

**ME563 – Fall 2014
Purdue University
West Lafayette, IN**

Final Examination

- You have two hours (120 minutes) to complete the exam.
- You are allowed to use your ME 563 lecture book during the exam. No other materials may be accessed during the exam.
- Calculators may be used during the exam. However, no cell phones, tablets or computers may be used.
- Please do NOT write on the back of the exam papers.

Name _____

PUID _____

Problem 1 (25 pts) _____

Problem 2 (15 pts) _____

Problem 3 (15 pts) _____

Problem 4 (15 pts) _____

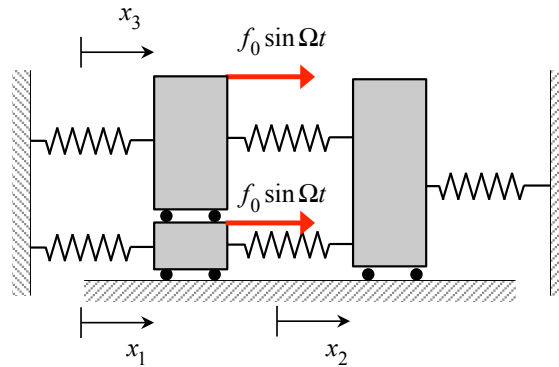
Problem 5 (15 pts) _____

Problem 6 (15 pts) _____

TOTAL _____

ME 563 – Fall 2014
Final Examination
Problem 1 – 25 pts

Name _____



The three-DOF system shown above has the following natural frequencies and mass-normalized modal vectors:

$$\omega_{1,2,3} = 1, 2, 5 \text{ rad/sec}$$

$$\hat{\mathbf{X}}^{(1)} = \begin{Bmatrix} 1 \\ 2 \\ 4 \end{Bmatrix} \quad \hat{\mathbf{X}}^{(2)} = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} \quad \hat{\mathbf{X}}^{(3)} = \begin{Bmatrix} 1 \\ -1 \\ 2 \end{Bmatrix}$$

- a) If the particular solution of the EOMs for this system is written as $\bar{\mathbf{x}}_p(t) = \bar{\mathbf{A}} \sin \Omega t$, determine $\bar{\mathbf{A}}$ as a function of Ω .
- b) Make sketches of $|A_j|$ vs. Ω on the plot axes provided below. Clearly indicate all resonances and anti-resonances in your sketches.



ME 563 – Fall 2014
Final Examination
Problem 1 (cont.)

Name _____

ME 563 – Fall 2014
Final Examination
Problem 1 (cont.)

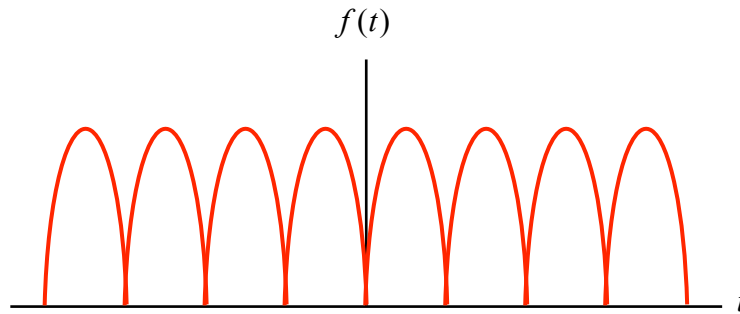
Name _____

ME 563 – Fall 2014
Final Examination
Problem 2 – 15 pts

Name _____

Consider the function $f(t) = 3|\sin 4t|$ shown below. Suppose that the Fourier series for

this function is written as: $f(t) = f_0 + \sum_{k=1}^{\infty} [f_{ck} \cos k\Omega t + f_{sk} \sin k\Omega t]$.



- Determine the fundamental frequency Ω for this periodic function.
- Set up the integral expressions for the Fourier coefficients f_0 , f_{ck} and f_{sk} . Do NOT evaluate these integrals.
- Which, if any, of the Fourier coefficients written down above in b) are zero?

