Test 2

Name______________________________________

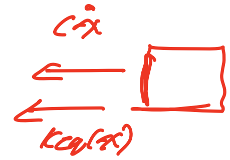
Pledge______________________________________

I have neither given nor received aid on this examination.

Instructions:

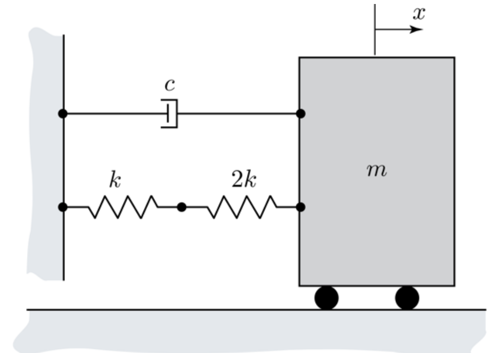
This is a closed-book, closed-notes exam.

Please read the question on this exam carefully and answer only the questions I ask. Don't waste your time doing extra work, and don't skip any of the smaller questions within a problem. I can't grade what I can't see and I can't give partial credit for something in your head.



ME 563 - Fall 2024

Test Problem 1 -30 points ~ In person and online



The single-DOF system shown below is made up of a particle of mass m, two springs (having stiffnesses of k and $2k$) and a dashpot with a damping coefficient c. Let x represent the motion of the particle measure from a position at which the springs are unstretched. If m = 100 kg, k = 1350 N/m and c = 40 kg/sec:

a) What is the *undamped natural frequency* ω_n?

$$k_{eq} = \frac{k(2k)}{k+2k} = \frac{2}{3}k \longrightarrow \omega_n = \sqrt{k_{eq}/m} = \sqrt{\tfrac{2}{3}k/m}$$

$$\omega_n = \sqrt{\tfrac{2}{3}\frac{(1350)}{100}} = 3 \text{ rad/s}$$

b) What is the *damping ratio* ξ?

$$2\zeta\omega_n = c/m \qquad \zeta = \frac{c}{2m\omega_n} = \frac{c}{2m\sqrt{k_{eq}/m}} = \frac{c}{2\sqrt{mk_{eq}}}$$

$$\zeta = \frac{40}{2\sqrt{100\cdot\tfrac{2}{3}\cdot 1350}} = 0.0667$$

c) What is the *damped natural frequency* ω_d?

$$\omega_n = 2\pi\frac{1}{T}$$

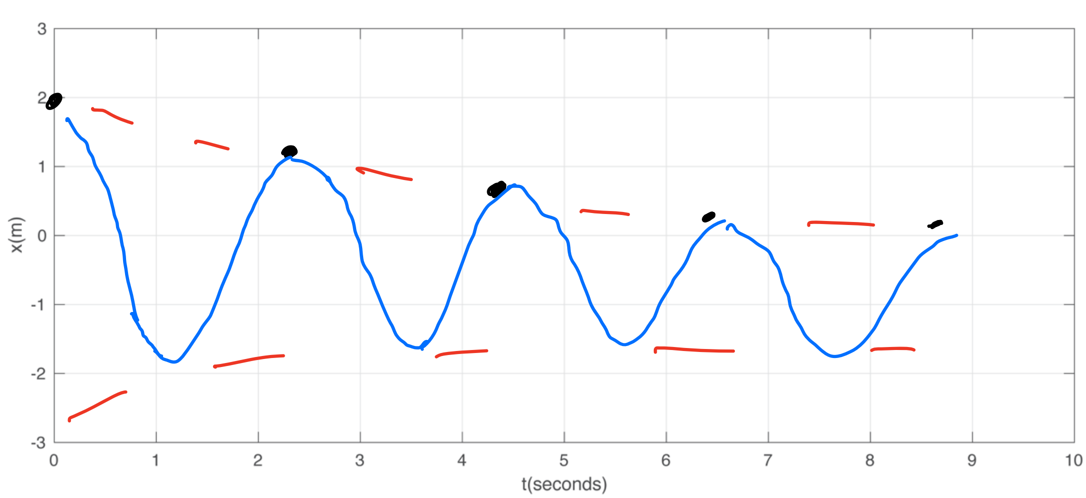
$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 2.9933 \text{ rad/s}$$

$$T = 2\pi/\omega_n = \tfrac{2}{3}\pi = 2.094 \text{ s}$$

$$T_D = 2\pi/\omega_d = 2.091 \text{ s}$$

d) Given an initial condition $x(0)$=2 m, $v(0)$=0 m/s, Sketch the response on the axis below.

