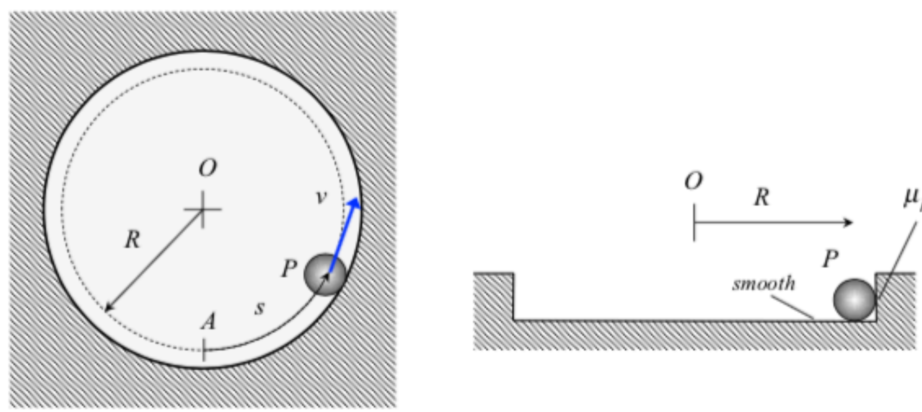
ME562 – Spring 2020
Purdue University
West Lafayette, IN

Homework Set No. 4 - SOLUTION

Problem 4.1 – 10 points

motion of P in the HORIZONTAL plane



TOP view **SIDE view**

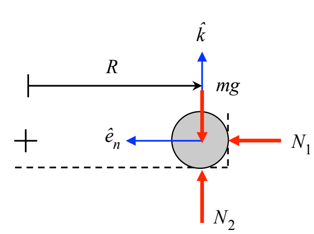
Particle P (having a mass of m) is constrained to move around the wall of a horizontal circular cavity, with the path of P in the cavity being a circle of radius R. The horizontal surface on which P moves is smooth, with the wall of the cavity along which P moves is rough having a coefficient of kinetic friction between the wall and P of μ_k. When at position A, P is known to have a speed of v_A.

a) Show that the speed of P as it moves around the cavity is governed by the differential equation: $\frac{dv}{ds} = -\mu_k \frac{v}{R}$, where s is the distance traveled by P.
b) Using the result of (a) above, determine the speed v of P as a function of s as it moves around the cavity wall.
c) How far does P travel before it comes to rest?

SOLUTION

$$\sum F_n = N_1 = m\frac{v^2}{R}$$

$$\sum F_t = -\mu_k N_1 = m\frac{dv}{dt} = mv\frac{dv}{ds} \quad \Rightarrow \quad mv\frac{dv}{ds} = -\mu_k m\frac{v^2}{R} \quad \Rightarrow \quad \frac{dv}{ds} = -\mu_k \frac{v}{R}$$

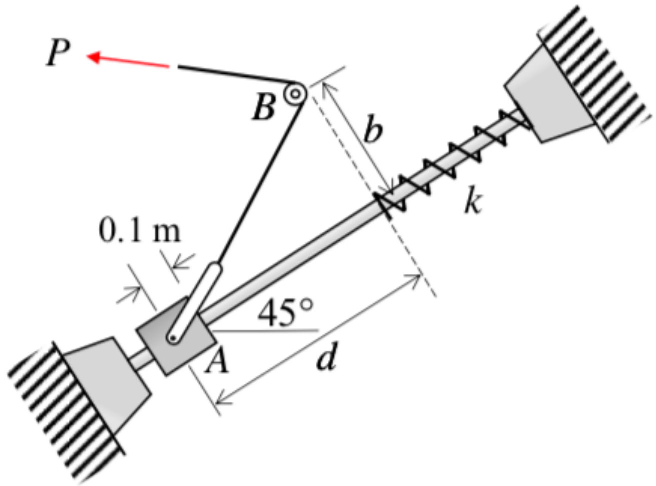


Integrating the above gives:

$$-\frac{\mu_k}{R}\int_0^s ds = \int_{v_A}^{v} \frac{dv}{v} \;\Rightarrow\; -\frac{\mu_k}{R}s = ln\,v\Big|_{v_A}^{v} = ln\left(\frac{v}{v_A}\right) \;\Rightarrow\; v = v_A e^{-\mu_k s/R}$$

The speed asymptotically approaches zero as s goes to infinity; that is, the particle never comes to rest over a finite distance.

Problem 4.2 – 10 points



A constant force P = 50 N acts at the free end of the cable as block A (having mass m = 30 kg) is pulled up the smooth rod. The system starts out from rest.

Determine the stiffness of k of the spring corresponding to a maximum spring compression of $\Delta_{max} = 0.03m$. Use b = 0.75 m and d = 1.5 m.

