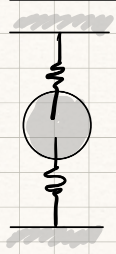
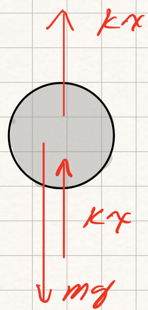
Problem #1

System (a)





$+\downarrow \ \Sigma F_x: \ -kx - kx + mg = m\ddot{x}$

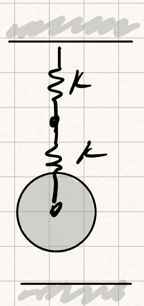
$m\ddot{x} + 2kx = +mg$

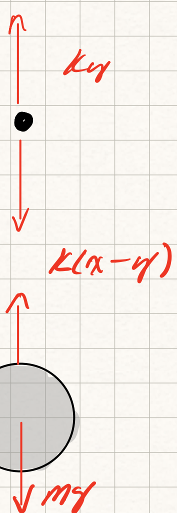
$k_{eq} = 2k$ in parallel

at equilibrium $\ddot{x} = \dot{x} = 0, \quad x = x_s$

$2k x_{ST} = mg \longrightarrow x_{ST} = \dfrac{mg}{2k}$ T

System (b)





$+\downarrow \ \Sigma F_y: \ -ky + k(x-y) = 0$

$\downarrow \ \Sigma F_x: \ -k(x-y) - mg = m\ddot{x}$

$-ky + kx - ky = 0$

$2ky = kx$

$y = \tfrac{1}{2}x$

$-k(x - \tfrac{1}{2}x) - mg = m\ddot{x}$

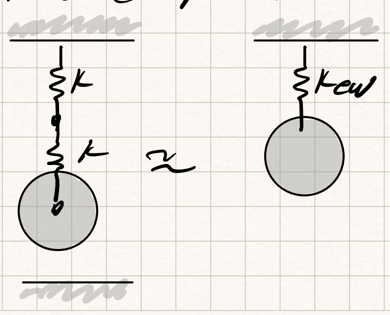
$-\tfrac{k}{2}(x) - mg = m\ddot{x}$

$m\ddot{x} + \tfrac{k}{2}x = mg \ , \quad k_{eq} = k/2$ in series

at equilibrium $\ddot{x} = \dot{x} = 0, \quad x = x_{ST}$

$\tfrac{k}{2} x_{ST} = mg \longrightarrow x_{ST} = 2mg/k$

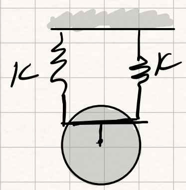
or directly realize

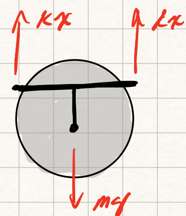


where: $1/k_{eq} = 1/k + 1/k$ series

$k_{eq} = \dfrac{kk}{k+k} = k/2$

System (c)





$+\downarrow \ \Sigma F_x : \ -kx \ -kx \ +mg = m\ddot{x}$

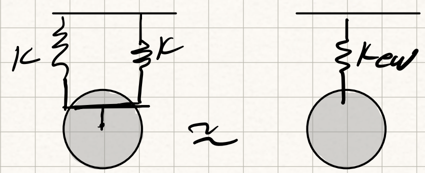
$m\ddot{x} + 2kx = +mg$

$k_{eq} = 2k$ in parallel

at equilibrium $\ddot{x} = \dot{x} = 0$

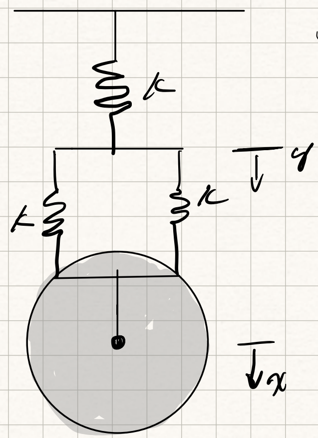
$2k x_{st} = mg \longrightarrow x_{st} = mg/2k$

