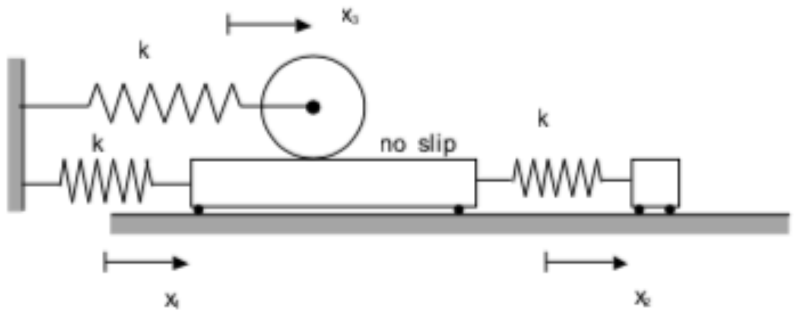
ME 563 - Fall 2002 **Name**____________________________

Midterm Exam

Problem No. 1 – 30 points

Use the influence coefficient method to determine the flexibility matrix for the three-DOF system shown below.



ME 563 - Fall 2002 **Name**____________________________

Midterm Exam

Problem No. 2 – 30 points

A two-DOF system has the following EOM's:

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Determine the natural frequencies and mode shapes for this system.

ME 563 - Fall 2002 **Name______________________________**
Midterm Exam
Problem No. 3 – 40 points (total)

Part (a) – 10 points

A three-DOF system having EOM's of:

$$[M]\ddot{\underline{x}} + [K]\underline{x} = \underline{0}$$

and a mass matrix of:

$$[M] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

has a mass-normalized second modal vector of:

$$\underline{\tilde{\phi}}^{(2)} = \frac{1}{\sqrt{11}}\begin{Bmatrix} 1 \\ 2 \\ -1 \end{Bmatrix}$$

If the system is released from rest ($\dot{\underline{x}}(0) = \underline{0}$), find a set of non-zero initial conditions $\underline{x}(0)$ for which the second mode does NOT contribute to the response of the system.

Part (c) – 10 points

The characteristic equation for a rod problem is known to be:

$$\cot(\beta L) = -0.5\,\beta L$$

where $\beta = \omega\sqrt{\frac{\rho}{E}}$. Find a set of upper and lower bounds on the *second natural frequency* for the rod. Express your answer in terms of $\sqrt{\frac{E}{\rho L^2}}$. HINT: Make a sketch of the above characteristic function in terms of βL.

Part (d) – 5 points

A five-DOF system has the following characteristic equation:

$$a_5\omega^{10} + a_4\omega^8 + a_3\omega^6 + a_2\omega^4 = 0$$

Show that the system has at least two "zero" natural frequencies.

Part (e) – 5 points

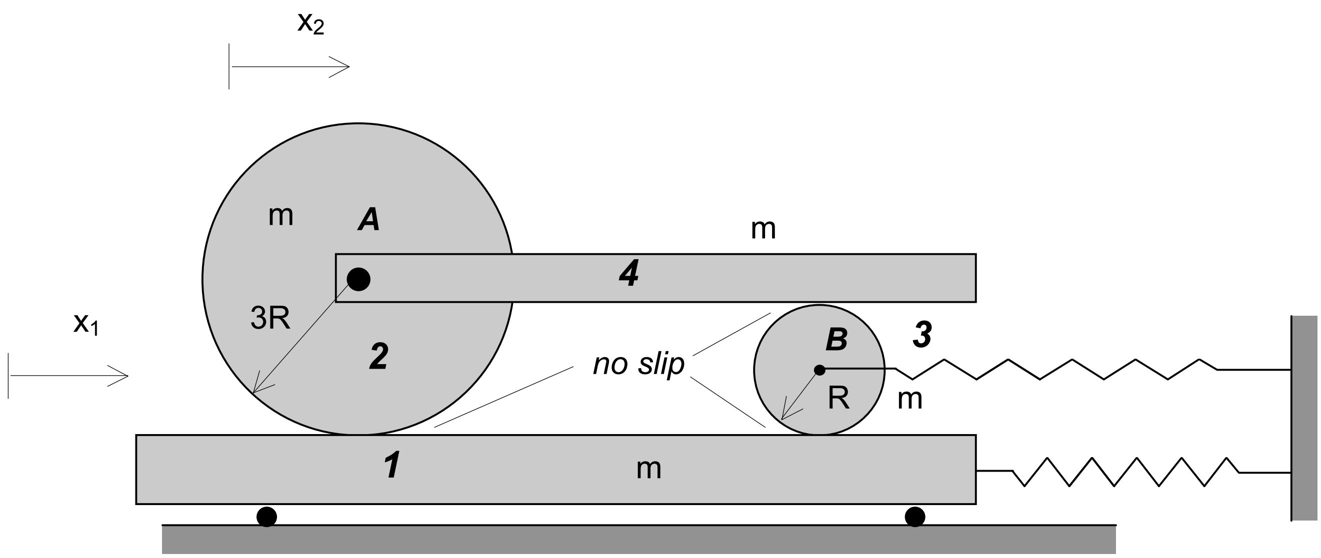
Explain in words what restrictions were placed on the mass [M] and stiffness [K] matrices in order to prove that the modal vectors are orthogonal through [M] and [K].

