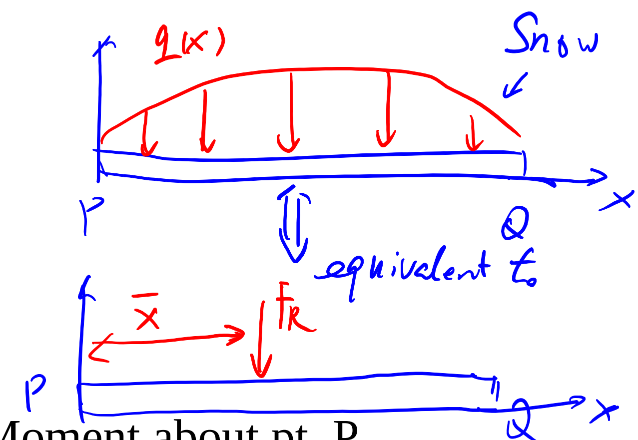
DISTRIBUTED LOADS

Learning Objectives

1). To determine the *resultant* of a given *line load* and to evaluate the *support reactions* acting on the body that carries such a load.

2). To do an *engineering estimate* of the equivalent load and its location.



Distributed Parallel "Line" Loads

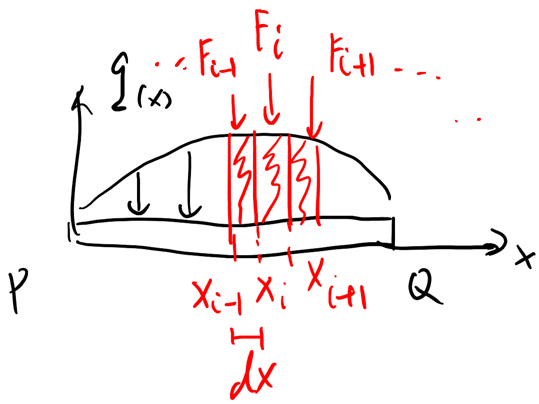
$$R = \int_o^L w(x)\,dx \quad = \text{ Resultant Force}$$

$$M_{rP} = \int_o^L x_c\, w(x)\,dx \quad = \text{Resultant Moment about pt. P}$$

$$\bar{x} = \frac{\int_o^L x_c\, w(x)\,dx}{\int_o^L w(x)\,dx} = \frac{M_{rP}}{R} \quad = \text{Line of action for equivalent force}$$

Note:

- F_R = area under the curve, q(x)
- $\bar{X}$ = centroid of the area under the curve, q(x)
- Determine direction of F_R and M_{R_O} by inspection
- Use your intuition to check your answer.

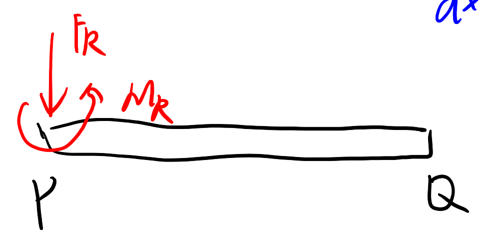


$q(x)$ — force per unit length (Force density). N/m.

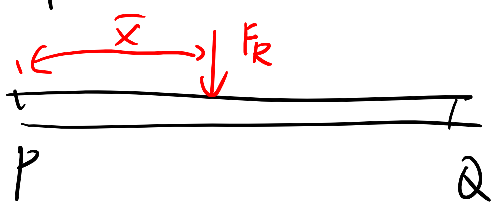
Replace $q(x)$ by a force and a moment at P.

$$\begin{cases} F_R = \sum F_i = \lim_{dx \to 0} \sum_i \left(q(x_i)\, dx \right) = \int_0^L q(x)\, dx \\ M_R = \sum -(F_i x_i) = \lim_{dx \to 0} \sum_i \left(-x_i\, q(x_i)\, dx \right) = -\int_0^L x\, q(x)\, dx \end{cases}$$

$\Longleftrightarrow$



$\Longleftrightarrow$



$$-F_R \bar{x} = M_R$$

$$\Rightarrow \bar{x} = \frac{\int_0^L x\, q(x)\, dx}{\int_0^L q(x)\, dx}$$

Manager of Syracuse's Carrier Dome worries about its roof with every snowstorm that's predicted

Published: Tuesday, December 14, 2010, 6:00 AM

By Mike Waters / The Post-Standard


View full sizeAnn Heisenfelt / APThe collapsed roof of the Metrodome is shown in this aerial view of Minneapolis on Sunday. The inflatable roof of the Metrodome collapsed Sunday after a snowstorm that dumped 17 inches on the city.

