ME 563 - Fall 2025

Homework No. 2

Due: September 26, 2025

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Homework Problem 2.1

A thin, homogenous bar having a mass of M and length of L is pinned to the ground at point O. A particle P of mass m is free to slide on the smooth surface of the bar. A spring of stiffness k and unstretched length of R_0 is attached at point O and particle P. Let r be the radial distance from O to P and θ be the rotation of the bar from a fixed vertical line.

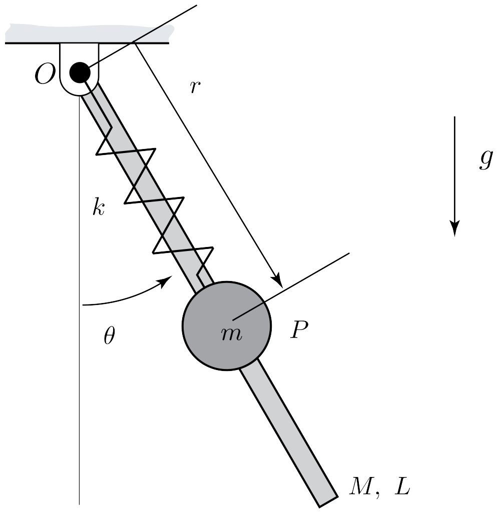

Figure 1: A particle of mass m sliding on a rotating bar.

(a) Equations of Motion

Use Lagrange's equation to develop the EOM's for this two-DOF system using generalized coordinates of r and θ. Hint: Use a polar coordinate system to describe the velocity of the particle m.

(b) Equilibrium Values

Using the equations of motion, determine the equilibrium values for r and θ.

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Homework Problem 2.2

Inspired by the power-amplifying jump of a click beetle (see this video reference), we model the beetle as a simplified two-link rigid body system as shown below. The system consists of two uniform links: link 1 (mass m_1, length L_1) and link 2 (mass m_2, length L_2). The end of link 1 is pinned to the ground at origin O.

The generalized coordinates are the absolute angles θ_1 and θ_2 measured from the **horizontal** axis. Gravity acts downwards.

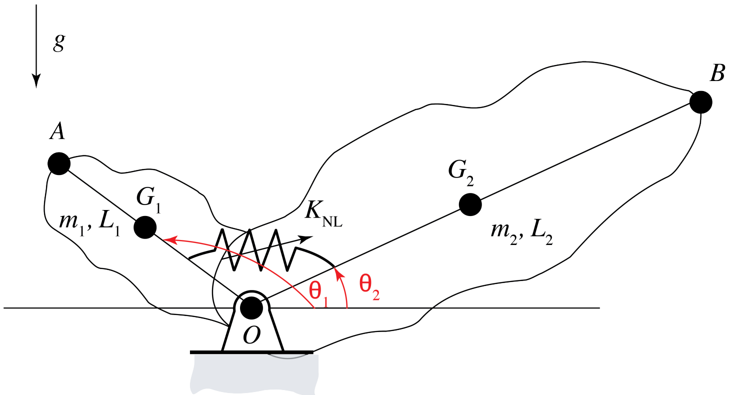

Figure 2: Simplified two-link model of a click beetle pinned at O.

The complex biomechanics of the hinge are modeled by two components:

1. A nonlinear torsional spring based on experimental data from the PNAS paper "Nonlinear elasticity and damping govern ultrafast dynamics in click beetles." The potential energy stored in the hinge is given by:
$$U_s(\theta_1, \theta_2) = \frac{1}{2}k_1(\theta_2 - \theta_1)^2 + \frac{1}{3}k_2(\theta_2 - \theta_1)^3 + \frac{1}{4}k_3(\theta_2 - \theta_1)^4$$

2. A **nonlinear Rayleigh dissipation function**, $\mathcal{R}$, which models both linear and nonlinear energy loss in the hinge:
$$\mathcal{R}(\dot\theta_1, \dot\theta_2) = \frac{1}{2}b_1(\dot\theta_2 - \dot\theta_1)^2 + \frac{1}{3}b_2|\dot\theta_2 - \dot\theta_1|^3$$
where b_1 and b_2 are the linear and nonlinear damping coefficients, respectively.

(a) Kinetic and Potential Energy

Determine the expressions for the total kinetic energy T and the total potential energy U of the system. The total potential energy is $U = U_g + U_s$.

(b) Nonlinear Equations of Motion

Using your results from part (a) and the provided dissipation function, apply the Lagrange equations with dissipative forces to derive the two coupled, nonlinear differential equations of motion for the system. Recall that the generalized dissipative force is $Q_i = -\frac{\partial \mathcal{R}}{\partial \dot q_i}$.

(c) Linearization and System Matrices

Determine the stable equilibrium position (s) of the beetle. Linearize the full equations of motion about this equilibrium point(s) to find the system mass $[M]$, damping $[C]$, and stiffness $[K]$ matrices for small oscillations.

(d) Liftoff

The dynamic model developed treats the pre-jump latched state as a set of initial conditions and is only valid while the beetle is on the ground. How can this model be used to determine the exact moment of liftoff? Specifically, which forces must be monitored, and what is the mathematical criterion that signals the transition from the ground phase to free-flight

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Consider the system below, whose motion is described by the absolute coordinates shown. The disk of mass m rolls without slip.