ME563 – Fall 2006
Midterm Examination
Problem No. 1 – 25 points

A system is made up of rigid bodies 1, 2, 3 and 4, with each body having a mass of m. Bodies 2 and 3 are homogeneous disks having outer radii of 3R and R, respectively. Bodies 2 and 3 roll without slipping on body 1, and body 3 rolls without slipping on body 4. Body 4 is pinned to the center A of body 2 and remains horizontal for all motion. x_1 and x_2 are absolute coordinates that describe the motion of body 1 and point A, respectively.

a) Write down the *kinetic energy* of the system in terms of the time derivatives of the generalized coordinates x_1 and x_2 described above.

b) Based on your results in a), determine the *mass matrix* for this system corresponding to the generalized coordinates of x_1 and x_2.



ME563 – Fall 2006
Midterm Examination
Problem No. 2 – 10 points

A four-DOF system has the following stiffness matrix:

$$[K] = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & -1 & 0 \\ 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix} k$$

One of the modal vectors for this system is known to be:

$$\underline{X}^{(1)} = \begin{Bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{Bmatrix}$$

Show that $\underline{X}^{(1)}$ is a *rigid body* (zero-frequency) mode.

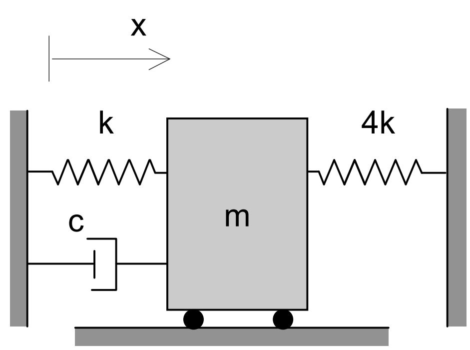
ME563 – Fall 2006
Midterm Examination
Problem No. 3 – 20 points

The single-DOF system shown below is made up of a particle of mass m, two springs (having stiffnesses of k and 4k) and a dashpot with a damping coefficient c. Let x represent the motion of the particle measure from a position at which the springs are unstretched. If m = 10 kg, k = 800 N/m and c = 40 kg/sec:

a) what is the *undamped natural frequency* ω_n?

b) what is the *damping ratio* ζ?

c) what is the *damped natural frequency* ω_d?



ME563 – Fall 2006
Midterm Examination
Problem No. 4 – 25 points

A two-DOF is known to have a mass matrix [M], natural frequencies ω_1 and ω_2, and modal vectors $\underline{X}^{(1)}$ and $\underline{X}^{(2)}$ where:

$$[M] = \begin{bmatrix} 1 & \\ & 2 \end{bmatrix} kg$$

$$\omega_1 = 10\ rad/sec \qquad \omega_2 = 15\ rad/sec$$

$$\underline{X}^{(1)} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} \qquad \underline{X}^{(2)} = \begin{Bmatrix} 1 \\ -0.25 \end{Bmatrix}$$

where coordinates x_1 and x_2 are used to describe the motion. *Determine the free response of the system* corresponding to initial conditions of:

$$\underline{x}(0) = \begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 0.2 \\ 0 \end{Bmatrix} meters$$