Syracuse, NY -- The impact of the Metrodome's roof collapse hit Pete Sala as hard as if 2 feet of snow had just been dumped in his own living room.

Sala, the managing director of Syracuse University's Carrier Dome, is close friends with Steve Maki, his counterpart with the Metrodome in Minneapolis. Sala was invited to the wedding of Maki's daughter. Sala sought Maki's counsel while preparing for the monster truck show at the Carrier Dome in March.

"It's our worst nightmare," Sala said of the Metrodome's collapse under 2 feet of snow early Sunday morning. "Those guys are going through hell right now."

Sala and Maki oversee the only working facilities in the United States with Teflon-coated Fiberglas, lattice-cabled roof systems. The two domes were built within two years of each other in 1980 and 1981.

"I told Steve, 'Anything you need, anything I can do, just ask,' " Sala said. Sala's offer included extra Fiberglas panels left over from the Carrier Dome's old roof, which was replaced in 1999, and key members of his staff to aid in the repair of the Metrodome's roof. On Monday, Sala and his staff loaded three old roof panels onto a Greater Syracuse Moving and Storage truck bound for Minneapolis.

The video of the Metrodome's collapse looks like a bag of sugar splitting open and the contents spilling everywhere.

"They had a major tear and the roof deflated," Sala said. "What you're seeing when you watch the video is as the roof is deflating and it goes into the inverted position, you get a huge avalanche of snow. The lowest point of the roof is going to take that impact. At that speed, something's got to give."

The Metrodome's roof sustained damage to one triangle- and two diamond-shaped panels. The triangle represents one of the largest panels in the roof.

The Carrier Dome's roof covers 6.5 acres compared to the 10-acre Metrodome roof. The Carrier Dome's 64 Teflon-coated fiberglass panels spread out over 287,000 square feet. The roof consists of 36 diamond panels in the center, 24 rectangles and four triangles.

Sala said what happened in Minnesota could just as easily happen in Syracuse. "I've been telling everybody at the university," said Sala, "you're as vulnerable as the next big snowstorm."

A major snowstorm walloped the Central New York region just last week, dumping roughly 4 feet of snow on Syracuse over the course of four days. "I didn't sleep from Sunday night to Thursday," Sala said. "You get so stressed out, it's just incredible. It's a nightmare."

Avoiding a tear in the roof and the resulting collapse starts before the snow starts to fly. The formula, said Sala, starts with pumping air heated to 140 degrees into the space between the Dome roof's two layers a day ahead of any storm.

Airport scrambles to remove ice after canopy collapse

Posted: Feb 07, 2011 12:43 PM EST Updated: Feb 07, 2011 4:37 PM EST

Crews are busy removing ice from the remaining panels.

This picture shows the damaged cars in the background.

Kris Kirschner/Eyewitness News

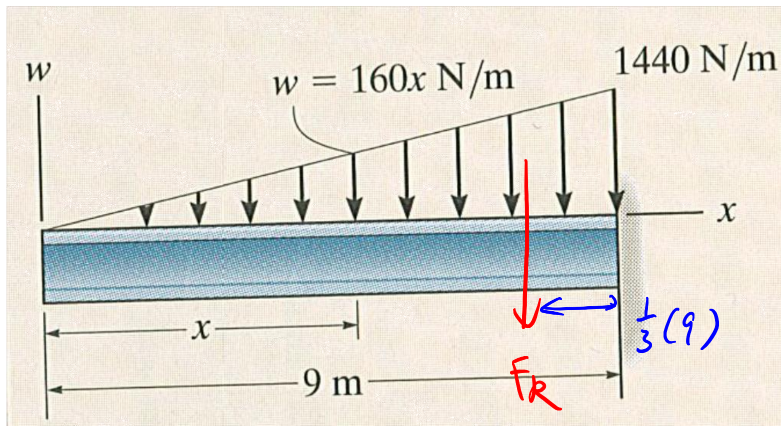
Distributed Loads
Example 1

Given: A distributed loading acting over the top surface of the beam is given by w(x) = 160x N/m.