**ME 563 – Fall 2014
Final Examination
Problem 2 (cont.)**

Name _____

ME 563 – Fall 2014
Final Examination
Problem 3 – 15 pts

Name _____

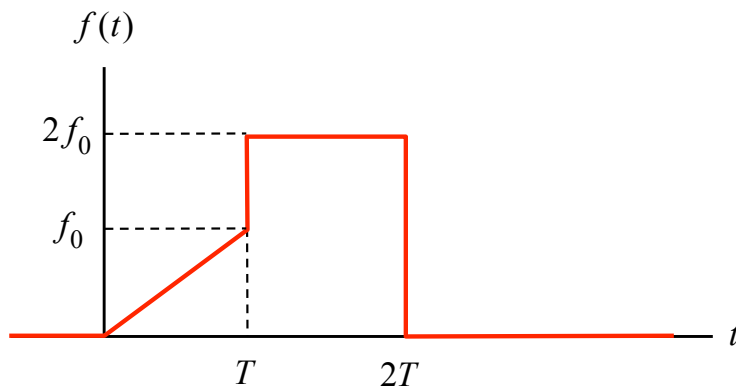
An undamped single-DOF system governed by the following EOM:

$$m\ddot{x} + kx = f(t)$$

has an excitation of $f(t)$ shown below. The system is given the initial conditions of: $x(0) = \dot{x}(0) = 0$. Set up the convolution integral solution for the response of this system. Clearly indicate the solutions that are valid for the time ranges of:

- $0 \leq t \leq T$
- $T \leq t \leq 2T$
- $t \geq 2T$

Do NOT evaluate the integrals above.



**ME 563 – Fall 2014
Final Examination
Problem 3 (cont.)**

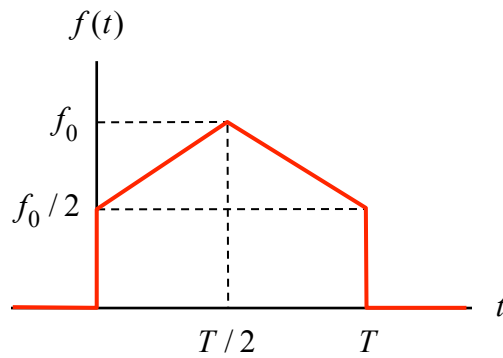
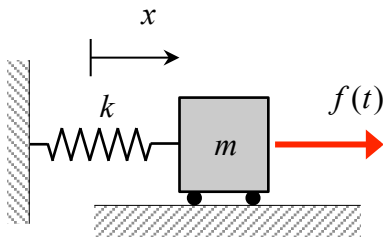
Name _____

ME 563 – Fall 2014
Final Examination
Problem 4 – 15 pts

Name _____

Forcing $f(t)$ acts on an undamped single-DOF system, with $f(t)$ as shown below. Use the following parameters: $m = 100\text{kg}$, $k = 10,000\text{N}$, $f_0 = 120\text{N}$ and $T = 0.2\text{sec}$.

- Treating $f(t)$ as a SHOCK loading, determine upper AND lower bounds on the value of x_{\max} .
- Treating $f(t)$ as an IMPACT loading, determine an upper bound on the value of x_{\max} .
- Does the system serve as an effective shock isolator?



ME 563 – Fall 2014
Final Examination
Problem 5 – 15 pts

Name _____

A five-DOF system has the following mass and flexibility matrices:

$$[M] = \begin{bmatrix} 10 & & & & \\ & 10 & & & \\ & & 10 & & \\ & & & 10 & \\ & & & & 10 \end{bmatrix} kg \quad [A] = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 4 & 4 & 4 \\ 2 & 4 & 6 & 6 & 6 \\ 2 & 4 & 6 & 8 & 8 \\ 2 & 4 & 6 & 8 & 9 \end{bmatrix} \times 10^{-4} \frac{m}{N}$$

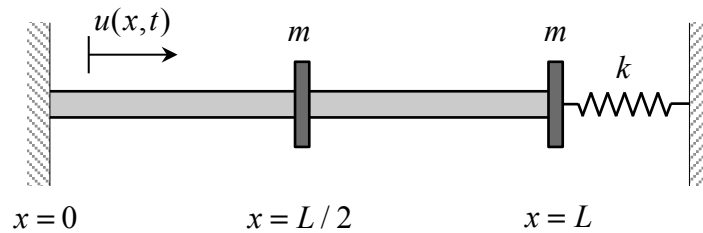
Determine an upper bound and a lower bound on the lowest natural frequency for the system. Do NOT perform any matrix inversions in your analysis.

ME 563 – Fall 2014
Final Examination
Problem 6 – 15 pts

Name _____

A rod of length L , Young's modulus E , cross-sectional area A and mass density ρ is attached to a fixed wall at its left end ($x = 0$). A rigid block of mass m and a spring of stiffness k are attached at the right end ($x = L$). A second rigid block, also of mass m , is attached to the rod at midlength ($x = L/2$). Use $m = \rho AL$ and $kL / EA = 1$.

- Write down the Rayleigh quotient for this rod system.
- Choose an admissible trial function $v(x)$ and calculate an upper bound for the lowest natural frequency for the rod system. Feel free to use any integration results from lecture examples in your work here.



ME 563 – Fall 2014
Final Examination
Problem 6 (cont)

Name _____