or directly realize

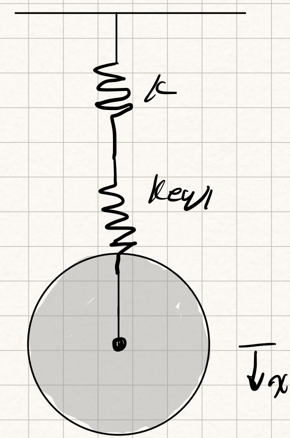


$k_{eq} = k + k = 2k$ parallel

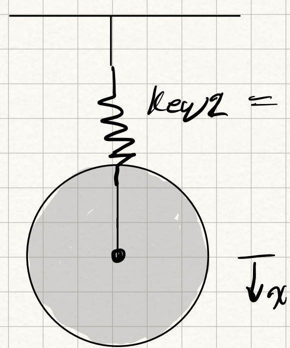
System (d)



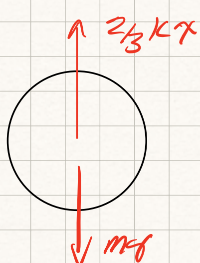
$\approx$



$k_{eq_1} = k + k = 2k$



$$k_{eq2} = \frac{k\, k_{eq1}}{k + k_{eq1}} = \frac{2k^2}{k+3k} = \tfrac{2}{3}k$$



$+\downarrow \sum F_{xi} \quad -\tfrac{2}{3}kx + mg = m\ddot{x}$

$m\ddot{x} + \tfrac{2}{3}kx = mg$

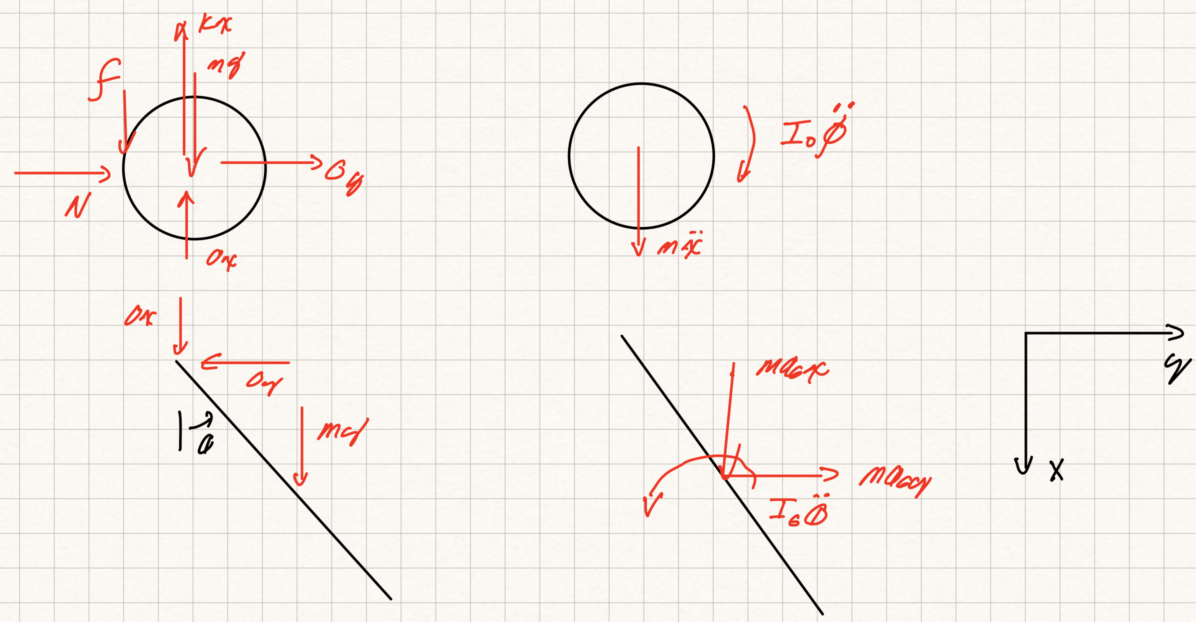
at equilibrium $\ddot{x} = \dot{x} = 0$

$\tfrac{2}{3}k x_{ST} = mg$

$x_{ST} = \tfrac{3}{2}\, mg/k$

System (b) deflects the most.