Find:

a) <u>Estimate</u> the equivalent load and its location along the beam.

b) <u>Calculate</u> the magnitude and location (from the left side of the beam) of the equivalent resultant of this load.



Solution:

F_R – Aera under Curve

$F_R = \frac{1}{2}(9)(1440)$

$= 6480$ N

$\bar{x}$ – Centroid of aera

$\bar{x} = \frac{2}{3} \cdot 9 = 6$ m

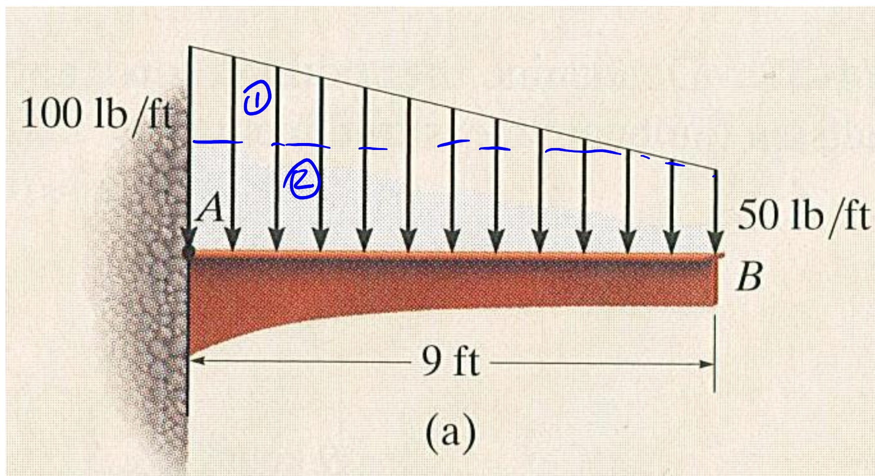
Distributed Loads
Example 2

Given: Granular material exerts the distributed loading shown on the beam.

Find:

a) <u>Estimate</u> the equivalent point load and its location along the beam.

b) <u>Calculate</u> the magnitude and location (from the left side of the beam) of the equivalent resultant of this load.

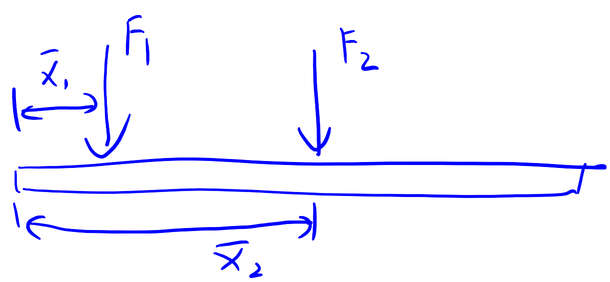


Solution:

$F_1 = \frac{1}{2}(100-50)(9)$

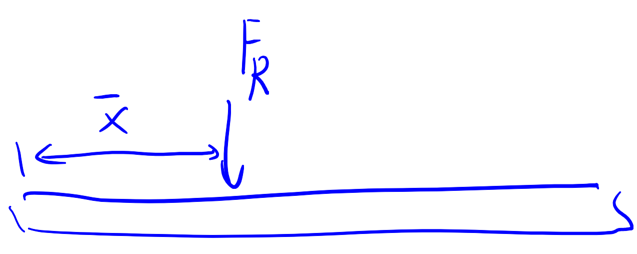
$= 225$ lb

$\bar{x}_1 = 3$ ft



$F_2 = 50 \times 9 = 450$ lb

$\bar{x}_2 = 4.5$ ft



$\Rightarrow \quad F_R = F_1 + F_2 = 675$ lb

$\bar{x} \cdot F_R = \bar{x}_1 F_1 + \bar{x}_2 \cdot F_2$

$\Rightarrow \quad \bar{x} = 4$ ft

Distributed Loads

Example 4

Given: Angled bar AB is loaded with a distributed load and a 4 kN point load as shown. The bar is held in static equilibrium by a pin joint at B and a roller support at A.