t	$x(t)$
2.091	1.3143
4.198	0.8637
6.30	0.5676
8.40	0.3730



$$x(t) = e^{-\zeta\omega_n t}x_0 \cos\omega_d t$$

ME 563 - Fall 2024

Test Problem 2 -30 points ~ In Person

Please answer the following

1. A 3 DOF system has stiffness matrix

$$K = \begin{bmatrix} k_1 & -2k_1 & k1 \\ -k_1 & k_1+3k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$$

   Can the system have a rigid body mode? Justify your answer

2. A lumped 2DOF models has the following stiffness and mass matrices

$$K = k\begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}, \quad M = m\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

   a) Determine the natural frequencies of the model
   b) Determine the mode shapes of the system

① if $\det(K) = 0$ or $K\Delta = 0$ ~ harder to show

either one will work

$$\Delta K = k_1 \begin{vmatrix} k_1+3k_2 & -k_2 \\ -k_2 & k_2 \end{vmatrix} + k_1 \begin{vmatrix} -2k_1 & k_1 \\ -k_2 & k_2 \end{vmatrix} + 0$$

$$\Delta K = k_1\left(k_1k_2 + 3k_2^2 - k_2^2\right) + k_1\left(-2k_1k_2 + k_1k_2\right)$$

$$= k_1^2k_2 + 3k_1k_2^2 - k_1k_2^2 - 2k_1^2k_2 + k_1^2k_2$$

$$= \underbrace{k_1^2k_2 - 2k_1^2k_2 + k_1^2k_2}_{0} + \underbrace{3k_1k_2^2 - k_1k_2^2}_{2k_1k_2^2} = 2k_1k_2^2$$

No rigid body modes

Option 2

$$[k]\Delta = \left\{ \begin{array}{l} k_1\Delta_1 - 2k_1\Delta_2 + k_1\Delta_3 \\ -k_1\Delta_1 + (k_1+3k_2)\Delta_2 - k_2\Delta_3 \\ \quad -k_2\Delta_2 + k_2\Delta_3 \end{array} \right\} = 0$$

$\Delta_2 = \Delta_3$ last equation

$k_1\Delta_1 - 2k_1\Delta_2 + k_1\Delta_2 = 0$ 1st equation

$\Delta_1 - \Delta_2 = 0 \longrightarrow \Delta_1 = \Delta_2$

$-k_1\Delta_1 + (k_1+3k_2)\Delta_2 - k_2\Delta_3 = -k_1\Delta_1 + (k_1+3k_2)\Delta_1 - k_2\Delta_1 = 0$

$= 2k_2\Delta = 0$

only solution is $\Delta_1 = \Delta_2 = \Delta_3 = 0$ if $k_2 \neq 0$

$$[M]\ddot{\bar{x}} + [K]\bar{x} = 0 \qquad x = X e^{i\omega t}$$

$$-\omega^2 \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \bar{X} + \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix} k/m\, \bar{X} = 0$$

$$\begin{bmatrix} -2\omega^2 + 2k/m & -4k/m \\ -4k/m & -4\omega^2 + 8k/m \end{bmatrix} \bar{X} = 0$$

$$(-2\omega^2 + 2k/m)(-4\omega^2 + 8k/m) - 16\,k/m = 0$$

$$8\omega^4 - 16\omega^2 k/m - 8\omega^2 k/m + 16k/m - 16k/m = 0$$

$$8\omega^4 - 24\omega^2 k/m = 0$$

$$\omega^2 (8\omega^2 - 24k/m) = 0 \qquad \omega = 0$$

$$8\omega^2 = 24k/m$$

$$\omega = \sqrt{3}\,k/m$$

~ only keep positive ω

$$(-2\omega^2 + 2k/m)X_1 = +4k/m\, X_2$$

$$\frac{X_2}{X_1} = \frac{4k/m}{-2\omega^2 + 2k/m}$$

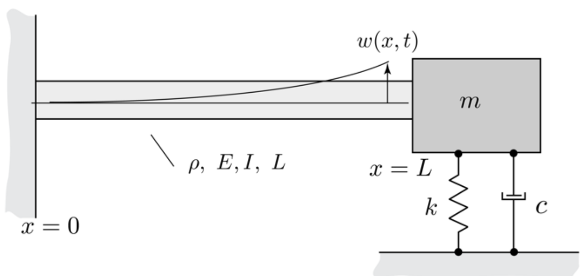
@ $\omega = 0$ $\quad \frac{X_2}{X_1} = 1/2 \qquad \bar{X} = \begin{Bmatrix} 1 \\ 1/2 \end{Bmatrix}$

$$\frac{X_2}{X_1} = \frac{4k/m}{-2(3)k/m + 2k/m} = \frac{4}{-4} = -1 \qquad \bar{X} = \begin{Bmatrix} -1 \\ +1 \end{Bmatrix}$$

