ME562 – Spring 2020
Purdue University
West Lafayette, IN

Homework Set No. 6

Due date: Tuesday, March 31, 11:59pm

- Please include this cover sheet as the first page of your homework submission.
- Submit homework file on Gradescope.

Name _____

PUID _____

Problem 6.1 _____

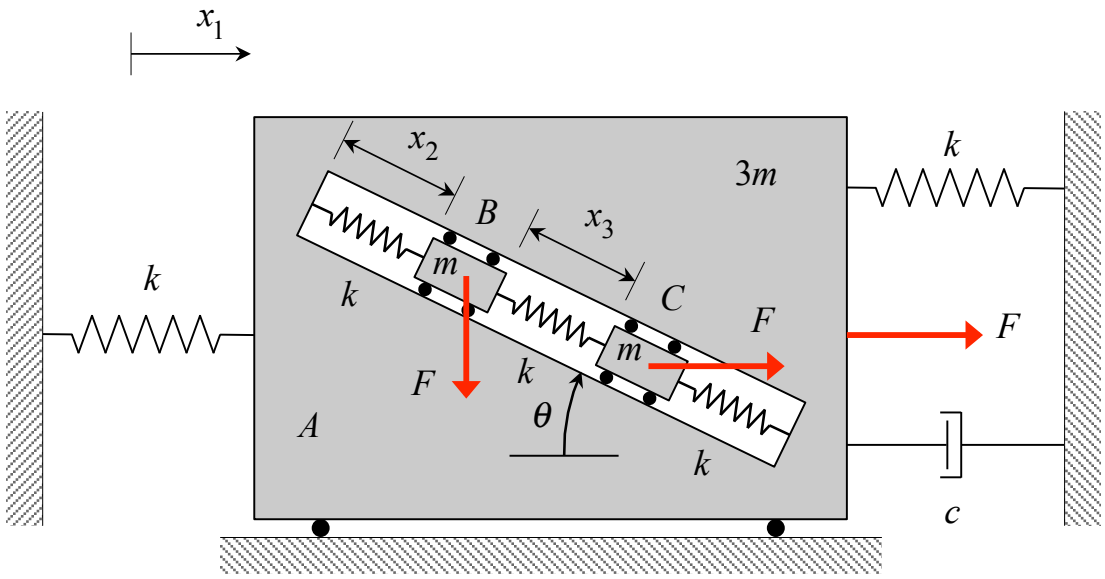
Problem 6.2 _____

Problem 6.3 _____

Problem 6.4 _____

TOTAL _____

Problem 6.1 – 10 points

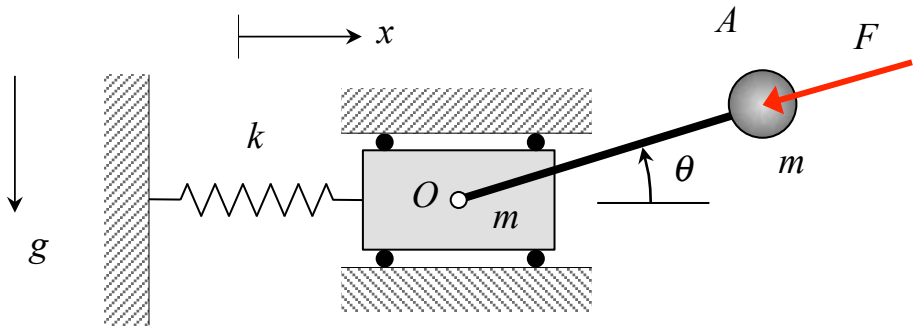


HORIZONTAL PLANE

A system is made up of particles A, B and C. Particle A moves along a smooth horizontal plane. Particles B and C move within a smooth, angled slot cut in particle A. Five identical springs, each of stiffness *k*, are interconnected as shown on the figure, and a dashpot with a damping coefficient *c* is attached between particle A and ground. Identical magnitude forces *F* act on all three particles, as shown. The position of particle A is described by the absolute coordinate x_1, with particle B being described by the coordinate x_2 measured relative to A and with particle C being described by the coordinate x_3 measured relative to particle B. All springs are unstretched when $x_1 = x_2 = x_3 = 0$.

a) How many degrees of freedom does this system possess?
b) Corresponding to the coordinates described above, write down the kinetic and potential energies for the system, and determine the generalized forces.
c) Using Lagrange's equations, derive the equations of motion of the system for the coordinates described above.