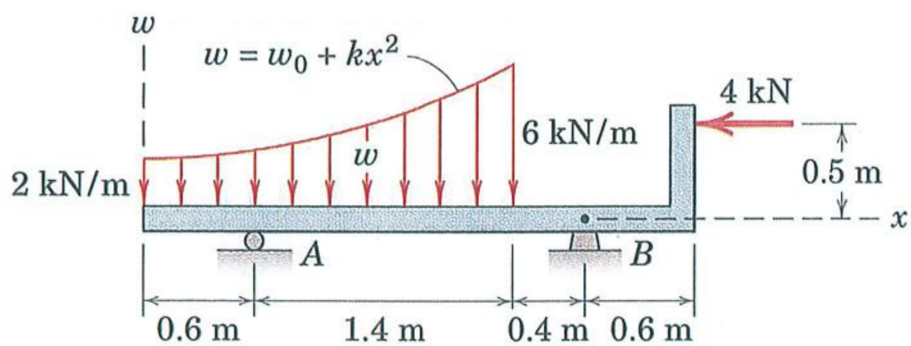
Find:

a) Estimate and then compute the force and location of an equivalent point load for the distributed load.

b) Draw a free body diagram of bar AB.

c) Estimate the reactions at supports A and B.

d) Determine the reactions at supports A and B.



Distributed Loads

Example 4

Given: Angled bar AB is loaded with a distributed load and a 4 kN point load as shown. The bar is held in static equilibrium by a pin joint at B and a roller support at A.

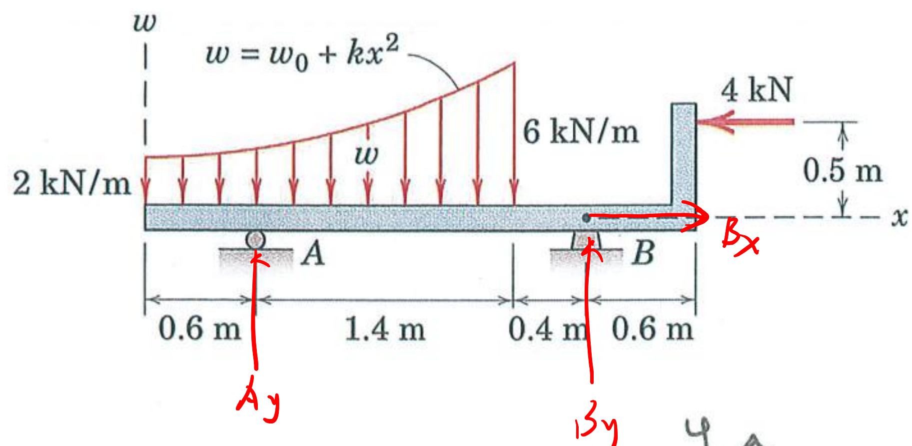
Find:

a) Estimate and then compute the force and location of an equivalent point load for the distributed load.

b) Draw a free body diagram of bar AB.

c) Estimate the reactions at supports A and B.

d) Determine the reactions at supports A and B.



$$\begin{cases} w(x) = 2 & \text{at } x = 0 \\ w(x) = 6 & \text{at } x = 2 \end{cases}$$