Problem # 2)



$$+\downarrow \Sigma F_x: -O_x - kx + f + mg = m\ddot{x}$$

$$\rightarrow \Sigma F_y: \quad N + O_y = 0$$

$$\circlearrowright \Sigma M_O: \quad -fR = I_O\ddot{\phi} \qquad\qquad I_O = \tfrac{1}{2}mR^2$$

$$m\ddot{x} + kx = mg + f - O_x$$

$$m\ddot{x} + kx = mg - \frac{I_O\ddot{\phi}}{R} - O_x$$

$$m\ddot{x} + kx = mg - \tfrac{1}{2}mR\ddot{\phi} - O_x$$

$$\rightarrow \Sigma F_y: \quad -O_y = ma_{Gy} \qquad\qquad O_y = -ma_{Gy}$$

$$+\downarrow \Sigma F_x: \quad O_x + mg = ma_{Gx} \qquad\qquad O_x = ma_{Gx} - mg$$

$$\circlearrowright \Sigma M_G: \quad +O_x \tfrac{L}{2}\sin\theta + O_y \tfrac{L}{2}\cos\theta = I_G\ddot{\theta}$$

$$\hat{\imath} \quad \hat{\jmath} \quad \hat{k}$$

$$\bar{a}_G = \bar{a}_O + \vec{\alpha} \times \vec{r}_{G/O} - \omega^2 \vec{r}_{G/O}$$

$$\bar{a}_G = +\ddot{x}\hat{\imath} + \ddot{\theta}\hat{k} \times \tfrac{L}{2}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) - \omega^2 \tfrac{L}{2}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$\bar{a}_G = \ddot{x}\hat{\imath} + \ddot{\theta}\tfrac{L}{2}(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}) - \omega^2 \tfrac{L}{2}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$a_{cx} = \ddot{x} - \ddot{\theta}\,\tfrac{L}{2}\sin\theta - \dot{\theta}^2\,\tfrac{L}{2}\cos\theta$$

$$a_{cy} = \ddot{\theta}\,\tfrac{L}{2}\cos\theta - \dot{\theta}^2\,\tfrac{L}{2}\sin\theta$$

$$I_G\ddot{\theta} = O_x\tfrac{L}{2}\sin\theta + O_y\tfrac{L}{2}\cos\theta$$

$$I_G\ddot{\theta} = (m a_{cx} - mg)\tfrac{L}{2}\sin\theta - m a_{cy}\tfrac{L}{2}\cos\theta$$

$$I_G\ddot{\theta} = \left(m\ddot{x} - m\ddot{\theta}\tfrac{L}{2}\sin\theta - m\dot{\theta}^2\tfrac{L}{2}\cos\theta\right)\tfrac{L}{2}\sin\theta$$
$$- m\ddot{\theta}\tfrac{L^2}{4}\cos^2\theta - m\dot{\theta}^2\tfrac{L}{2}\cos\theta\,\tfrac{L}{2}\sin\theta - mg\tfrac{L}{2}\sin\theta$$

$$= m\ddot{x}\tfrac{L}{2}\sin\theta - m\ddot{\theta}\tfrac{L^2}{4}\sin^2\theta - m\dot{\theta}^2\tfrac{L^2}{4}\cos\theta\sin\theta$$
$$- m\ddot{\theta}\tfrac{L^2}{4}\cos^2\theta - m\dot{\theta}^2\tfrac{L^2}{4}\cos\theta\sin\theta - mg\tfrac{L}{2}\sin\theta$$

$$I_G\ddot{\theta} = m\ddot{x}\tfrac{L}{2}\sin\theta - m\ddot{\theta}\tfrac{L^2}{4} - m\dot{\theta}^2\tfrac{L}{2}\cos\theta\sin\theta - mg\tfrac{L}{2}\sin\theta$$