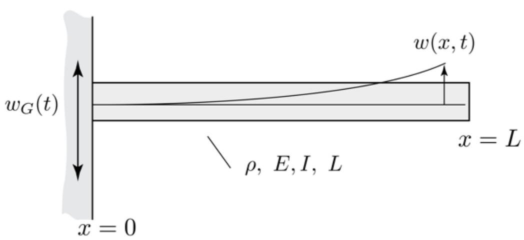
ME 563 - Fall 2024

Test Problem 3 - 40 points ~ In person and Online

The transverse motion of the beam shown above is to be described by $w(x,t)$.



System 1



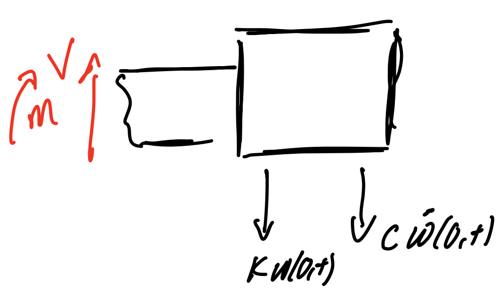
System 2

1. Write down/derive the set of boundary conditions for System 1.

2. In System 2 the beam is subject to base motion at its fixed end. The base motion is purely temporal $w_G(t)$. The total deflection of the beam can be written as $w_T(x, t) = w_G(t) + w(x,t)$, where $w(x,t)$ is the deflection relative to $w_G(t)$.

   a) Derive the governing equation and boundary conditions for the system.
   b) Discuss how the base excitation changes your system.

① $w(0,t) = 0 \qquad \frac{\partial w}{\partial x}(0,t) = 0$



$+\uparrow \sum F_y: V - kw(0,t) - c\dot{w}(0,t) = m\frac{\partial^2 w}{\partial t^2}(0,t)$

$EI\frac{\partial w^3}{\partial x^3}(0,t) = m\frac{\partial^2 w}{\partial t^2} + kw(0,t) + c\dot{w}(0,t)$

$+\curvearrowleft \sum M: \quad M = 0 \rightarrow EI\dfrac{\partial^2 w}{\partial x^2}(0,t) = 0$

(2)

(a) $EI\dfrac{\partial^4 w}{\partial x^4} + \rho A\dfrac{\partial^2 w}{\partial t^2} = 0$ for our system

$$EI\frac{\partial^4 w_T}{\partial x^4} + \rho A\frac{\partial^2 w_T}{\partial t^2} = 0 \qquad w_T(x,t) = w(x,t) + w_b(t)$$

$$\frac{\partial^4 w_T}{\partial x^4} = \frac{\partial^4 w}{\partial x^4} \quad \text{and} \quad \frac{\partial^2 w_T}{\partial t^2} = \frac{\partial^2 w}{\partial t^2} + \ddot{w}_b$$

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = -\rho A\ddot{w}_b$$

only a function of time

$w_T(x,t) = w_b(t) + w(x,t)$

at $x = 0 \rightarrow w_T(0,t) = w_b(t) \rightarrow w(0,t) = 0$

$\partial w_T(x,t) = \partial w(x,t)$ at $x = 0 \rightarrow \partial w(0,t) = 0$

same as if the wall was not moving

at $x = L$

$EI\dfrac{\partial^2 w_T(L,t)}{\partial x^2} = EI\dfrac{\partial^2 w(L,t)}{\partial x^2} = 0$ ~ No moment

$EI\dfrac{\partial^3 w_T(L,t)}{\partial x^3} = EI\dfrac{\partial^3 w(L,t)}{\partial x^3} = 0$ ~ No shear

The base excitation acts a forcing function and doesn't change the modal properties of the system.

