STATIC EQUILIBRIUM OF RIGID BODIES (3-D)

Learning Objectives

1). To evaluate the *unknown reactions* holding a rigid body in equilibrium by solving the *equations of static equilibrium*.

2). To recognize situations of *partial* and *improper constraint*, as well as *static indeterminacy*, on the basis of the solvability of the equations of static equilibrium.

Newton's First Law

Given no net force, a body at rest will remain at rest and a body moving at a constant velocity will continue to do so along a straight path $(\overline{F_R} = \sum \overline{F} = \overline{0},\ \overline{M_{R_O}} = \sum \overline{M_O} = \overline{0})$.

Definitions

Zero-Force Members: structural members that support no loading but aid in the stability of the truss.

Two-Force Members: structural members that are: a) subject to no applied or reaction moments, and b) are loaded only at two pin joints along the member.

Multi-Force Members: structural members that have a) applied or reaction moments, or b) are loaded at more than two points along the member.

Vector Equations

$$\overline{F_R} = \sum \overline{F} = \overline{0}$$

$$\overline{M_{R_O}} = \sum \overline{M_O} = \overline{0}$$ where O is any arbitrary point

Component Equations

For 3-D problems, there are SIX "component" equations

$$\sum F_X = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Static Determinacy/Partial and Improper Constraints

Static Indeterminacy: occurs when a system has *more* constraints than is necessary to hold the system in equilibrium (i.e., the system is *overconstrained* and thus has *redundant* reactions).

Static Determinancy: occurs when a system has a *sufficient* number of constraints to prevent motion without any redundancy.

Partial Constraint: occurs when there is an *insufficient* number of reaction forces to prevent motion of the system (i.e., the system is *partially constrained*).

Improper Constraint: occurs when a system has a *sufficient* number of reaction forces but one or more are *improperly applied* so as not to prevent motion of the system (i.e., the system is *improperly constrained*).

Problem Solving

1). Select body (or bodies) to be isolated in a FBD.

2). Choose an xyz coordinate system.

3). Complete FBD slowing all external reaction forces/moments.

4). $\sum \overline{M_O} = \overline{0}$, select point O to eliminate some unknown constraint forces and simplify the cross products.

5). $\sum \overline{F} = \overline{0}$

6). Solve simultaneous equations.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

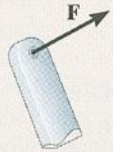
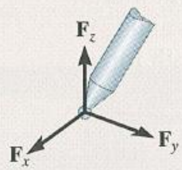
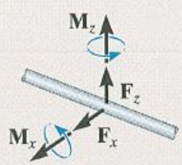
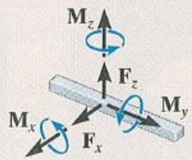
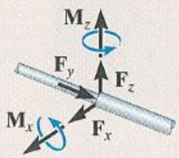
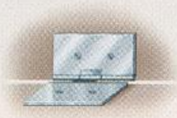
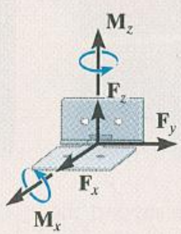
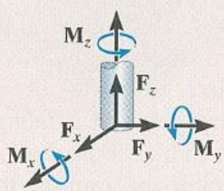
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are generally not applied if the body is supported elsewhere. See the examples.

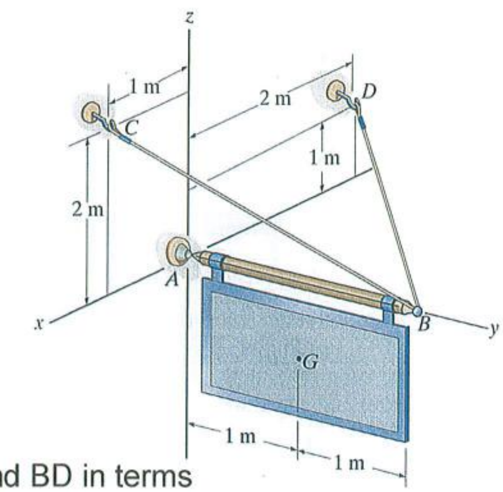
TABLE 5–2 Continued

	Types of Connection	Reaction	Number of Unknowns
(6)	 single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(7)	 single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(8)	 single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(9)	 single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments are generally not applied if the body is supported elsewhere. See the examples.
(10)	 fixed support		Six unknowns. The reactions are three force and three couple-moment components.