$$m\ddot{x} + kx = mg - \tfrac{1}{2}m\cancel{\ddot{\theta}}\ \ddot{x} - O_x$$

$$\tfrac{3}{2}m\ddot{x} + kx = mg - \cancel{O_x}$$
$$\left(\ddot{x} - \ddot{\theta}\tfrac{L}{2}\sin\theta - \dot{\theta}^2\tfrac{L}{2}\cos\theta\right)m$$

$$\tfrac{5}{2}m\ddot{x} + m\ddot{\theta}\tfrac{L}{2}\sin\theta + m\dot{\theta}^2\tfrac{L}{2}\cos\theta + kx = mg$$

$$\tfrac{5}{2}m\ddot{x} + m\ddot{\theta}\tfrac{L}{2}\sin\theta + m\dot{\theta}^2\tfrac{L}{2}\cos\theta + kx = mg$$

$$\underbrace{I_G\ddot{\theta} + m\ddot{\theta}\tfrac{L^2}{4}}_{I_O\ddot{\theta}\,L^2/4} - m\ddot{x}\tfrac{L}{2}\sin\theta + m\dot{\theta}^2\tfrac{L}{2}\cos\theta\sin\theta = -mg\tfrac{L}{2}\sin\theta$$

$$I_G = \tfrac{1}{12}mL^2$$

$$I_O = \tfrac{1}{3}mL^2$$

$$\frac{5}{2}m\ddot{x} + m\frac{L}{2}\sin\theta\ddot{\theta} + m\frac{L}{2}\cos\theta\dot{\theta}^2 + kx = mg$$

$$I_0\ddot{\theta} - m\frac{L}{2}\sin\theta\ddot{x} + m\frac{L}{2}\cos\theta\sin\theta\dot{\theta}^2 + mg\frac{L}{2}\sin\theta = 0$$

$\theta_{eq} \rightarrow \ddot{\theta} = \dot{\theta} = 0$

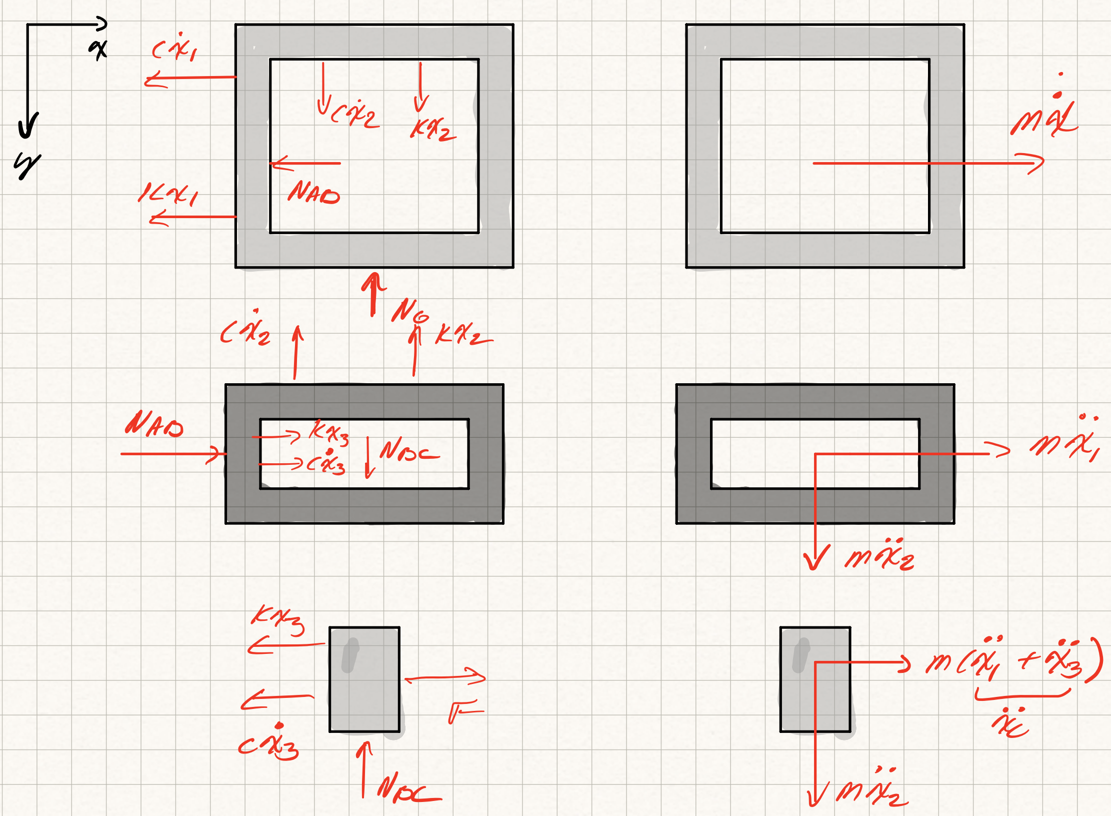
$x_{eq} \rightarrow \ddot{x} = \dot{x} = 0$