Newton Euler

$$\sum \vec{F} = m\vec{a}_g$$
$$\sum \vec{M}_A = I_A \vec{\alpha}$$

Where A is a fixed point or center of gravity

SDOF Response

$$m\ddot{x} + c\dot{x} + kx = 0,$$
$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0,$$
$$2\zeta\omega_n = \frac{c}{m},\ \omega_n^2 = \frac{k}{m}$$

$$0 \leq \zeta \leq 1,$$
$$x(t) = \exp -\zeta\omega_n t\,(C\cos\omega_d t + S\sin\omega_d t)$$

Power Equation

$$T + U = T_o + U_o + W^{(nc)}$$
$$Power = \frac{dW^{(}nc)}{dt} = \frac{dT}{dt} + \frac{dU}{dt}$$

Lagrange's Equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial U}{\partial q_i} = Q_i$$

Eigenvalue Problem

$$[M]\ddot{\vec{x}} + [K]\vec{x} = \vec{0},$$
$$\vec{x}(t) = \vec{X}\exp i\omega t,$$
$$\left(-\omega^2[M] + [K]\right)\vec{X} = \vec{0}$$

MDOF Response

$$\vec{x}(t) = \sum_{j=1}^{N} \vec{X}^{(}j)[c_j\cos\omega_j t + s_j\sin\omega_j t]$$
$$c_j = \frac{\vec{X}^{(j)T}[M]\vec{x}(0)}{\vec{X}^{(j)T}[M]\vec{X}^{(j)}}$$
$$sj = \frac{\vec{X}^{(j)T}[M]\dot{\vec{x}}(0)}{\omega_j\vec{X}^{(j)T}[M]\vec{X}^{(j)}}$$

Linearized Lagrange's Equations

$$[M]\ddot{\overrightarrow{z}} + [C]\dot{\overrightarrow{z}} + [K]\overrightarrow{z} = \overrightarrow{0}$$

$$\overrightarrow{z}(t) = \overrightarrow{q}(t) - \overrightarrow{q}_0$$
$$M_{ik} = (m_{ik})_{\overrightarrow{q}_0} = M_{ki}$$
$$C_{ik} = \left(\frac{\partial^2 R}{\partial \dot{q}_i \partial \dot{q}_k}\right)_{\overrightarrow{q}_0} = C_{ki}$$
$$K_{ik} = \left(\frac{\partial^2 U}{\partial q_i \partial q_k}\right)_{\overrightarrow{q}_0} = K_{ki}$$

Mass Normalized Eigenvectors

$$\alpha_j = \frac{1}{\sqrt{\vec{X}^{(j)T}[M]\vec{X}^{j}}}$$
$$\vec{X}_m^j = \alpha_j\vec{X}^j$$
$$c_j = \vec{X}_m^{(j)T}[M]\vec{x}(0)$$
$$s_j = \frac{\vec{X}_m^{(j)T}[M]\dot{\vec{x}}(0)}{\omega_j}$$

Orthogonality of Mass Normalized Eigenvectors

$$\vec{X}_m^{(i)T}[M]\vec{X}_m^j = \delta_{ij}$$

Log Decrement SDOF

$$\delta = \ln\left(\frac{x_j}{x_{j+1}}\right)$$
$$\zeta = \frac{\delta/2\pi}{\sqrt{1 + (\delta/2\pi)^2}}$$
$$\zeta << 1 \rightarrow \zeta = \frac{\delta}{2\pi}$$

Second-Order Continuous Systems

$$\frac{\partial}{\partial x}\left(p(x)\frac{\partial u}{\partial x}\right) + f(x,t) = r(x)\frac{\partial^2 u}{\partial t^2}$$

Solution form

$$u(x,t) = \sum U_n(x)(C_n \cos\omega_n t + S_n \sin\omega_n t)$$

$$C_n = \frac{\int_0^L r(x)U_n(x)u(x,0)dx}{\int_0^L r(x)U_n^2(x)dx}$$

$$S_n = \frac{1}{\omega_n}\frac{\int_0^L r(x)U_n(x)\frac{\partial u}{\partial t}(x,0)dx}{\int_0^L r(x)U_n^2(x)dx}$$

Fourth-Order Continuous Systems

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 w}{\partial x^2}\right) + \rho(x)A(x)\frac{\partial^2 w}{\partial t^2} = 0$$