3-D Static Equilibrium

Example 1

Given: A 100 kg vertical sign with a center of mass at G hangs from mast AB. The mast is held in static equilibrium by a ball-and-socket support at A and cables BC and BD.



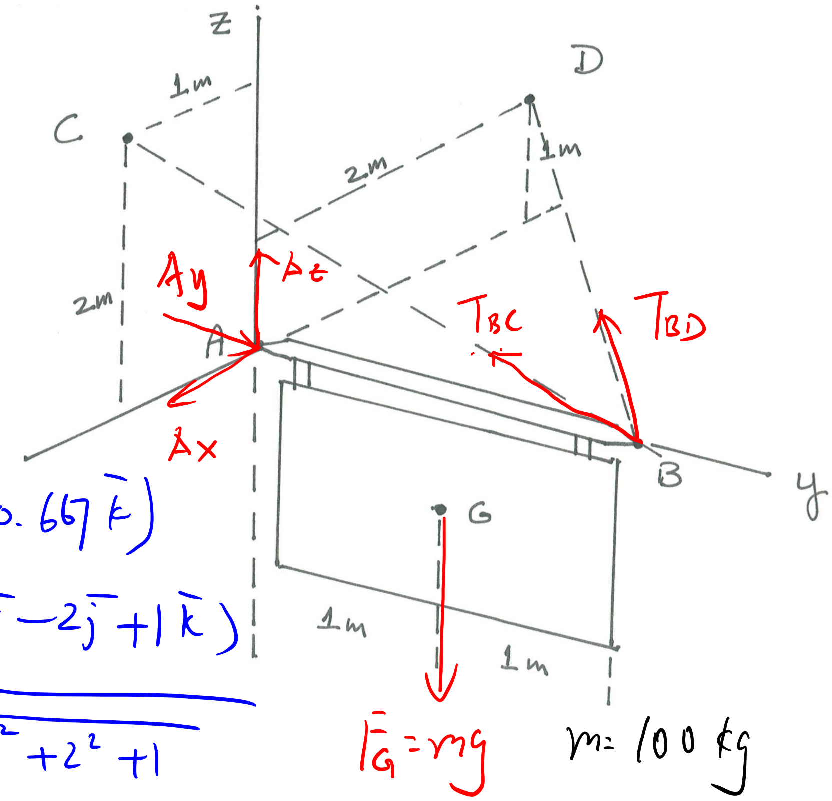
Find:

a) Sketch a free body diagram of the mast AB.

b) Write vector expression for the tensions in cables BC and BD in terms of their unknown magnitudes and their known unit vectors.

c) Determine the magnitudes of the tensions in cables BC and BD.

d) Determine the reactions at the ball-and-socket A.



b) $\bar{T}_{BC} = T_{BC}(\bar{u}_{BC})$

$$= T_{BC}\frac{1\bar{i} - 2\bar{j} + 2\bar{k}}{\sqrt{1 + 2^2 + 2^2}}$$

$$\bar{T}_{BC} = T_{BC}(0.333\bar{i} - 0.667\bar{j} + 0.667\bar{k})$$

$$\bar{T}_{BD} = T_{BD}(\bar{u}_{BD}) = T_{BD}\frac{(-2\bar{i} - 2\bar{j} + 1\bar{k})}{\sqrt{2^2 + 2^2 + 1}}$$

