ME 579 Fourier Methods in Digital Signal Processing
Homework Set 3

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Problem 3-1

A window function is defined as

$$x(t) = 1 \text{ for } -0.4 \leq t \leq 0.4 \text{ seconds}$$

$$x(t) = 5 + 10t \text{ for } -0.5 \leq t \leq -0.4 \text{ seconds}$$

$$x(t) = 5 - 10t \text{ for } 0.4 \leq t \leq 0.5 \text{ seconds}$$

$$x(t) = 0 \text{ for } |t| > 0.5 \text{ seconds}$$

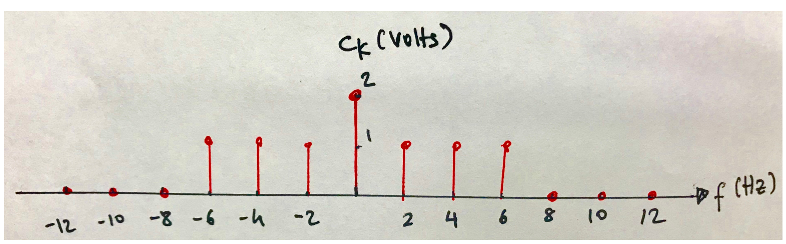
(a) Calculate its Fourier transform.

(b) Plot $|X(f)|$ for f=0 to 10 Hz with 0.01 Hz steps.

(c) Compare this window's properties to that of a rectangular window.

Problem 3-2

The spectrum of a signal that has even symmetry in time, $x(t) = x(-t)$, is shown in the figure below



If the signal was sampled so that there are exactly N points in one period and the C_k were estimated by using

$$\hat{C}_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n\Delta) e^{-j\frac{2\pi nk}{N}}.$$

Plot the estimated spectrum over the range k=-18,-16,...,-1,0,1,...,16,18. Do this for N=8 and N=6 and comment on what the sampling frequency (f_s) is for each value of N. Also, explain the differences between the true spectrum (C_k vs f_s) and the two estimated spectra.

Problem 3-3

(a) Sample a signal $h(t)$ every Δ seconds. The signal starts at t=0 seconds after which it is defined as

$$h(t) = e^{-20\pi t}.$$

Calculate, by hand, the Fourier transform of this sampled signal: $H_s(f)$. Note this signal is not of finite length. As t gets large, it gets smaller, but only approaches 0 as $t \to \infty$.

(b) Plot the magnitude and phase of $H_s(f)$ from 0 to 50 Hz. Plot the magnitude in dB: $20\, log_{10}|H_s(f)|$. Choose sample rates: 10 Hz, 20 Hz, 50 Hz, 100 Hz and 200 Hz. What changes in the spectra as the sample rate increases?

(c) Calculate the Fourier transform of $h(t)$ (continuous not sampled signal) by hand, plot the resulting function of f and compare it with the Fourier transform of the sampled signals divided by their sampling rates.

Reminder: The summation of an $N-$point geometric series:

$$a_0,\quad a_0.a,\quad a_0.a^2,\quad a_0.a^3,\quad a_0.a^4.....,\quad a_0.a^{N-1} \text{ is } a_0\frac{1-a^N}{1-a}$$

If $N \to \infty$ and $|a| < 1$ then the sum becomes:

$$a_0\frac{1}{1-a}.$$

Problem 3-4

(a) With reference to the notation used in lectures, explain the relationships between $X(f)$, $X_s(f)$ and X_k. These are, respectively, the Fourier transform of a continuous signal, the Fourier transform of a continuous signal $x(t)$ after sampling it every Δ seconds, and the discrete Fourier transform of N samples of $x(n\Delta)$ for $n = 0, 1, 2, ..., N-1$, and $T = N\Delta$.

(b) If one performs a DFT on the computer to obtain X_k and then uses it to estimate $X(f)$, what are the sources of error?

(c) Now let's examine the effect of windowing on this approximation to $X(f)$ for the case of a damped exponential:

Set f_s=20 Hz, and examine results for $T = 10$ seconds and $T = 100$ seconds, where T is the width of the window in seconds. The corresponding values for N are 200 and 2000, respectively.

$$y(t) = 5e^{-0.1t} \text{ Newtons for } t \geq 0 \text{ seconds and,}$$
$$= 0 \text{ Newtons for } t < 0 \text{ seconds.}$$

For each value of T,

Notes: You can use semilogy when plotting the magnitude of the various Fourier Transforms.

Problem 3-5

Two discrete signals, the result of sampling continuous signals at 20 samples per second are defined as follows:

$$x_n = 10/n \text{ for } n = 2, 3, 4, 5 \text{ and is 0 elsewhere.}$$

$$h_n = 4 \text{ for } n = -1, 0, 1, 2 \text{ and is 0 elsewhere.}$$

Discrete convolution is given, in general, by:

$$y_n = \Delta \sum_{m=-\infty}^{\infty} x_m \, h_{n-m}.$$

(a) What is the duration of y_n (points and time in seconds), and when does it start and stop (n values and corresponding times)?

(b) Calculate, by hand, the result of convolving x_n and h_n. Plot the resulting, y_n.

(c) Now do the calculation by using MATLAB's *fft* and *ifft* programs, i.e., by doing convolution in time via multiplication in the frequency domain and inverse transforming. Plot the result. Note: you will need to set up the time-axis correctly.

(d) Compare the results in parts (a) and (b).