Solution form

$$w(x,t) = \sum W_n(x)(C_n \cos\omega_n t + S_n \sin\omega_n t)$$

$$C_n = \frac{\int_0^L \rho(x)A(x)W_n(x)w(x,0)dx}{\int_0^L \rho(x)A(x)W_n^2(x)dx}$$

$$S_n = \frac{1}{\omega_n}\frac{\int_0^L \rho(x)A(x)W_n(x)\frac{\partial w}{\partial t}(x,0)dx}{\int_0^L \rho(x)A(x)W_n^2(x)dx}$$

Properties of Even and Odd functions

The product of two even functions is an even function.
The product of two odd functions is an odd function.
The product of an even and an odd function is an odd function

Test 2

***Name*___________________________________**

***Pledge*_________________________________**

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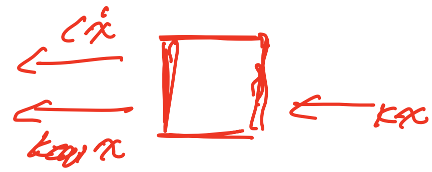
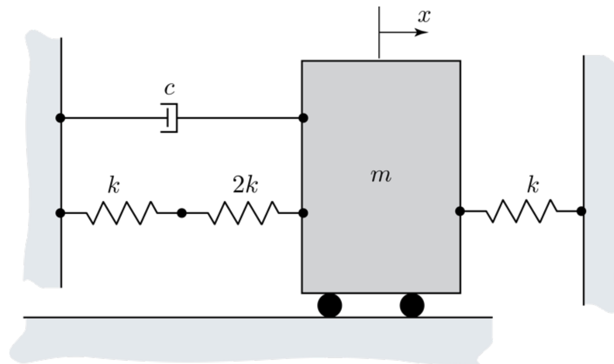


ME 563 - Fall 2024

~ Alternate

Test Problem 1 -30 points

The single-DOF system shown below is made up of a particle of mass m, two springs (having stiffnesses of k and $2k$) and a dashpot with a damping coefficient c. Let x represent the motion of the particle measure from a position at which the springs are unstretched. If m = 100 kg, k = 1350 N/m and c = 40 kg/sec:



a) What is the *undamped natural frequency* ω_n?

$$k_{eq1} = \frac{k(2k)}{k+2k} = \frac{2}{3}k \qquad k_{eq2} = k + k_{eq1} = \frac{5}{3}k$$

$$\omega_n = \sqrt{\tfrac{5}{3}k/m} = \sqrt{\tfrac{5}{3}\,\tfrac{1350}{100}} = 4.74 \text{ rad/s}$$

b) What is the *damping ratio* ξ?

$$2\zeta\omega_n = c/m \qquad \zeta = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{\tfrac{5}{3}mk}} = \frac{40}{2\sqrt{\tfrac{5}{3}\cdot 100\cdot 1350}} = 0.0422$$

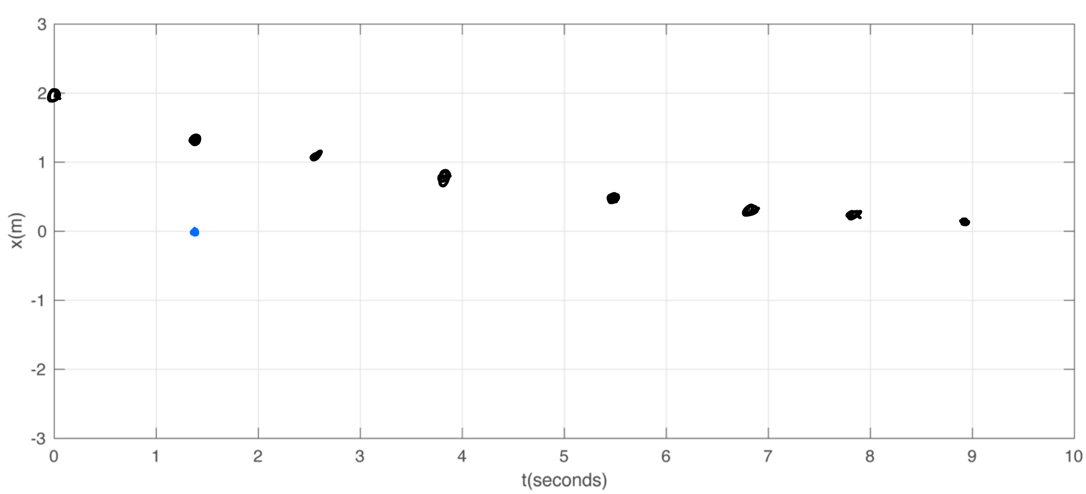
c) What is the *damped natural frequency* ω_d?

