ME 563 - Fall 2025

Homework No. 5

Due: December 7, 2025

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Homework Problem 5.1

A forcing $F(t) = F_0 \sin(\Omega t)$ acts at the end of the thin, homogeneous bar of the two-DOF system shown below. The wheel can be modeled as a cylinder with mass m, radius R, and rolls without slipping. The response of the system is to be described by the coordinates $x(t)$ and $\theta(t)$. Let $\dfrac{g}{L} = \dfrac{2k}{m}$.

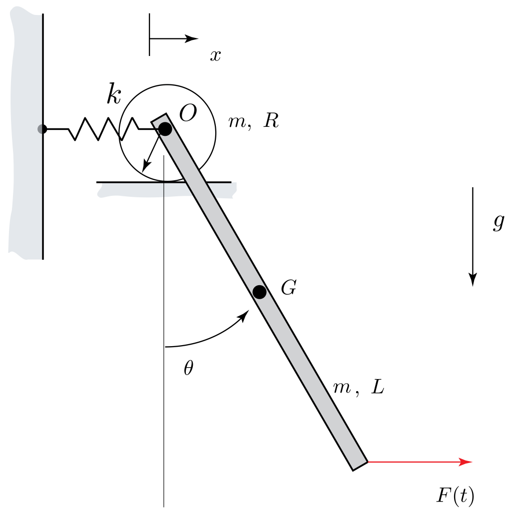

Figure 1: Forced excitation of two degree of freedom system.

(a) Derive the equations of motion for the system.

(b) Derive the particular solutions $x_p(t)$ and $\theta_p(t)$ for the system.

(c) At what values of the temporal frequency Ω does resonance occur in the system?

(d) Show that the "shape" (ratio of amplitudes) of the response is that of the first mode when excited at the first natural frequency, and that the shape is that of the second mode when excited at the second natural frequency.

(e) At what values (if any) of the temporal frequency Ω do anti-resonances occur for $x_p(t)$? For $\theta_p(t)$?

(f) Make plots for the amplitudes of $x_p(t)$ and $\theta_p(t)$ versus the temporal frequency Ω.

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Homework Problem 5.2

The undamped, single-DOF system above is given a base excitation $y(t)$. The base motion $y(t)$ is T-periodic in time, as shown above.

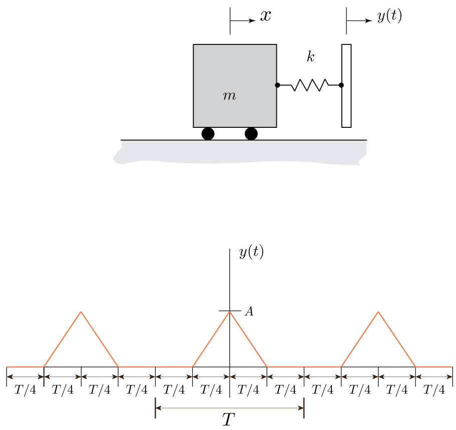

Figure 2: Forced excitation of two degree of freedom system.

(a) Determine the Fourier series of $y(t)$.

(b) Use Matlab to make a plot of $y(t)$ vs. t/T for your Fourier series. Use a sufficient number of terms in your Fourier series to ensure that the series has converged.

(c) Derive the equation of motion for the system.

(d) Determine the particular solution for the response $x_p(t)$.

(e) Use Matlab to plot $x_p(t)$ vs. t/T for your Fourier series corresponding to

$$T = 0.87\,T_n, \qquad T_n = 2\pi\sqrt{\frac{m}{k}}$$

using the same number of terms in $x_p(t)$ as in the Fourier series for $y(t)$.

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Homework Problem 5.3

Consider a damped, single-DOF system with equation of motion

$$m\ddot{x} + c\dot{x} + kx = f(t).$$

It was shown in lecture that the convolution integral for the undamped case ($\zeta = 0$) with zero initial

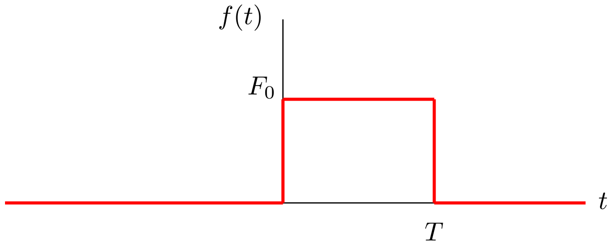

Figure 3: Discontinuous force

conditions can be written as

$$x(t) = \int_0^t h(t-\tau)\, f(\tau)\, d\tau,$$

where the impulse response function is

$$h(t-\tau) = \frac{1}{m\omega_n} \sin\left[\omega_n(t-\tau)\right].$$

(a) Show that the convolution integral solution for zero initial conditions for the *critically damped* case ($\zeta = 1$) can be written in the same general form,

$$x(t) = \int_0^t h(t-\tau)\, f(\tau)\, d\tau.$$

Determine the impulse response function $h(t-\tau)$ in this case. You may begin with the general convolution form in the lecturebook written in terms of the fundamental solutions $u(t)$ and $v(t)$.

(b) Use the convolution integral derived in part (a) to determine the response of the system to the forcing shown above, where

$$T = 0.50\, T_n, \qquad T_n = 2\pi\sqrt{\frac{m}{k}}.$$

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Bonus: 10 points on HW

In this assignment we will look at various forms of the transfer function. Given the equation of motions for a single degree fo freedom system

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t).$$

Let the mass is $m = 1$ kg, stiffness $k = 1 \times 10^4$ N/m, and damping ratio $\zeta = 0.005$, complete the following:

(a) Receptance Transfer Function (Laplace Domain)

Write the receptance transfer function

$$H(s) = \frac{X(s)}{F(s)}$$

by taking the Laplace transform of the equation of motion.

(b) Receptance in the Fourier Domain

Convert $H(s)$ to the Fourier domain by substituting:

$$s = i\omega.$$

(c) Pole–Zero Form

Write the receptance transfer function from (a) in explicit **pole–zero form**:

$$H(s) = \frac{(s - z_1)(s - z_2)\cdots}{(s - p)(s - p^{)}.}$$

where poles p, p^* are roots of the denominator, zeros (z_i) are roots of the numerator. Note p^* is the complex conjugate of p.

(c) Partial Fraction Expansion

Write the receptance transfer function in **partial fraction form**:

$$H(s) = \frac{a}{s - p} + \frac{a^*}{s - p^*}.$$

Note: You must compute a and a using partial fraction expansion.

(d) Real–Imaginary Form (Fourier Domain)

Using $s = i\omega$, write the receptance transfer function explicitly in **real–imaginary form**:

$$H(i\omega) = H_R(\omega) + iH_I(\omega),$$

by eliminating the imaginary part from the denominator.

(e) Magnitude and Phase Form

Using the result from (d), write the transfer function in **magnitude** and **phase** form:

$$|H(i\omega)| = \sqrt{H_R^2(\omega) + H_I^2(\omega)}, \qquad \phi(\omega) = \tan^{-1}\left(\frac{H_I(\omega)}{H_R(\omega)}\right).$$

(f) Physical Interpretation: Explain physically what the poles and zeros of the receptance transfer function represent for this system. Show why in a SDOF the receptance has no zeros. In a MDOF system what do the zeros represent.