$$\bar{T}_{BD} = T_{BD}(-0.667\bar{i} - 0.667\bar{j} + 0.333\bar{k})$$

$$\bar{F}_G = F_G(\bar{u}_G) = -100(9.81)\bar{k}$$

c) $\Sigma \bar{M}_A = 0 : (\bar{r}_{AB} \times \bar{T}_{BC}) + (\bar{r}_{AB} \times \bar{T}_{BD}) + (\bar{r}_{AG} \times \bar{F}_G) = 0$

$$[1.333 T_{BC}\bar{i} - 0.667 T_{BC}\bar{k}] + [0.667 T_{BD}\bar{i} + 1.333 T_{BD}\bar{k}] - 981\bar{i} = 0$$

$$\Sigma (M_A)_x = 0 : \quad 1.333 T_{BC} + 0.667 T_{BD} - 981 = 0$$

$$\Sigma (M_A)_y = 0 : \quad -0.667 T_{BC} + 1.333 T_{BD} = 0$$

$$\Rightarrow \quad T_{BC} = 589 \text{ N}, \quad T_{BD} = 294 \text{ N}$$

d) $\sum F_x = 0$: $A_x + 0.333 T_{BC} - 0.667 T_{BD} = 0$

$\Rightarrow A_x = 0 \ N$

$\sum F_y = 0$: $A_y - 0.667 T_{BC} - 0.667 T_{BD} = 0$

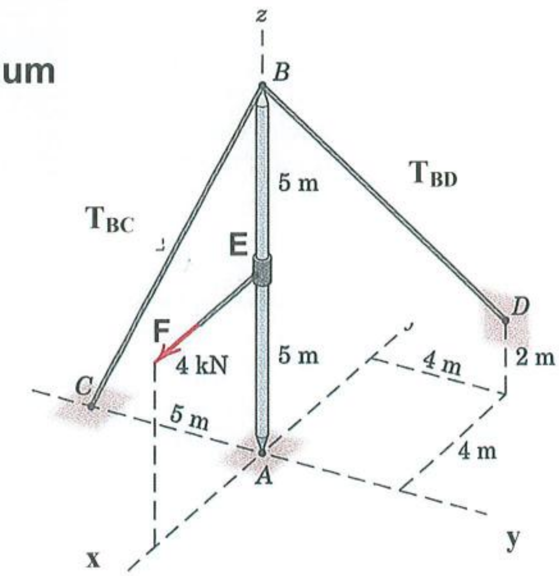
$\Rightarrow A_y = 589 \ N$

$\sum F_z = 0$: $A_z + 0.667 T_{BC} + 0.333 T_{BD} - 981 = 0$

$\Rightarrow A_z = 491 \ N$

3-D Static Equilibrium

Example 2



Given: Vertical mast AB supports a 4 kN load utilizing two fixed cables BC and BD at a ball-and-socket connection at base A.

Find:

a) Sketch the free body diagram of mast AB.

b) Express the force vectors $\overline{T_{BC}}$ and $\overline{T_{BD}}$ in terms of their unknown magnitudes and their known unit vectors.

c) Estimate the magnitude of the tension in cables $\overline{T_{BC}}$ and $\overline{T_{BD}}$.

d) Using your equations of static equilibrium, determine the magnitudes tension in cables $\overline{T_{BC}}$ and $\overline{T_{BD}}$.

e) Again using your equations of static equilibrium, determine the reaction forces at base A.