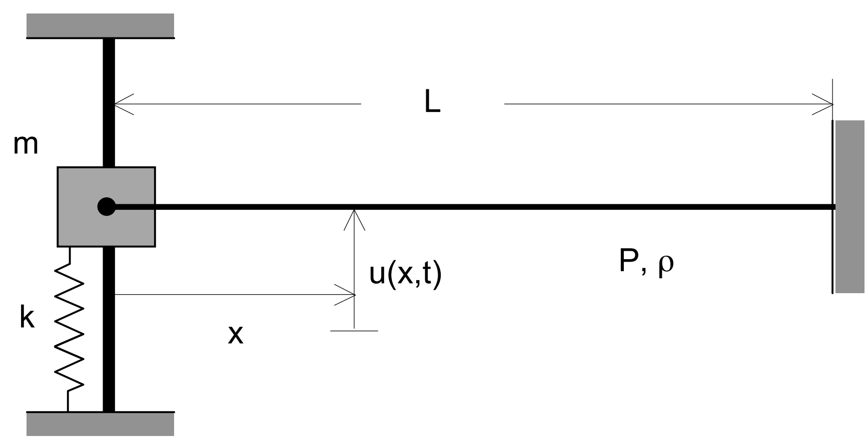
$$\underline{\dot{x}}(0) = \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix} = \begin{Bmatrix} 2 \\ 0 \end{Bmatrix} meters/sec$$

ME563 – Fall 2006
Midterm Examination
Problem No. 5 – 20 points

A string having a tension of P and mass/length ρ is stretched between two supports. The support on the left end (x = 0) is made up of a particle of mass m that is constrained to slide on a smooth vertical guide with a spring of stiffness k connecting the particle to ground. The support on the right end (x = L) is fixed.

Derive the characteristic equation for this system. For each natural boundary condition, an accurate free body diagram must be provided and used in this derivation. Ignore gravity in your analysis.

Use $m / \rho L = 1$ and $kL / P = 1$.



ME 563 - Fall 2004 **Name**___________________________

Midterm Exam

Problem No. 2 – 20 points

A summer intern working under your direction has found the natural frequencies and modal vectors for a three-DOF system where the mass matrix $[M]$ for the system is known to be:

$$[M] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} kg$$

In the process, he has chosen to "*stiffness normalize*" the modes such that $\underline{\hat{\phi}}^{(i)T}[K]\underline{\hat{\phi}}^{(i)} = 1 \quad (i = 1,2,3)$ where $[K]$ is the (unknown) stiffness matrix. The results of his work are:

$$\underline{\hat{\phi}}^{(1)} = \begin{Bmatrix} -2 \\ 1 \\ -2 \end{Bmatrix} \sqrt{\frac{meter}{newton}} \qquad \underline{\hat{\phi}}^{(2)} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \sqrt{\frac{meter}{newton}} \qquad \underline{\hat{\phi}}^{(3)} = \begin{Bmatrix} 1 \\ 2 \\ -1 \end{Bmatrix} \sqrt{\frac{meter}{newton}}$$

a) As his supervisor, you are checking over his work. As a result of this check, you have reason to believe that his results for the modal vector $\underline{\hat{\phi}}^{(3)}$ are incorrect. Explain why the modal vector $\underline{\hat{\phi}}^{(3)}$ cannot be correct. Support your explanation with calculations, if necessary.

b) Assuming that the modal vector $\underline{\hat{\phi}}^{(1)}$ is correct, what is the *natural frequency* corresponding to this mode?