$$\Rightarrow w_0 = 2, \quad k = 1$$

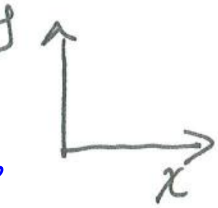
b) $F_{eq} = \int_0^2 w(x)\,dx$

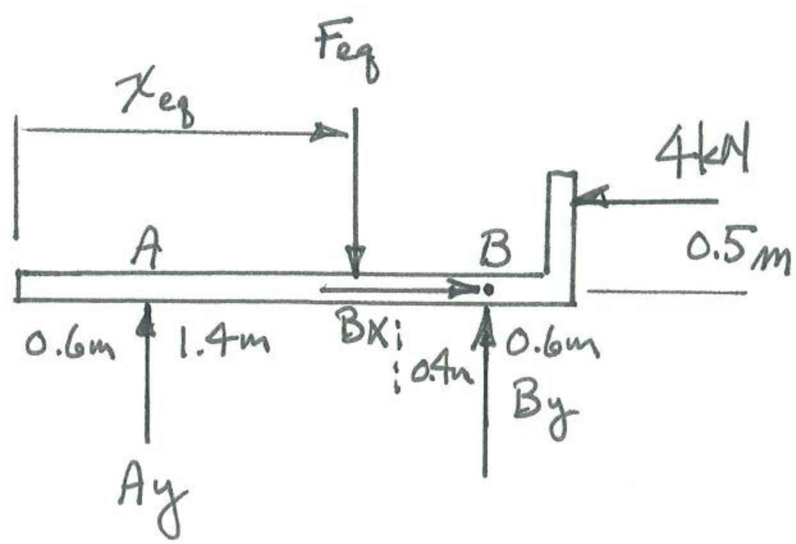
$= \int_0^2 (2 + x^2)\,dx$

$= \left(2x + \frac{1}{3}x^3\right)\Big|_0^2$

$= 6.67$ kN

Replace w(x) by F_{eq} at X_{eq}





$$X_{eq} \cdot F_{eq} = M_o = \int_0^2 x\, w(x)\, dx = \int_0^2 2x + x^3\, dx$$

$$= \left(x^2 + \frac{x^4}{4} \right) \Big|_0^2$$

$$\Rightarrow X_{eq} = 1.2\ m.$$

d). $\Sigma M_B = 0:\quad -A_y(1.8) - F_{eq}(2.4 - X_{eq}) + 4(0.5) = 0$

$$\Rightarrow A_y = 5.56\ kN$$

$\Sigma F_x = 0:\quad B_x - 4 = 0$

$$\Rightarrow B_x = 4\ kN$$

$\Sigma F_y = 0:\quad A_y + B_y - F_{eq} = 0$

$$\Rightarrow B_y = 1.11\ kN$$

Equivalent Systems, Distributed Loads
Group Quiz 1

Group #: ____________ Group Members: 1) ______________________________

(Present Only)

Date: ______________ Period: _________ 2) ______________________________

3) ______________________________

4) ______________________________

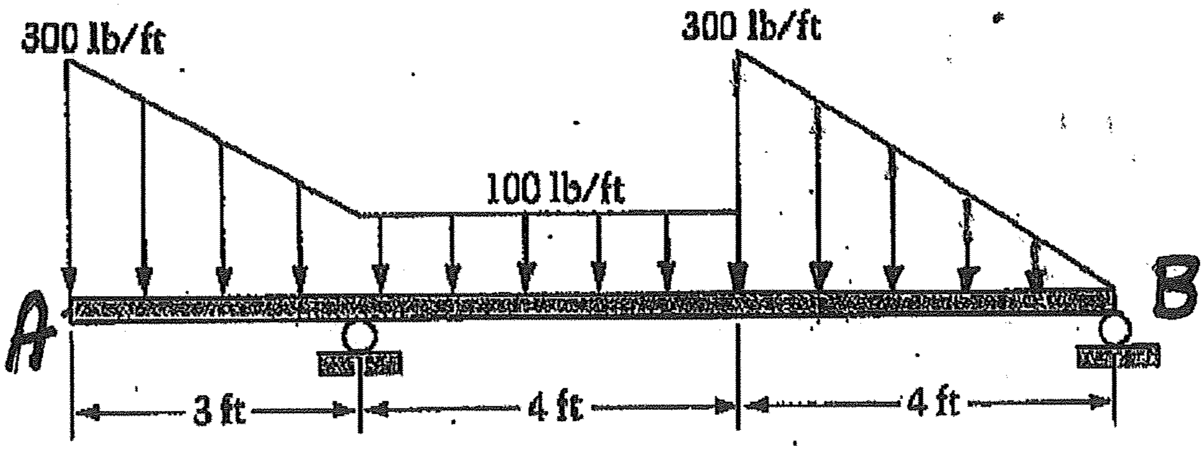
Given: The given beam is located as shown.

Find:

a) Replace the distributed loading on the beam with an equivalent set of point loads and identify the location and direction of each of the point loads.

b) Combine the set of point loads into a single equivalent point load and determine the point of application along the beam.