b) $\bar{T}_{BC} = T_{BC}(\bar{u}_{BC})$

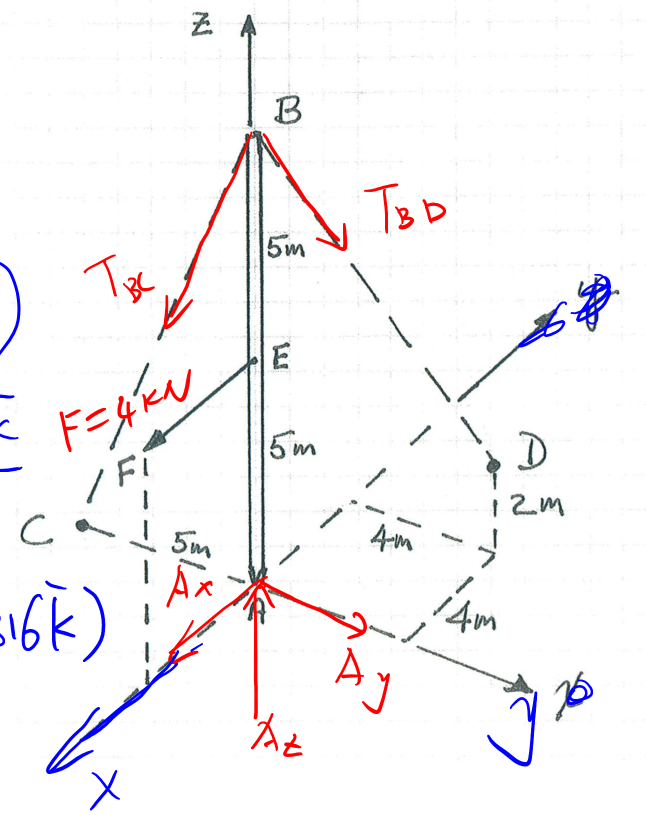
$$= T_{BC}\frac{(-5\bar{j}-10\bar{k})}{\sqrt{(-5)^2+(-10)^2}}$$

$$\Rightarrow \bar{T}_{BC} = T_{BC}(-0.447\bar{j}-0.894\bar{k})$$

$$\bar{T}_{BD} = T_{BD}(\bar{u}_{BD}) = T_{BD}\frac{-4\bar{i}+4\bar{j}-8\bar{k}}{\sqrt{4^2+4^2+8^2}}$$

$$\Rightarrow \bar{T}_{BD} = T_{BD}(-0.408\bar{i}+0.408\bar{j}-0.816\bar{k})$$

$$\bar{F}_{EF} = F_{EF}(\bar{u}_{EF}) = 4\bar{i}\ kN$$



c) $T_{BC} \approx T_{BD} \approx 4\,kN$

d) $\sum \bar{M}_A = 0: (\bar{r}_{AE}\times\bar{F}_{EF}) + (\bar{r}_{AB}\times\bar{T}_{BC}) + (\bar{r}_{AB}\times\bar{T}_{BD}) = 0$