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4.74\sqrt{1-0.0422^2} \approx 4.7392 \simeq 4.74$$

$$T_d = 2\pi/\omega_d = 1.3258$$

d) Given an initial condition $x(0)$=2 m, $v(0)$=0 m/s, Sketch the response on the axis below.

n	nT	$x(T)$
1	1.33	1.53
2	2.66	1.18
3	3.98	0.90
4	5.30	0.70
5	6.63	0.53



$$x(t) = e^{-\zeta\omega_n t} x_0 \cos\omega_d t$$

ME 563 - Fall 2024

Test Problem 2 -30 points ~ Alternate and Online Test

Please answer the following

1. A 3 DOF system has stiffness matrix

$$K = \begin{bmatrix} k_1 & -2k_1 & k1 \\ -k_1 & k_1 + 3k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$$

   Can the system have a rigid body mode? Justify your answer

2. A lumped 2DOF models has the following stiffness and mass matrices

$$K = k \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}, \quad M = m \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

   a) Determine the natural frequencies of the model
   b) Determine the mode shapes of the system

① if $\det(K) = 0$ or $K\Delta = 0$ ~ harder to show

either one will work

$$\Delta K = k_1 \begin{vmatrix} k_1 + 3k_2 & -k_2 \\ -k_2 & k_2 \end{vmatrix} + k_1 \begin{vmatrix} -2k_1 & k_1 \\ -k_2 & k_2 \end{vmatrix} + 0$$

$$\Delta K = k_1 (k_1 k_2 + 3k_2^2 - k_2^2) + k_1 (-2k_1 k_2 + k_1 k_2)$$

$$= k_1^2 k_2 + 3k_1 k_2^2 - k_1 k_2^2 - 2k_1^2 k_2 + k_1^2 k_2$$

$$= \underbrace{k_1^2 k_2 - 2k_1^2 k_2 + k_1^2 k_2}_{0} + \underbrace{3k_1 k_2^2 - k_1 k_2^2}_{2k_1 k_2^2} = 2k_1 k_2^2$$

No rigid body modes

Option 2

$$[k]\Delta = \left\{ \begin{array}{l} k_1\Delta_1 - 2k_1\Delta_2 + k_1\Delta_3 \\ -k_1\Delta_1 + (k_1+3k_2)\Delta_2 - k_2\Delta_3 \\ -k_2\Delta_2 + k_2\Delta_3 \end{array} \right\} = 0$$

$\Delta_2 = \Delta_3$ last equation

$k_1\Delta_1 - 2k_1\Delta_2 + k_1\Delta_2 = 0$ 1st equation

$\Delta_1 - \Delta_2 = 0 \longrightarrow \Delta_1 = \Delta_2$

$$-k_1\Delta_1 + (k_1+3k_2)\Delta_2 - k_2\Delta_3 = -k_1\Delta_1 + (k_1+3k_2)\Delta_1 - k_2\Delta_1 = 0$$
$$= 2k_2\Delta = 0$$

only solution is $\Delta_1 = \Delta_2 = \Delta_3 = 0$ if $k_2 \neq 0$

$[m]\ddot{\bar{x}} + [k]\bar{x} = 0 \qquad x = X e^{i\omega t}$

$$-\omega^2 \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \bar{\bar{X}} + \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix} k/m \; \bar{X} = 0$$

$$\begin{bmatrix} -2\omega^2 + 2k/m & -4k/m - \omega^2 \\ -4k/m - \omega^2 & -4\omega^2 + 8k/m \end{bmatrix} \bar{X} = 0$$

$7m^2\omega^4 - 32km\omega^2 = 0$

$\omega^2 (7m^2\omega^2 - 32k) = 0$

$\omega = 0 \qquad \omega = \sqrt{32/7 \, k/m}$ only keep +ve roots

$(+2\omega^2 + 2k/m) X_1 = +4k/m \, X_2$

$\frac{X_2}{X_1} = \frac{4k/m}{-2\omega^2 + 2k/m}$ @ $\omega = 0 \quad \frac{X_2}{X_1} = 1/2 \quad \bar{X} = \begin{Bmatrix} 1 \\ 1/2 \end{Bmatrix}$

$\frac{X_2}{X_1} = \frac{4k/m}{-2(3)k/m + 2k/m} = \frac{4}{-4} = -1 \quad \bar{X} = \begin{Bmatrix} -1 \\ +1 \end{Bmatrix}$