Solution:



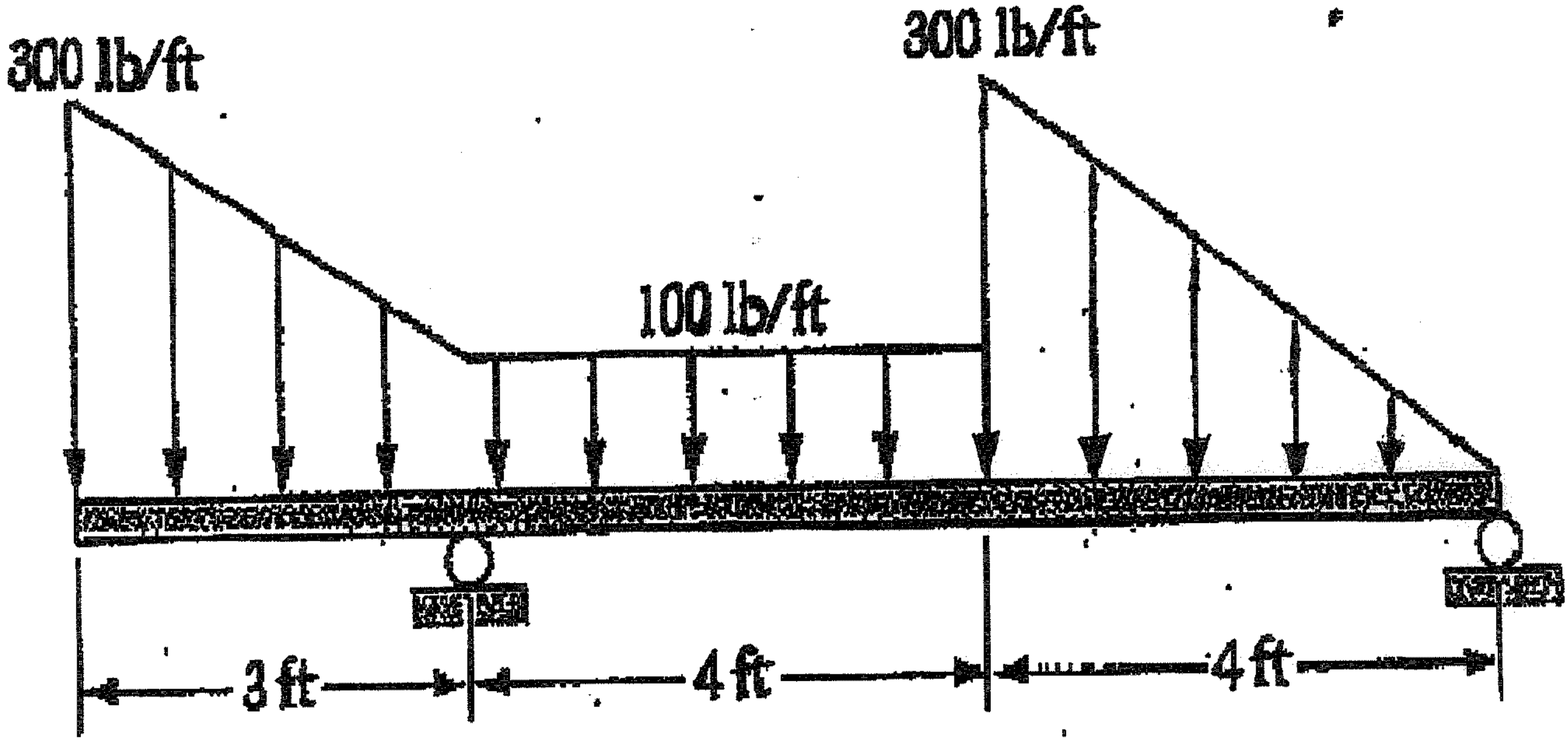


Figure P3.155

ME 270 – Basic Mechanics I – Group Quiz

Your Name: SOLUTION Group Members: 1) ____________

Date: ____________ Period: ____________ 2) ____________

3) ____________

4) ____________

Given: The given beam is loaded as shown. Note the triangular loading at the end of the beam is in the "upward" direction.

