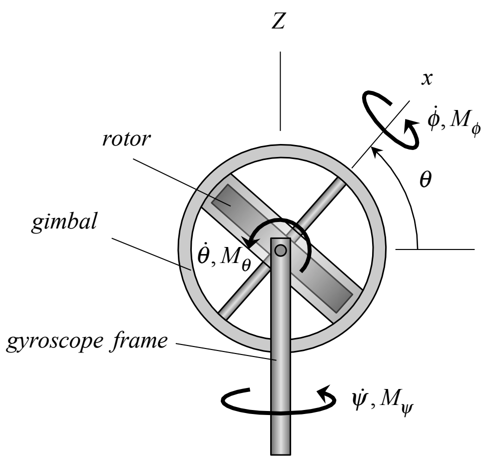
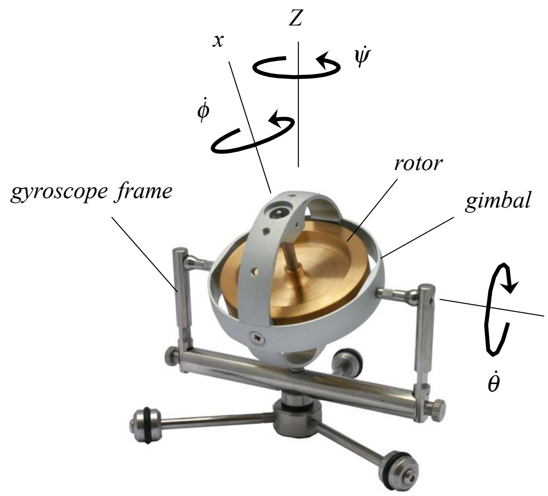
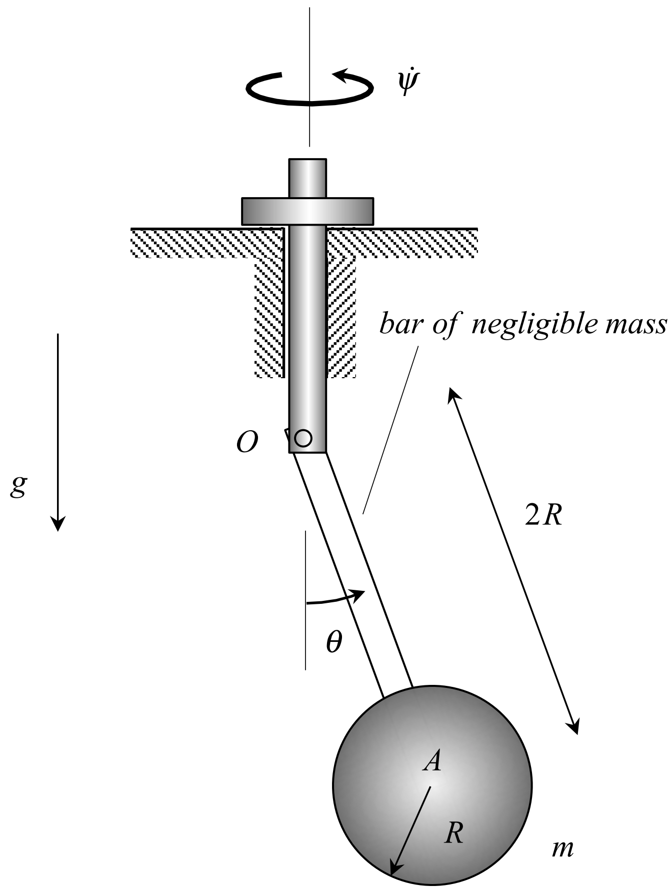
### *Problem 9.1 – 25 points*




Consider the rotor of the above gyroscope to be a thin disk of mass m and radius R. Ignore the mass of the gimbal and gyroscope frame. The center of mass of the disk lies on the fixed Z-axis. At some instant in time when the gyroscope has three non-zero rotation rates $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$, a torque of $M_\psi$ is applied to the frame. Determine $\ddot{\theta}$ as a result of this applied torque.

