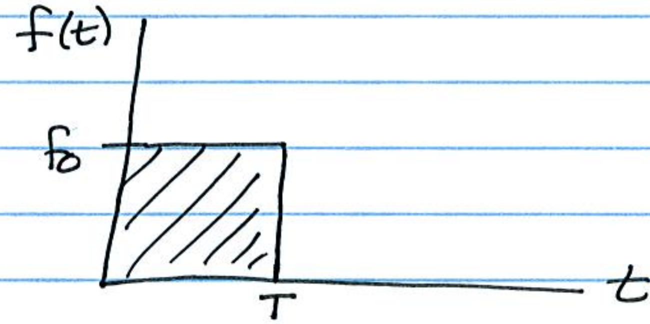
$$x(t) = \begin{cases} \frac{f_0}{k}\left[1 - \cos\omega_n t\right] & t < T \\ \frac{f_0}{k}\left[\cos\omega_n (t-T) - \cos\omega_n t\right] & t > T \end{cases}$$

where



Let $T_n = \frac{2\pi}{\omega_n} \Rightarrow \omega_n = \frac{2\pi}{T_n}$

$\therefore$

$$x(t) = \begin{cases} \frac{f_0}{k}\left[1 - \cos 2\pi \frac{t}{T_n}\right] & t < T \\ \frac{f_0}{k}\left[\cos 2\pi\left(\frac{t}{T_n} - \frac{T}{T_n}\right) - \cos 2\pi\left(\frac{t}{T_n}\right)\right] & t > T \end{cases}$$

Consider $t < T \Rightarrow \frac{t}{T_n} < \frac{T}{T_n}$

$$x(t) = \frac{f_0}{k}\left[1 - \cos 2\pi \frac{t}{T_n}\right]$$

We know x_{max} occurs at t_{max}, where

$$\dot{x}(t_{max}) = 0$$

$$\dot{x}(t) = \frac{2\pi f_0}{kT_n} \sin\left(\frac{2\pi t}{T_n}\right)$$

$$\dot{x} = 0 \Rightarrow \sin\frac{2\pi t}{T_n} = 0 \Rightarrow t_{max} = \frac{jT_n}{2}, \; j = 1, 2, \ldots$$

$$x\left(t = \frac{jT_n}{2}\right) = \frac{f_0}{K}\left[1 - \cos 2\pi \left(\frac{1}{T_n}\right)\left(\frac{jT_n}{2}\right)\right]$$

$$= \frac{f_0}{K}\left[1 - \cos j\pi\right]$$

$$= \frac{2f_0}{K} \quad \text{or } 0$$

$\therefore$ If $t < T$, $x_{max} = \frac{2f_0}{K}$

Consider $t > T$

$$x(t) = \frac{f_0}{k}\left[\cos 2\pi\left(\frac{t-T}{T_n}\right) - \cos 2\pi \frac{t}{T_n}\right]$$

$$= \frac{f_0}{k}\left(2\sin\frac{\pi T}{T_n}\right)\sin\left(\frac{2\pi t}{T_n} - \frac{\pi T}{T_n}\right)$$

$$\dot{x} = \frac{2f_0}{k}\sin\frac{\pi T}{T_n}\left(\frac{2\pi}{T_n}\right)\cos\frac{\pi(2t-T)}{T_n}$$

$$\dot{x} = 0 \Rightarrow \cos\frac{\pi(2t-T)}{T_n} = 0$$

$$\Rightarrow \frac{\cancel{\pi}(2t-T)}{T_n} = \frac{j\cancel{\pi}}{2}, \quad j = 1, 2, 3 \ldots$$

$$\Rightarrow 2t - T = \frac{jT_n}{2}$$

$$\Rightarrow t_{max} = \frac{T}{2} + \frac{jT_n}{4}$$

$$x(t_{max}) = \frac{2f_0}{k}\sin\frac{\pi T}{T_n}\sin\left[\frac{2\pi}{T_n}\left(\frac{T}{2} + \frac{jT_n}{4}\right) - \frac{\pi T}{T_n}\right]$$

$$= \frac{2f_0}{k}\sin\frac{\pi T}{T_n}\sin\left[\frac{2\pi j}{4}\right]$$

$$x_{max} = \frac{2f_0}{k}\sin\frac{\pi T}{T_n}$$

So

$$x_{max} = \begin{cases} \dfrac{2f_0}{k} & t < T \\ \dfrac{2f_0}{k} \sin \dfrac{\pi T}{T_n} & t > T \end{cases}$$

Note $x_{max,1} = x_{max,2}$ when $\frac{T}{T_n} = \frac{1}{2}$

By inspecting $x(t)$, one cansee

- If $\frac{T}{T_n} > \frac{1}{2}$, $x_{max} = \frac{2f_0}{k}$
- If $\frac{T}{T_n} < \frac{1}{2}$, $x_{max} = \frac{2f_0}{k} \sin \frac{\pi T}{T_n}$