Find: a) Replace the distributed loading on the beam with an equivalent set of point loads and identify the location and direction of each of the point loads.

b) Combine the set of point loads into a single equivalent point load and determine the point of application along the beam.

Solution:

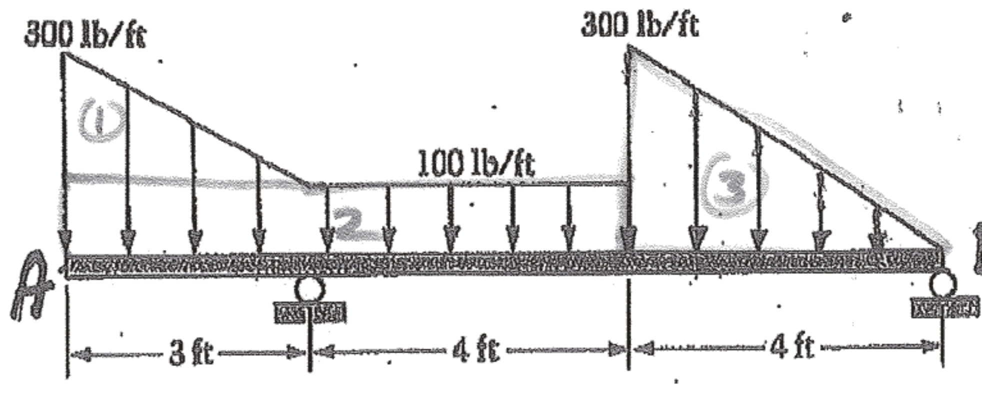


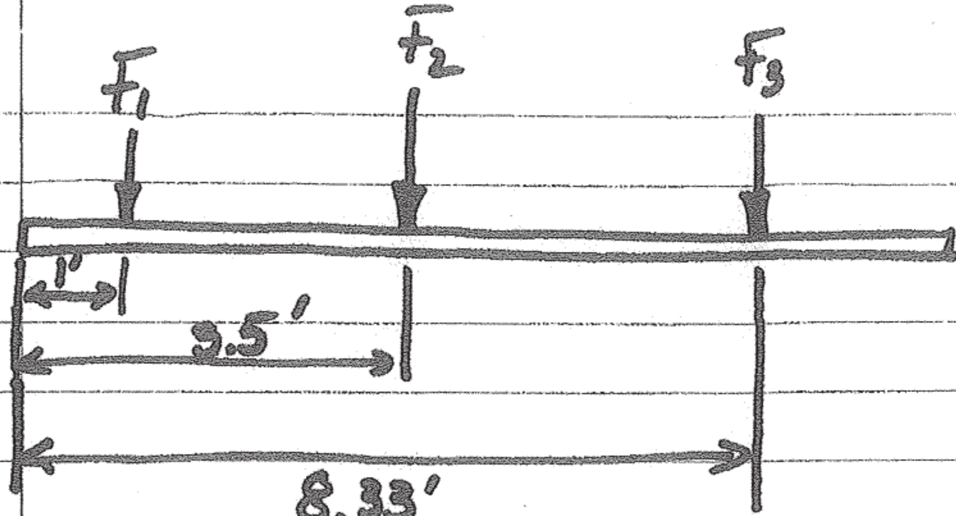
Figure P3.155

a) $F_1 = \frac{1}{2}(3)(200) = 300 \text{ lb} \quad @ \; x = 2 \text{ ft}$

$F_2 = (7)(100) = 700 \text{ lb} \quad @ \; x = 3.5 \text{ ft}$

$F_3 = \frac{1}{2}(4)(300) = 600 \text{ lb} \quad @ \; x = 7 + \frac{4}{3} \text{ ft}$

$x = 8.33 \text{ ft}$



$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

$$\bar{R} = -300\bar{j} - 700\bar{j} - 600\bar{j} = -1600\bar{j} \text{ lb.}$$

$$R(x) = F_1(x_1) + F_2(x_2) + F_3(x_3)$$

$$-1600x = -(300)(1) - (700)(3.5) - (600)(8.33)$$

$$\therefore x = 4.84 \text{ ft}$$