$$[20\bar{j}] + [4.47\bar{i}]T_{BC} + [-4.08\bar{i}-4.08\bar{j}]T_{BD} = 0$$

$$\sum(\bar{M}_A)_x = 0: \quad 4.47\,T_{BC} - 4.08\,T_{BD} = 0$$

$$\sum(\bar{M}_A)_y = 0: \quad 20 - 4.08\,T_{BD} = 0$$

$$\Rightarrow \quad T_{BD} = 4.90\,kN \qquad T_{BC} = 4.47\,kN$$

e) $\sum F_x = 0$: $A_x + 4 - 0.408\, T_{BD} = 0$

$\Rightarrow A_x = -2.00$ kN

$\sum F_y = 0$: $A_y + 0.408\, T_{BD} - 0.447\, T_{BC} = 0$

$\Rightarrow A_y = 0$ kN

$\sum F_z = 0$: $A_z - 0.894\, T_{BC} - 0.816\, T_{BD} = 0$

$\Rightarrow A_z = 8.00$ kN

$\bar{A} = -2.00\,\bar{i} + 8.00\,\bar{k}$ kN

3-D Static Equilibrium of Rigid Bodies
Group Quiz

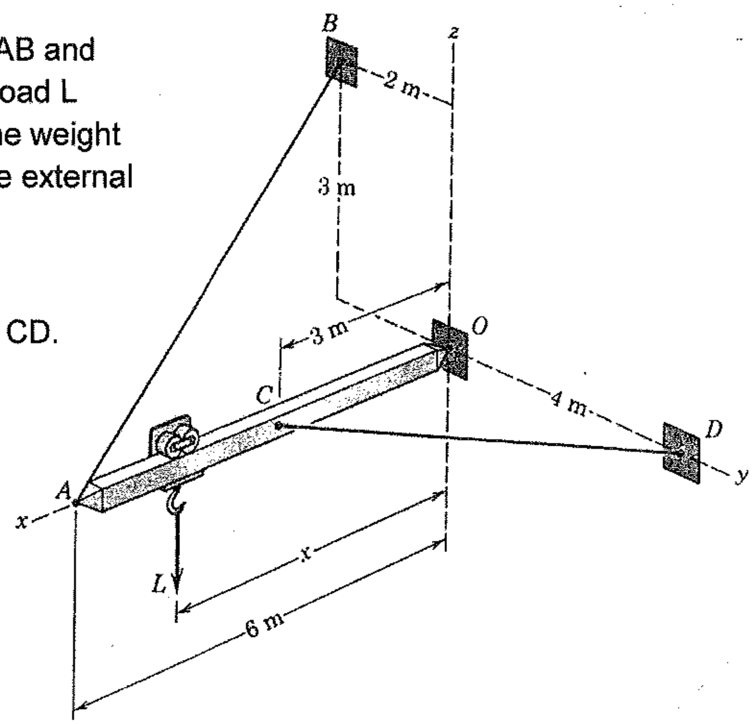
Group #: ____________ Group Members: 1) ______________________________
(Present Only)

Date: ______________ Period: _________ 2) ______________________________

3) ______________________________

4) ______________________________

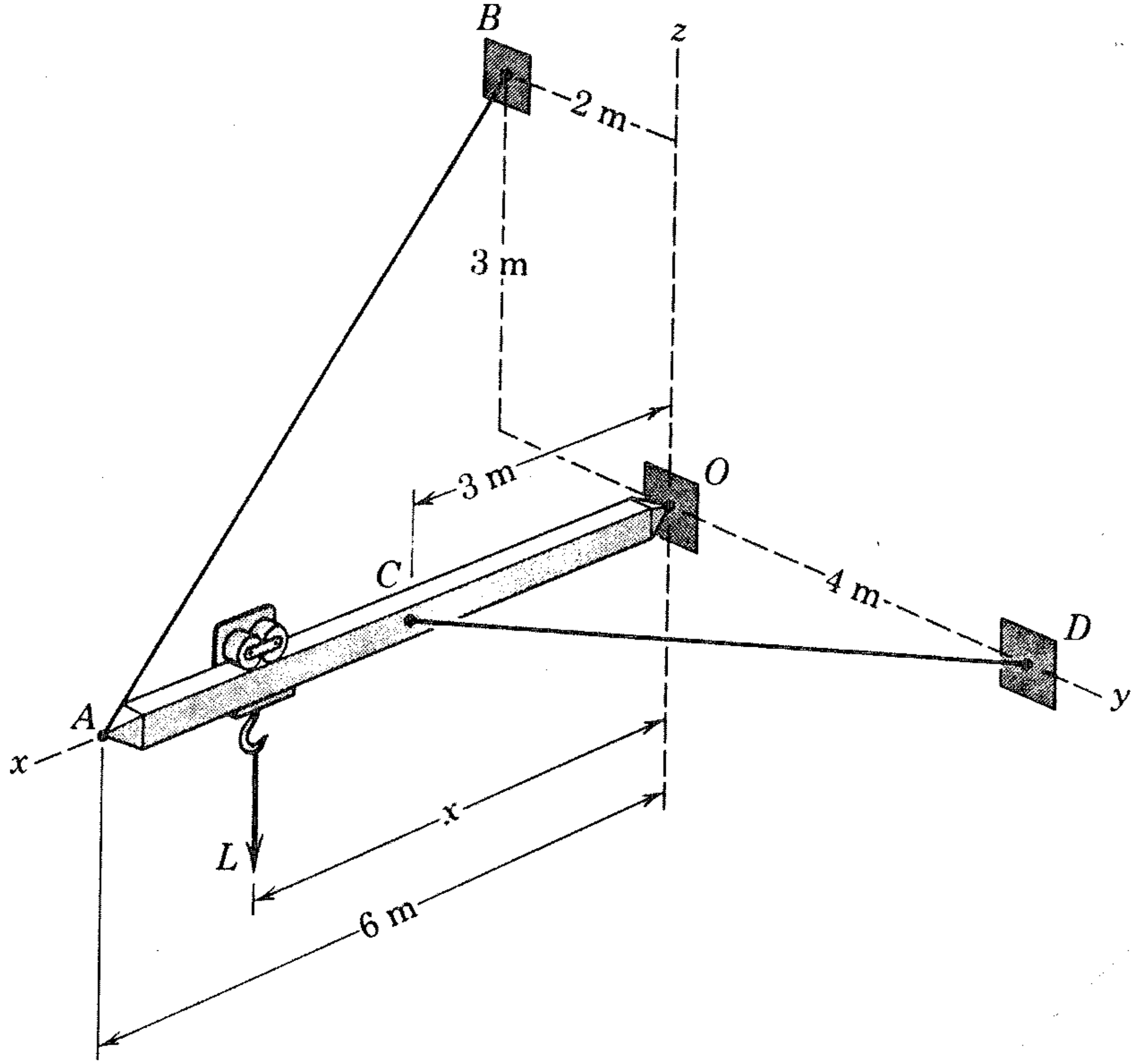
Given: A horizontal boom is supported by cables AB and CD and by a ball and socket joint at O. A load L = 5 kN is applied at a distance x = 5 m. The weight of the boom is negligible compared with the external load.