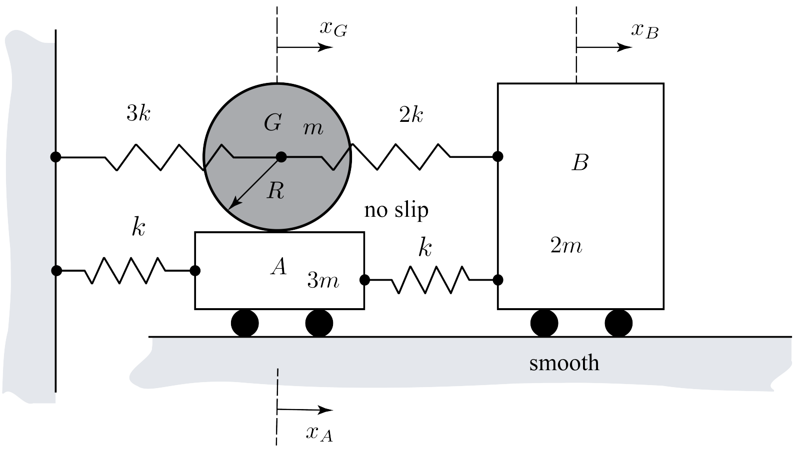

Figure 3: A three-degree-of-freedom spring-mass system.

(a) Stiffness Matrix

Write down the potential energy function U for this four-DOF system and use it to develop the stiffness matrix $[K]$ for the system.

(b) Flexibility Matrix

Use the method of influence coefficients to develop the flexibility matrix $[A] = [K]^{-1}$.

(c) Verification

Check your results in a) and b) above by verifying that $[A][K] = [I]$, where $[I]$ is the identity matrix.

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Homework Problem 2.4

Bullent B of mass βm, where β is a constant, is initially moving to the right with a speed of v. Rod AO of length L and mass m with center of gravity at G. The rod is at rest and attached to a spring and damper. The spring is unstretched. Upon impact, at time $t = 0$, block B sticks to rod AO. The system is viewed in the horizontal plane, i.e., gravity is into the page. For this problem use: $v = 100$ m/s, $m = 10$ kg, $c = 60$ kg/s, and $k = 300$ N/m.

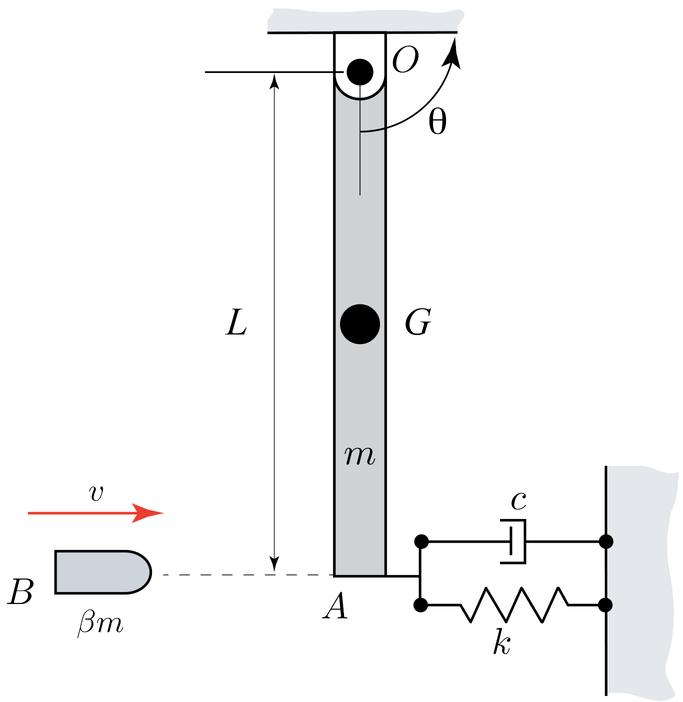

Figure 4: An inelastic collision with a spring-damper system.

(a) Equation of Motion

Derive the differential equation of motion (EOM) for the system corresponding to $t > 0$.

(b) Natural Frequency and Damping Ratio

Determine the numerical values for the undamped natural frequency and the damping ratio of the system when $\beta = 1$. How do these values change when $\beta > 1$ and $\beta < 1$?

(c) System Response

Determine the response of the system for $t > 0$. HINT: You can use conservation of momentum to determine the speed of blocks A and B immediately after sticking when $\beta = 1$. How does the value change when $\beta > 1$ and $\beta < 1$?

(d) Maximum Displacement

What is the maximum displacement of blocks A and B in the response found in c) when $\beta = 1$? How does the value change when $\beta > 1$ and $\beta < 1$?

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Homework Problem 2.5

(a) Logarithmic Decrement

Show that the logarithmic decrement is equal to

$$\delta = \frac{1}{n} \ln \left(\frac{x_0}{x_n} \right)$$

where x_n is the amplitude of vibration after n cycles have elapsed.

(b) Trigonometric Identity

Show that by calculation that

$$A \cos(\omega t - \phi)$$

can be represented as

$$B \sin(\omega t) + C \cos(\omega t)$$

where B and C are functions of A and ϕ.

(c) ODE Solution and Sketch

Solve

$$2\ddot{x} + 4\dot{x} + 16x = 0$$

with initial conditions $x(0) = 1$ and $v(0) = \dot{x}(0) = 0$ for $x(t)$ and sketch the time waveform.