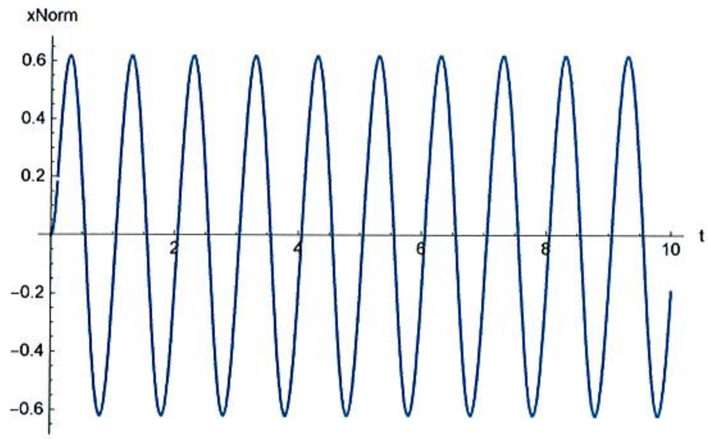
* $T = 0$ is an interesting case, as it depends a bit on what type of impulse is considered.

```
In[74]:= Tn = 1;
xNorm1 = 1 - Cos[2 * Pi * t / Tn];
xNorm2 = Cos[2 * Pi * (t - P) / Tn] - Cos[2 * Pi * t / Tn];
xNorm = Piecewise[{{xNorm1, t < P}, {xNorm2, t > P}}]
```

Out[77]= $\begin{cases} 1 - \mathrm{Cos}[2\pi t] & t < P \\ -\mathrm{Cos}[2\pi t] + \mathrm{Cos}[2\pi(-P+t)] & t > P \\ 0 & \mathrm{True} \end{cases}$

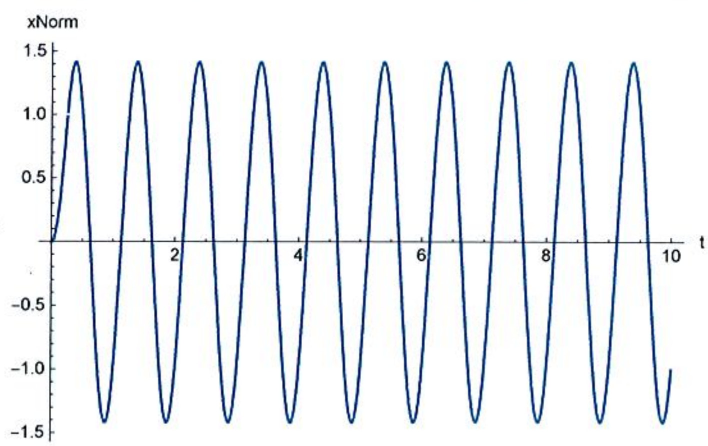
```
In[89]:= Case1 = xNorm /. P → 0.1;
Plot[Case1, {t, 0, 10}, AxesLabel → {"t", "xNorm"}]
```

Out[90]=




```
In[94]:= Case2 = xNorm /. P → 0.25;
Plot[Case2, {t, 0, 10}, AxesLabel → {"t", "xNorm"}]
```

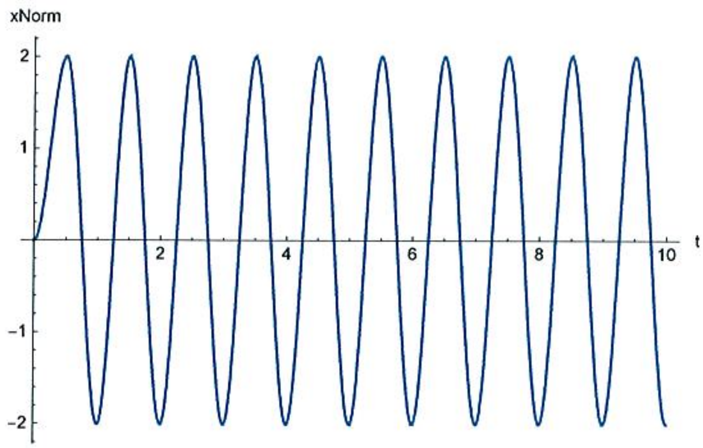
Out[95]=



6

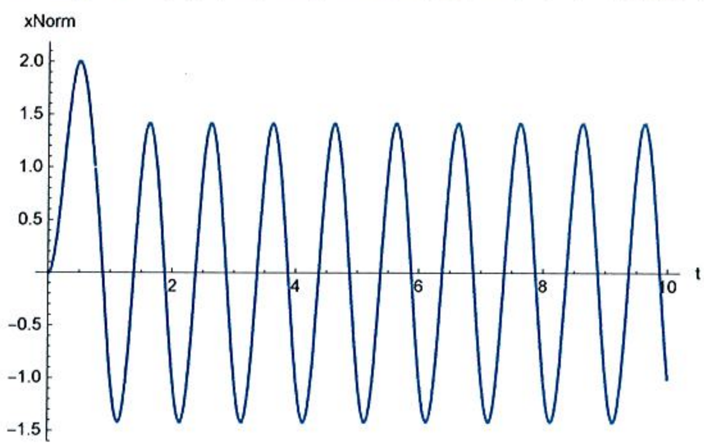
In[96]:=
```
Case3 = xNorm /. P → 0.5;
Plot[Case3, {t, 0, 10}, AxesLabel → {"t", "xNorm"}]
```