***Problem 9.2 – 25 points***



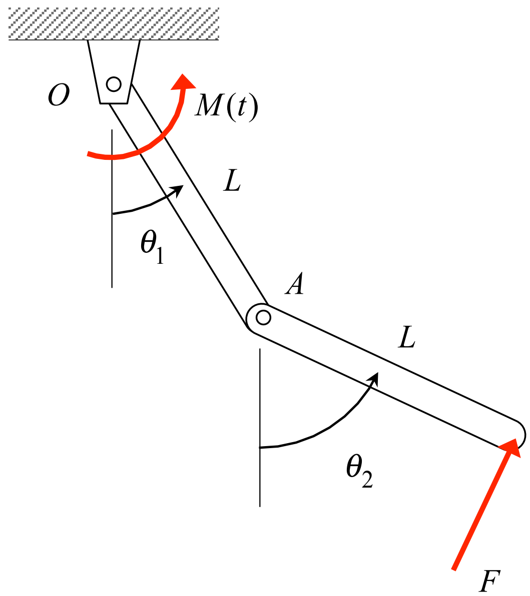

A homogeneous sphere of mass m and radius R is attached to a bar having negligible mass. This bar is pinned to a vertical shaft at end O, with the shaft rotating about its fixed axis with a constant rate of $\dot{\psi}$ . Let $\theta$ represent the angle between OA and a fixed vertical line.

a) Using Lagrange's equations, derive the dynamical equation of motion for the system using the generalized coordinate $\theta$ .

b) From the EOM in a) above, show that the system can precess at a constant rate with $\theta = constant = 0$ for all rotation rates. By considering small changes in $\theta$ from this steady value, evaluate the stability of this steady motion as a function of $\dot{\psi}$ .

c) From the EOM in a) above, show that the system can precess at a constant rate with $\theta = constant \neq 0$ for a sufficiently large rotation rate of $\dot{\psi}$ . What is the critical value of $\dot{\psi}$ for the appearance of this non-zero precession angle? By considering small changes in $\theta$ from this steady value, evaluate the stability of this steady motion when it exists.

***Problem 9.3 – 25 points***

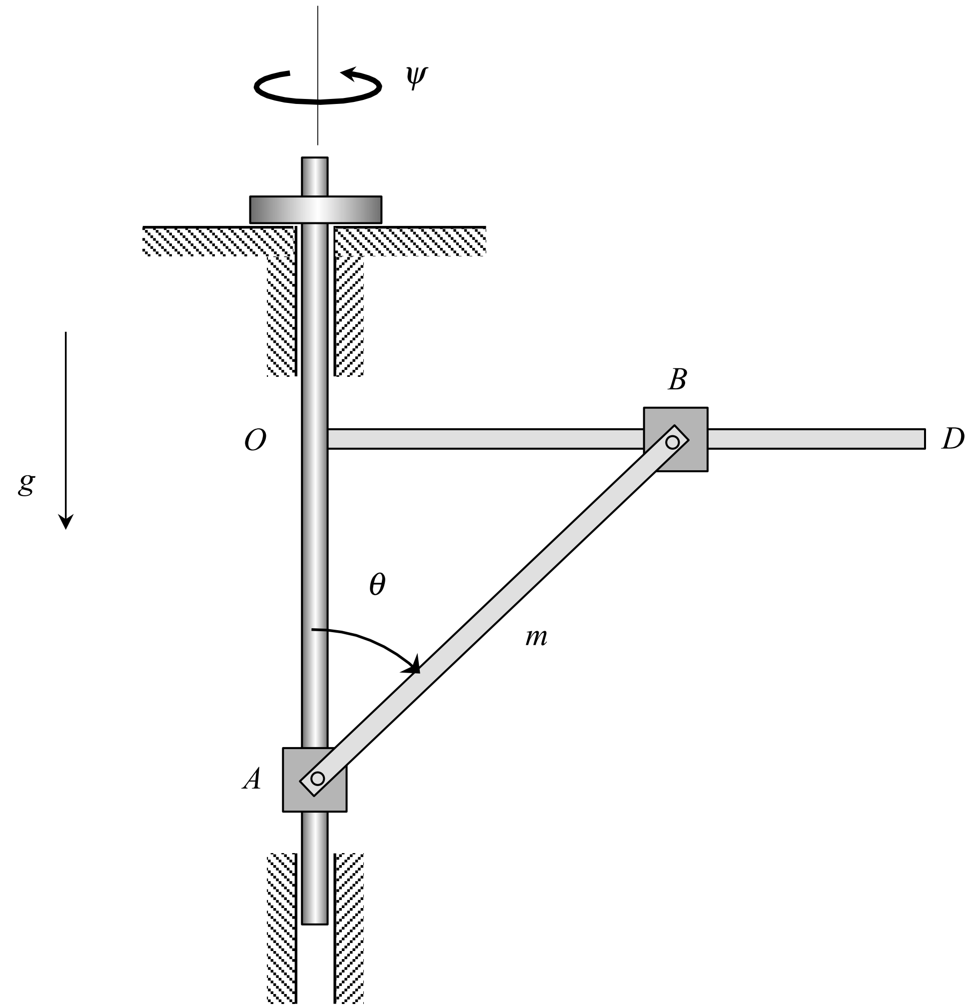



A two-link pendulum is made of two thin, homogeneous bars, OA and AB, each of mass m and length L. A known couple M(t) acts on bar OA. A force F acts in a direction that is perpendicular to bar AB, with the magnitude of F being such that the velocity of end B always collinear with line AB. The absolute angles $\theta_1$ and $\theta_2$ are to be used as the generalized coordinates in describing the motion of the system. The system moves in a horizontal plane (i.e., gravity does not affect the motion of the system).

a) Use the method of Lagrange multipliers with Lagrange's equations to derive the differential equations of motion for the system.

b) How is the Lagrange multiplier in these EOMs related to the applied force F?