$k x_{eq} = mg \qquad\qquad x_{eq} = {}^{k}/mg$

$mgL/_2 \sin\theta_{eq} = 0 \qquad\qquad \theta_{eq} = 0, \pi, 2\pi$

$= n\pi \quad n = 0, 1, 2 \ldots.$

Problem #3)



Block A: $\xrightarrow{+} \Sigma F_x: \; -c\dot{x}_1 - kx_1 - N_{AB} = m\ddot{x}_1$ 1)

Block B: $\xrightarrow{+} \Sigma F_x: \; N_{AB} + kx_3 + c\dot{x}_3 = m\ddot{x}_1$ 2)

$+\downarrow \Sigma F_y: \; -c\dot{x}_2 - kx_2 + N_{BC} = m\ddot{x}_2$ 3)

Block C: $\xrightarrow{+} \Sigma F_x: \; -kx_3 - c\dot{x}_3 + F = m(\ddot{x}_1 + \ddot{x}_3)$ 4)

$+\downarrow \Sigma F_y: \; -N_{BC} = m\ddot{x}_2$ 5)

1) + 2)

$$2m\ddot{x}_1 = kx_3 + c\dot{x}_3 - kx_1 - c\dot{x}_1$$

3) + 5)

$$2m\ddot{x}_2 = -c\dot{x}_2 - kx_2$$

Rearranging

$$2m\ddot{x}_1 + c(\dot{x}_1 - x_3) + k(x_1 - x_3) = 0$$

$$2m x_2 + c\dot{x}_2 + kx_2 = 0$$

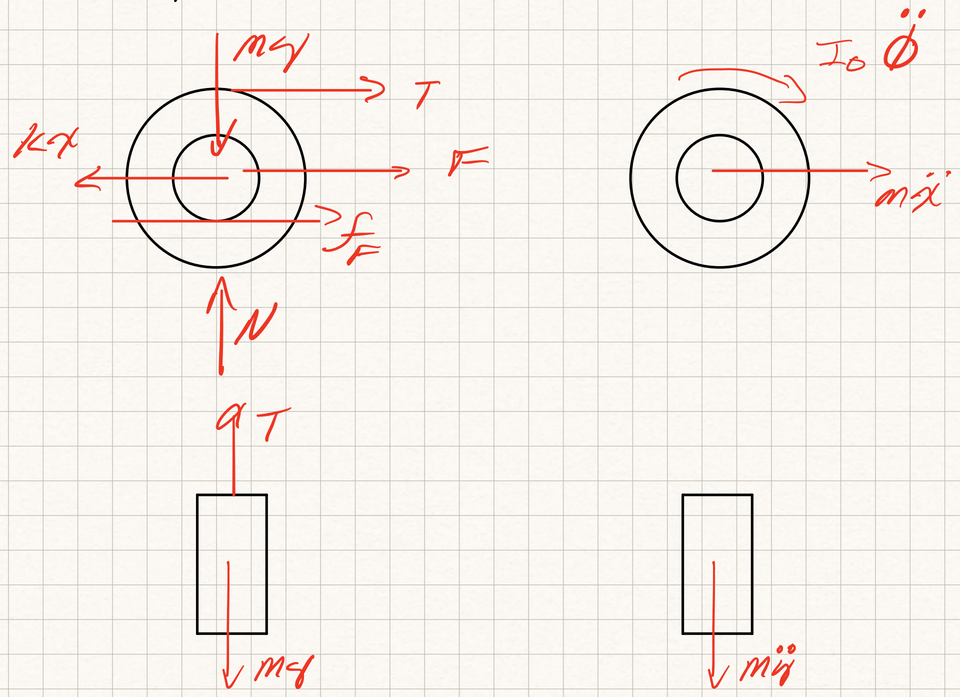
$$m(\dot{x}_1 + \ddot{x}_3) + c\dot{x}_3 + kx_3 = F$$