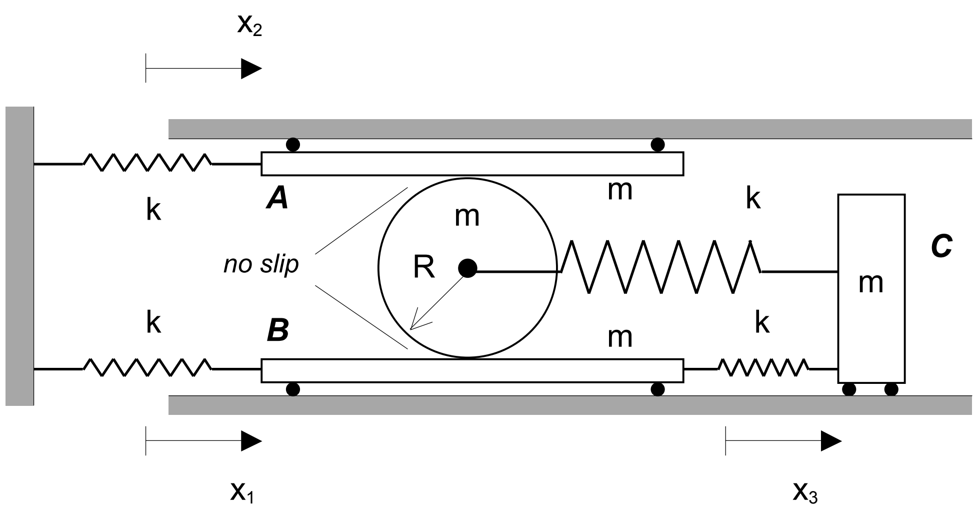
ME 563 - Fall 2004 **Name**____________________________

Midterm Exam

Problem No. 3 – 20 points

Consider the vibrational system shown below where the generalized coordinates $(q_1,q_2,q_3)=(x_1,x_2,x_3)$ are to be used to describe the motion of the system. All springs are unstretched when $x_1 = x_2 = x_3 = 0$. The disk is homogeneous ($I_G = \frac{1}{2}mR^2$) and rolls without slipping on blocks A and B.

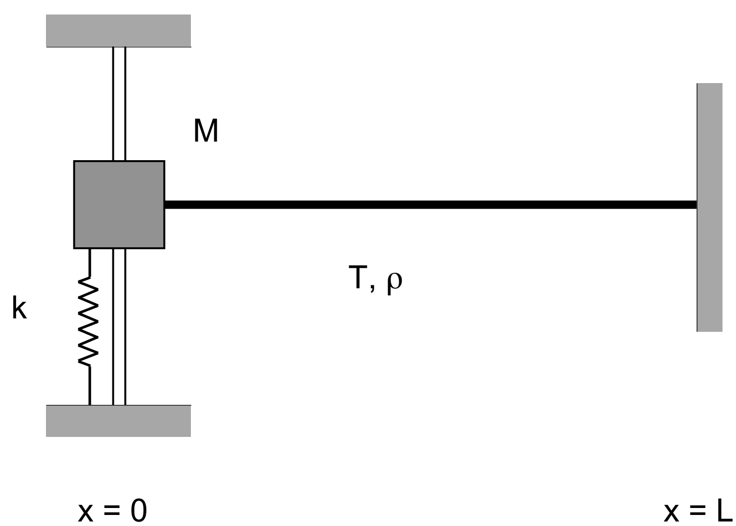
a) Write down the kinetic and potential energy expressions for the system.
b) Determine the elements M_{12} and K_{12} of the mass $[M]$ and stiffness $[K]$ matrices, respectively.



ME 563 - Fall 2004 **Name**___________________________
Midterm Exam
Problem No. 4 – 20 points

A taut string having a mass per length of ρ and tension T is stretched between two supports separated by a distance of L. On the left end support, the string is attached to a block of mass M that is free to slide on a smooth vertical guide. A spring of stiffness is attached between this block and ground. On the right end support, the string is attached to ground. Ignore the influence of gravity in your analysis of this system.

a) *Derive* the boundary condition for the left end of the string. *You must present a correct free body diagram to receive full credit for this derivation.*
b) Determine the characteristic equation that governs the natural frequencies of this system. Use $M/\rho L = 1$ and $kL/T = 2$. Your characteristic equation should be in terms of functions of βL, where $\beta = \omega\sqrt{\rho/T}$.



ME 563 - Fall 2004 **Name**____________________________
Midterm Exam
Problem No. 5 – 20 points

A two-DOF system has the equations of motion of:

$$\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k & -2k \\ -2k & 4k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where m and k have units of kg and N/m, respectively. This system is given a set of initial conditions of $x_1(0) = x_2(0) = \dot{x}_1(0) = 0$ and $\dot{x}_2(0) = v$, where v has units of m/sec.

Find the response $x_1(t)$ for $t > 0$.