Find:

a) Determine the tensions in cable AB and CD.
b) Determine the reactions at joint O.

Solution:

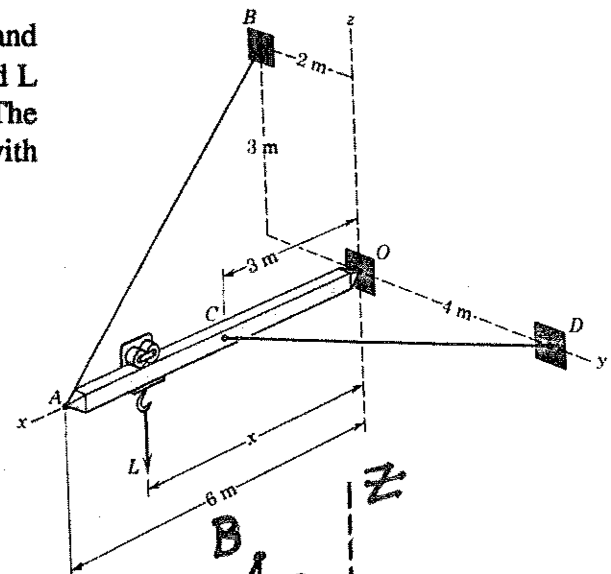

B
z
2 m
3 m
3 m
O
C
4 m
D
A
y
x
x
L
6 m

ME 270 - Basic Mechanics I - Group Quiz

Name/Group #: ______________ Group Members: 1) ______________ 2) ______________

Date: ______________ Period: ______________ 3) ______________ 4) ______________

Given: A horizontal boom is supported by cables AB and CD and by a ball-and-socket joint at O. A load L = 5 kN is applied at a distance x = 5 m. The weight of the boom is negligible compared with the external load.