In matrix form

$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & 2m & 0 \\ m & 0 & m \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} c & 0 & -c \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} k & 0 & -k \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F \end{Bmatrix}$$

Problem 1.4



Drum: $\circlearrowright + \ \Sigma M_c: \quad -kRx + FR + T(3R) = I_c \ddot{\phi} \qquad 1)$

$$I_c = I_o + mR^2$$

Block: $+\downarrow \ \Sigma F_y: \quad -T + mg = m\ddot{y} \qquad 2)$

Kinematics

$$x = R\phi, \quad \dot{x} = R\dot{\phi}, \quad \ddot{x} = R\ddot{\phi}$$

$$x_T = y_T, \quad \ddot{x}_T = \ddot{y}_T \qquad x_T = 3R\phi \qquad x_T$$

$$-kR^2\phi + FR + 3TR = (I_o + mR^2)\ddot{\phi} \qquad 1)$$

$$-T + mg = 3mR\ddot{\phi} \qquad 2)$$

$1) + 3R\ 2)$

$$-kR^2\phi + FR + 3mgR = (I_o + 10mR^2)\ddot{\phi}$$

Rearrange

$$(I_0 + 10mR^2)\ddot{\phi} + kR^2\phi = 3mgR + FR$$

Now, using power

$$T = \tfrac{1}{2} I_C \dot{\phi}^2 + \tfrac{1}{2} m \dot{y}$$

$$U = -mgy + \tfrac{1}{2} kx^2$$

$$dW^{NC} = \vec{F} \cdot d\vec{x} = F\hat{i} \cdot dx\hat{i} = F\,dx$$

Kinematics

$y = 3R\phi \qquad x = R\phi \qquad dx = R\,d\phi$

$\dot{y} = 3R\dot{\phi}$

$$T = \tfrac{1}{2} I_C \dot{\phi}^2 + \tfrac{1}{2} m\, 9R\dot{\phi}^2$$

$$U = -mg3R\phi + \tfrac{1}{2} kR^2\phi^2$$

$$\frac{dT}{dt} + \frac{dU}{dt} = \frac{dW^{NC}}{dt}$$

$$\frac{d}{dt}\left(\tfrac{1}{2} I_C \dot{\phi}^2 + \tfrac{1}{2} m\, 9R\dot{\phi}^2\right) = (I_C + m9R)\ddot{\phi}\dot{\phi}$$

$$= (I_0 + 10mR)\ddot{\phi}\dot{\phi}$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(-3mgR\phi + \tfrac{1}{2}kR^2\phi^2\right)$$

$$= \left(-3mgR\dot{\phi} + kR^2\phi\dot{\phi}\right)$$

$$\frac{dU}{dt} = (-3mgR + kR^2\phi)\dot{\phi}$$

$$\frac{dW^{nc}}{dt} = F\,dx/dt = FR\dot{\phi}$$

$$(I_0 + 10mR)\ddot{\phi}\cancel{\dot{\phi}} + (kR^2\phi - 3mgR)\cancel{\dot{\phi}} = FR\cancel{\dot{\phi}}$$

$$(I_0 + 10mR)\ddot{\phi} + kR^2\phi = 3mgR + FR$$

same as before