ME 563-Fall 2021

Homework No. 3

Due: October 12, 2020 11:59 pm on Gradescope

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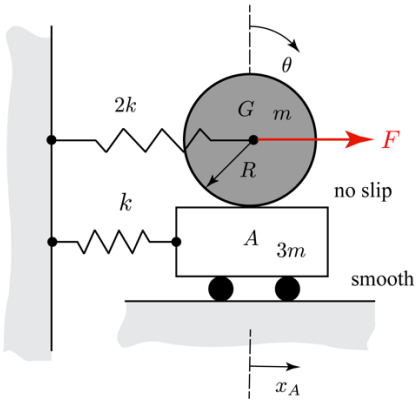
Homework Problem 3.1

A disk of mass m and radius r rolls without slip on a cart of mass 3m. A force F is applied to the disk. The disk has an angular displacement θ. Let x, describe the absolute motion of the cart. The springs is unstretched when x_A=0. Assume the surface between the cart and ground is smooth.

a) Use a *Newton Euler formulation* to find the equations of motion. Your final equations should not include any forces of reaction. The final differential equation are in terms of x_A and θ and should be in matrix form.

b) Use *Lagrange's Equations* to find the equations of motion. The generalize coordinates are x_A and θ. The final differential equation are in terms of x_A and θ and should be in matrix form.

c) Compare the form of the mass matrix, stiffness matrix and forcing vector of the *Newtonian* formulation with the *Lagrangian* formulation. In particular, are they the same? If not, in which formulation are the matrices symmetric ? Do the forcing vectors differ? Explain any similarities or differences.



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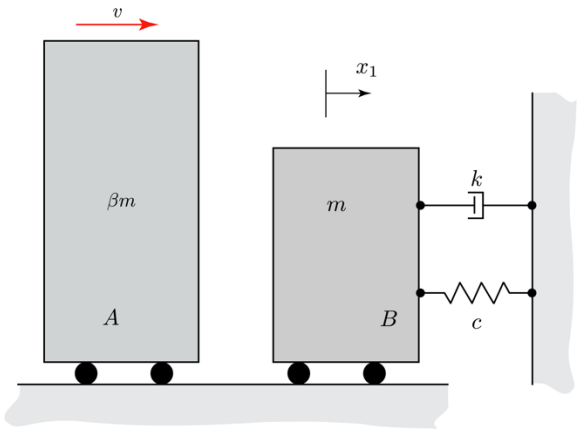
Homework Problem 3.2

Block A is initially moving to the right on a smooth, horizontal surface with a speed of v, with B at rest and the spring unstretched. Upon impact, at time $t = 0$, block A sticks to block B. For this problem use: v = 10m/ sec , m = 10 kg , c = 60 kg / sec and k = 300 N / m.



a) Derive the differential equation of motion (EOM) for the system corresponding to t > 0.
b) Determine the numerical values for the undamped natural frequency and the damping ratio of the system corresponding to the EOM found in a) when β=1. How does the value change when β>1 and β<1.
c) Determine the response of the system for t > 0 . HINT: You can use conservation of momentum to determine the speed of blocks A and B immediately after sticking when β=1. How does the value change when β>1 and β<1.
d) What is the maximum displacement of blocks A and B in the response found in c)? when β=1. How does the value change when β>1 and β<1.

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Homework Problem 3.3

a) Show that the logarithmic decrement is equal to

$$\delta = \frac{1}{n} \ln \frac{x_0}{x_n}$$

where x_n is the amplitude of vibration after n cycles have elapsed.

b) Show that by calculation that

$$A \sin(\omega_n t + \phi)$$

can be represented as

$$B \sin(\omega_n t) + C \cos(\omega_n t)$$

where B and C are functions of A and ϕ.

c) Solve

$$\ddot{x} - \dot{x} + x = 0$$

with initial conditions $x(0)=1$, and $v(0)=0$ for $x(t)$ and sketch the time waveform.

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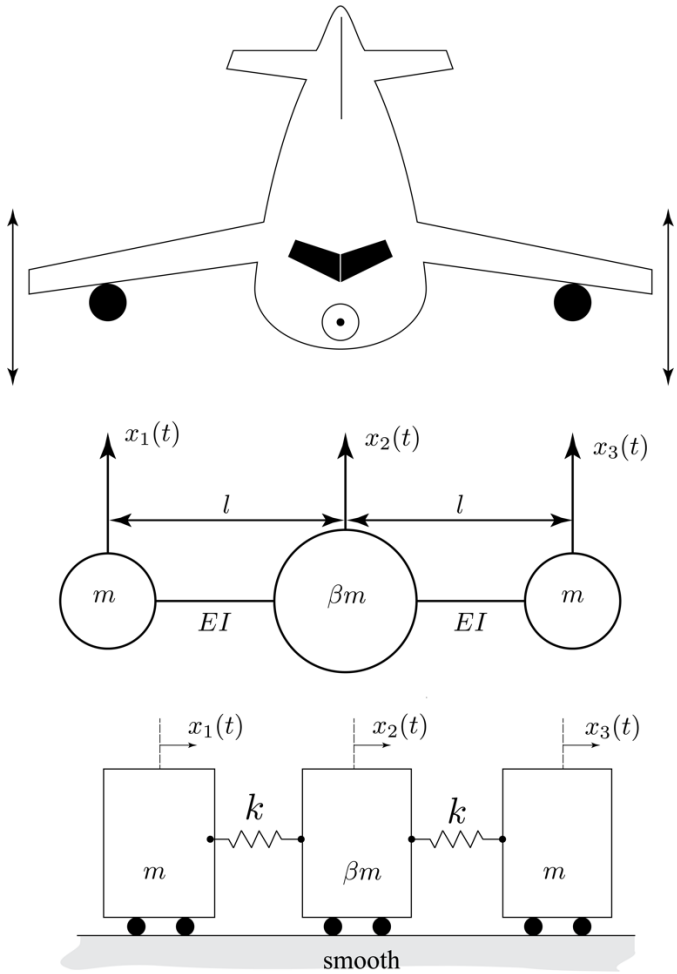
Homework Problem 3.4

The vibration in the vertical direction of an airplane and its wings can modeled as a three-degree-of-freedom system with one mass corresponding to the right wing (m), one mass for the left wing (m), an one mass for the fuselage (βm), where β is a constant greater than one. The stiffness connecting the three masses correspond to that of the wing and is a function of the modulus E of the wing. The generalized coordinates are the absolute positions $x_1(t)$, $x_2(t)$, and $x_3(t)$. The stiffness is $k = 3EI/l^3$ where E is the modulus of the wing and l is the length from the main body to fuselage. The figures show the wing vibration, the lumped mass/beam deflection model and the spring mass model.



a) Use *Lagrange's Equations* to find the equations of motion.
b) Calculate the natural frequencies an mode shapes of the system. One natural frequency is zero discuss the significance of this result.
c) Calculate the total time response of the system and find the initial conditions on displacement such that 1) the system only responds at the second mode of vibration, and 2) the system only responds at the third mode of vibration.