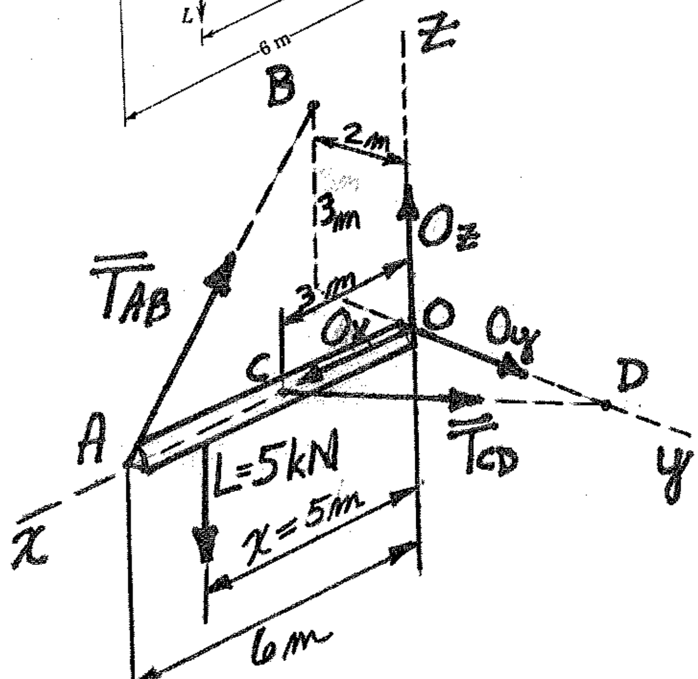


Find:

(a) Determine the tensions in cable AB and CD.

(b) Determine the reactions at joint O.

Solution:



(a) $$\overline{T}_{AB} = T_{AB}\left[\frac{-6\bar{i} - 2\bar{j} + 3\bar{k}}{[(-6)^2 + (-2)^2 + 3^2]^{1/2}}\right]$$

$$\overline{T}_{AB} = T_{AB}\left[-\frac{6}{7}\bar{i} - \frac{2}{7}\bar{j} + \frac{3}{7}\bar{k}\right] kN$$

$$\overline{T}_{CD} = T_{CD}\left[\frac{-3\bar{i} + 4\bar{j}}{[(-3)^2 + (4)^2]^{1/2}}\right]$$

$$\overline{T}_{CD} = T_{CD}\left[-\frac{3}{5}\bar{i} + \frac{4}{5}\bar{j}\right] kN$$

$$\overline{L} = -5\bar{k}\ kN$$

$$\sum M_O = 0 = (\vec{r}_{OA} \times \vec{T}_{AB}) + (\vec{r}_{OC} \times \vec{T}_{CD}) + (\vec{r}_{OL} \times \vec{L})$$

$$= \left[(6\vec{i}) \times T_{AB}\left(-\tfrac{6}{7}\vec{i} - \tfrac{2}{7}\vec{j} + \tfrac{3}{7}\vec{k}\right) + (3\vec{i}) \times T_{CD}\left(-\tfrac{3}{5}\vec{i} + \tfrac{4}{5}\vec{j}\right) + (5\vec{i}) \times (-5\vec{k}) \right]$$

$$= \left[-\tfrac{18}{7} T_{AB}\,\vec{j} - \tfrac{12}{7} T_{AB}\,\vec{k}\right] + \left[\tfrac{12}{5} T_{CD}\,\vec{k}\right] + \left[25\vec{j}\right]$$

$\vec{j}$ **eqn**: $-\frac{18}{7} T_{AB} + 25 = 0 \Rightarrow \boxed{T_{AB} = 9.72\ kN}$

k **eqn**: $-\frac{12}{7} T_{AB} + \frac{12}{5} T_{CD} = 0 \Rightarrow \boxed{T_{CD} = 6.94\ kN}$

(b) $\sum F_x = 0 = O_x - \frac{6}{7} T_{AB} - \frac{3}{5} T_{CD} \Rightarrow \boxed{O_x = 12.5\ kN}$

$\sum F_y = 0 = O_y - \frac{2}{7} T_{AB} + \frac{4}{5} T_{CD} \Rightarrow \boxed{O_y = -2.77\ kN}$

$\sum F_z = 0 = O_z + \frac{3}{7} T_{AB} - 5 \Rightarrow \boxed{O_z = 0.834\ kN}$

∴ In vector form,

$$\boxed{\vec{O} = 12.5\vec{i} - 2.77\vec{j} + 0.834\vec{k}\ kN}$$