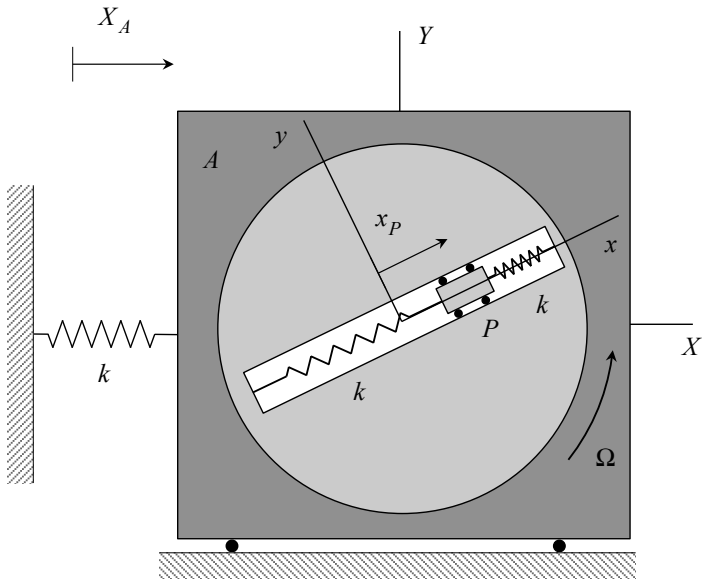
Problem 6.2 – 10 points



A system of two particles is shown above, with particle O constrained to move along a smooth, horizontal surface, and particle A connected to O with a rigid arm having negligible mass. A spring of stiffness k is attached between particle O and ground. A force F of constant magnitude is applied at A in such a way that the force is directed toward O for all positions. Let the absolute coordinate x describe the motion of particle O, and θ represent the angle of arm OA measured counterclockwise from the horizontal. The spring is unstretched when $x = 0$.

a) How many degrees of freedom does this system possess?
b) Corresponding to the coordinates described above, write down the kinetic and potential energies for the system, and determine the generalized forces.
c) Using Lagrange's equations, derive the equations of motion of the system for the coordinates described above.

Problem 6.3 – 10 points

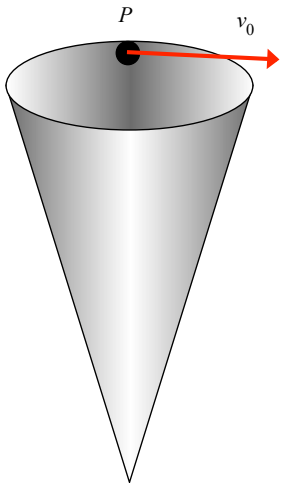


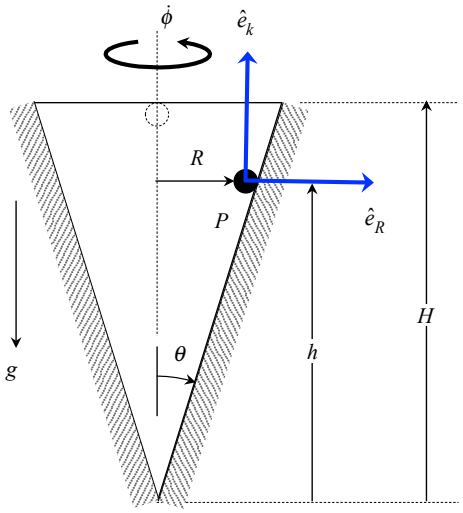
HORIZONTAL PLANE

A system of two particles is shown above, with particle A (having a mass of $2m$) constrained to move along a smooth surface, and particle P (having a mass of m) constrained to move within a smooth slot cut into a rotating turntable (with the turntable having negligible mass) attached to particle A. Two springs, each of stiffness k, are attached between P and the ends of the slot. Let XYZ represent a set of space-fixed coordinate axes, whereas the xyz-axes are attached to the turntable. Let the absolute coordinate X_A describe the motion of particle A, and x_P represent the position of P measured from the rotation axis of the turntable. The springs are unstretched when $X_A = x_p = 0$.

a) How many degrees of freedom does this system possess?
b) Corresponding to the coordinates described above, write down the kinetic and potential energies for the system, and determine the generalized forces.
c) Identify the T_2 portion of the kinetic energy. From this, write down the mass coefficients m_{ij}.
d) Identify the T_1 portion of the kinetic energy. From this, write down the gyroscopic coefficients H_{ij}.
e) Identify the T_0 portion of the kinetic energy. From this and the potential energy, write down the dynamic potential P.
f) Using the above, use the explicit Lagrangian formulation to derive the equations of motion for the system. Please do not use the standard Lagrange procedure for the derivation; we want you to learn and become accustomed to the explicit formulation.

Problem 6.4 – 10 points





SIDE VIEW

Particle P, having a mass of m, is constrained to move on the inner surface of a smooth cone. Let the position of the position of P be described by the height h and the angular position ϕ. At the initial state, P is given a speed of v_0 tangent to the upper surface of the cone at a height of H.

a) How many degrees of freedom does this system possess?
b) Corresponding to the coordinates described above, write down the kinetic and potential energies for the system, and determine the generalized forces.
c) Show that ϕ is an ignorable coordinate. Write down the initial generalized momentum of P corresponding to ϕ in terms of the system parameters and the initial conditions.
d) Write down the Routhian for the system in terms of the non-ignorable coordinate(s).
e) Use the Routhian formulation to derive the equation(s) of motion for the non-ignorable coordinate(s).