Out[97]=



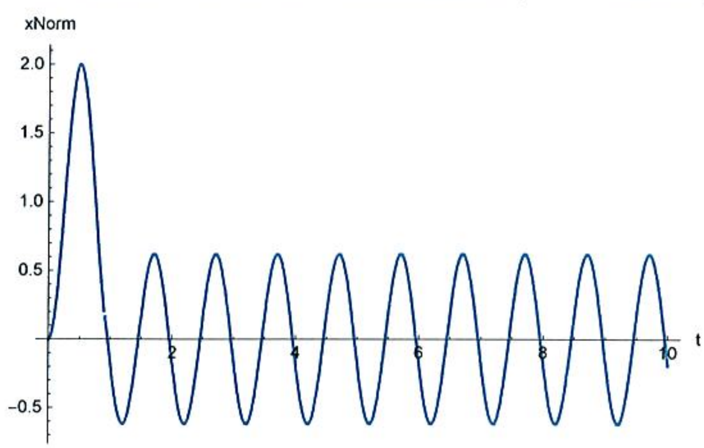
In[98]:=
```
Case4 = xNorm /. P → 0.75;
Plot[Case4, {t, 0, 10}, AxesLabel → {"t", "xNorm"}]
```

Out[99]=



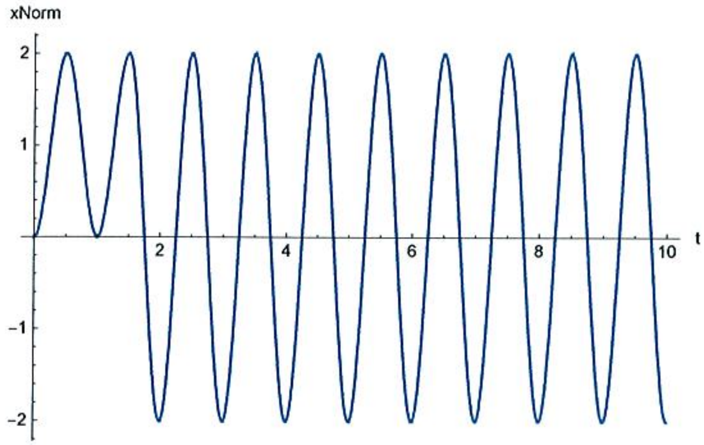
In[108]:=
```
Case5 = xNorm /. P → .9;
Plot[Case5, {t, 0, 10}, AxesLabel → {"t", "xNorm"}]
```

Out[109]=



In[132]:=
```
Case6 = xNorm /. P → 1.5;
Plot[Case6, {t, 0, 10}, AxesLabel → {"t", "xNorm"}]
```

Out[133]=



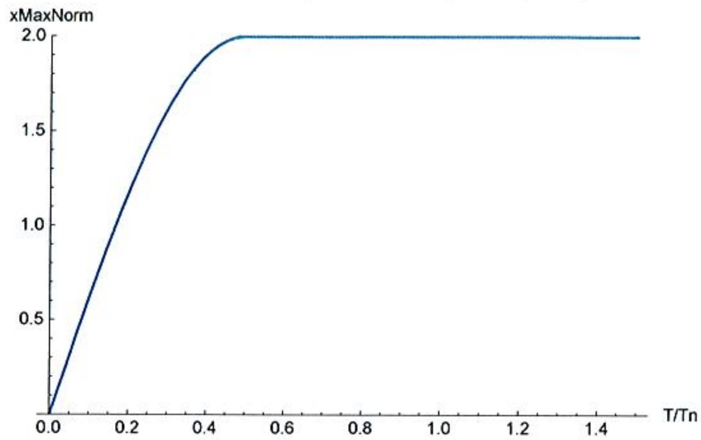
In[162]:=
```
xmaxnorm1 = 2;
xmaxnorm2 = Abs[2 * Sin[Pi * r]];
xmax = Piecewise[{{xmaxnorm1, r > 0.5}, {xmaxnorm2, r < 0.5}}]
```

Out[164]=

$$\begin{cases} 2 & r > 0.5 \\ 2\,\mathrm{Abs}[\mathrm{Sin}[\pi\, r]] & r < 0.5 \\ 0 & \mathrm{True} \end{cases}$$

In[166]:=
```
Plot[xmax, {r, 0, 1.5}, PlotRange → {0, 2}, AxesLabel → {"T/Tn", "xMaxNorm"}]
```

Out[166]=



In[161]:=
```
TrigReduce[Cos[a - b] - Cos[a]]
```

Out[161]= - Cos[a] + Cos[a - b]