***Problem 9.4 – 25 points***

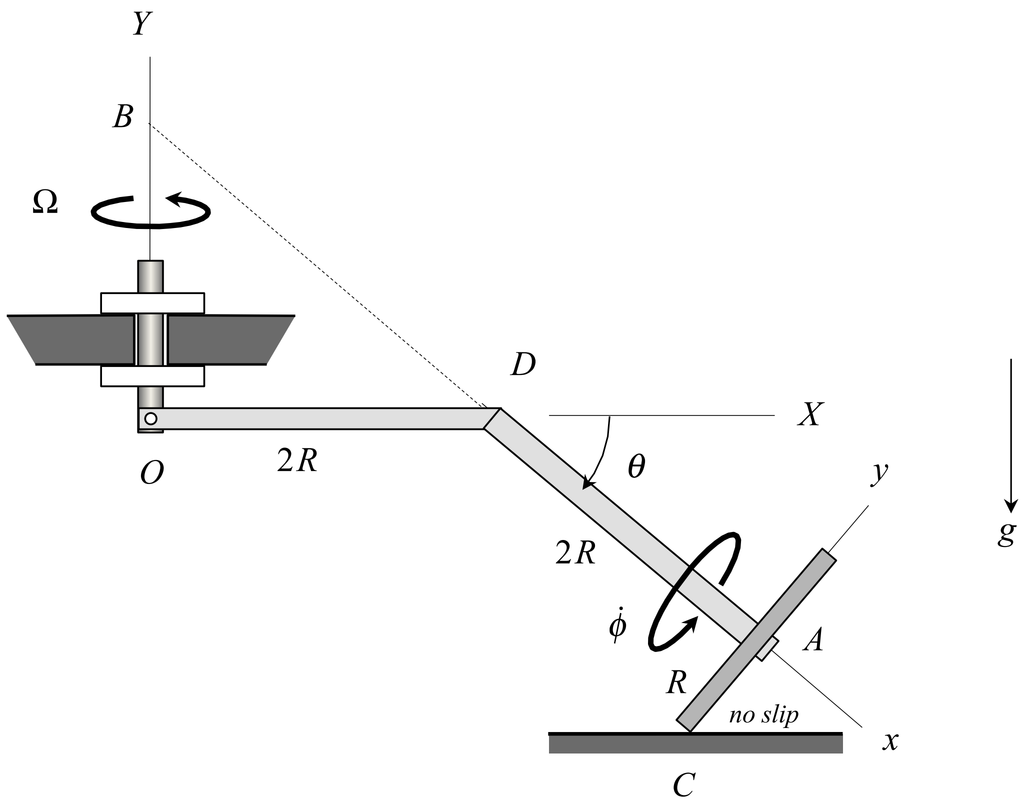



Arm OD is welded to a thin shaft at a right angle to the fixed rotation axis of the shaft, with the shaft being able to rotate freely about its fixed vertical axis. Let $I_O$ represent the mass moment of inertia of the shaft and arm OD about the rotation axis of the shaft. Arm AM (of mass m an length L) is free to slide along the vertical shaft at end A and along arm OD at end B. Consider the mass of sliders A and B to be negligible. Let $\theta$ represent the angle that bar AB makes with the rotation axis of the shaft, and let $\psi$ be the angle of rotation of the shaft. The system is given initial conditions of: $\psi(0) = \dot{\theta}(0) = 0$, $\dot{\psi}(0) = \Omega$ and $\theta(0) = \theta_0$.

a) Use Lagrange's equations to derive the differential equations of motion for the system using the generalized coordinates of $\theta$ and $\psi$.

b) Which, if any, of the coordinates in this problem are ignorable? What is the generalized momentum of the system corresponding to the ignorable coordinate (if one exists)?

***Problem 9.5***




A thin disk of mass m rolls without slipping on a flat, horizontal surface as it freely rotates about the bent arm, as shown, with the arm having negligible mass. The arm, in turn, is pinned to a shaft at O with the shaft being constrained to rotate about its vertical axis with a constant speed of $\Omega$. Determine the normal force acting on the disk by the horizontal surface on which it rolls.