SOLUTION

$$T_1 = 0$$

$$V_1 = 0 \;;\; \textit{choose datum at bottom most position of block}$$

$$T_2 = \frac{1}{2}mv^2$$

$$V_2 = mg(d+\Delta) + \frac{1}{2}k\Delta^2$$

$$U_{1\to 2}^{(nc)} = P\Delta s_C \;;\; \textit{where C is the end point of the cable}$$

where:

$$\Delta_C = \sqrt{b^2+d^2} - \sqrt{(\Delta-0.1)^2+d^2}$$

Using the work/energy equation we have:

$$T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)} \;\Rightarrow$$

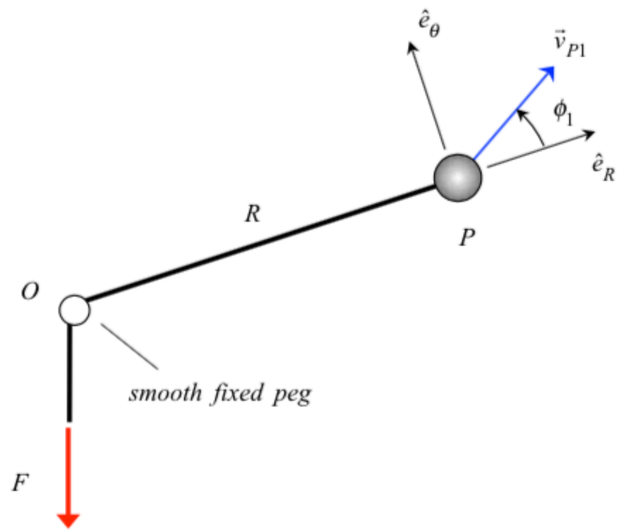
$$\frac{1}{2}mv^2 + mg(d+\Delta) + \frac{1}{2}k\Delta^2 = 0 + 0 + \left[\sqrt{b^2+d^2} - \sqrt{(\Delta-0.1)^2+d^2}\right]P$$

For $\Delta = \Delta_{max}$ we have $v = 0$:

$$mg\left(d + \Delta_{max}\right) + \frac{1}{2}k\Delta_{max}^2 = \left[\sqrt{b^2 + d^2} - \sqrt{\left(\Delta_{max} - 0.1\right)^2 + d^2}\right]P \quad \Rightarrow$$

$$k = \frac{2}{\Delta_{max}^2}\left\{\left[\sqrt{b^2 + d^2} - \sqrt{\left(\Delta_{max} - 0.1\right)^2 + d^2}\right]P - mg\left(d + \Delta_{max}\right)\right\}$$

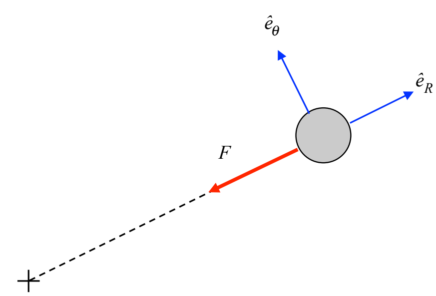
Problem 4.3 – 10 points



HORIZONTAL PLANE

A rope is attached to particle P (having a mass of m = 6 kg) with the rope being pulled over a fixed, smooth peg by a constant force F = 60 N applied at the other end of the rope. At the initial state, P has a speed of $v_{P1} = 20\,m/s$ with $\phi_1 = 30°$, and is at a distance $R = R_1 = 3m$ from the peg. The particle moves on a smooth horizontal plane.

Determine $\dot{R}$ when $R = R_2 = 4m$.



SOLUTION

Conservation of angular momentum

Since $\sum \vec{M}_O = \vec{0} \;\Rightarrow\; \vec{H}_{O2} = \vec{H}_{O1}$

Calculating the angular momentum about O:

$$\vec{H}_{O1} = m\vec{r}_{P/O} \times \vec{v}_{P1} = m\left(R_1\hat{e}_R\right) \times \left(v_{P1}cos\phi_1\hat{e}_R + v_{P1}sin\phi_1\hat{e}_\theta\right)$$
$$= mR_1 sin\phi_1 \hat{k}$$
$$\vec{H}_{O2} = m\vec{r}_{P/O} \times \vec{v}_{P2} = m\left(R_2\hat{e}_R\right) \times \left(\dot{R}_2\hat{e}_R + R\dot{\theta}_2\hat{e}_\theta\right) = mR_2^2\dot{\theta}_2\hat{k}$$

Therefore:

$$mR_1 v_{P1} sin\phi_1 = mR_2^2\dot{\theta}_2 \;\Rightarrow\; \dot{\theta}_2 = \frac{R_1 v_{P1} sin\phi_1}{R_2^2}$$

Conservation of energy

$$T_1 = \frac{1}{2} m v_{P1}^2$$

$$T_2 = \frac{1}{2} m v_{P2}^2 = \frac{1}{2} m \left[\dot{R}_2^2 + R_2^2 \dot{\theta}_2^2 \right]$$

$$U_{1 \to 2}^{(nc)} = F\left(R_1 - R_2\right)$$

Therefore:

$$T_2 = T_1 + U_{1 \to 2}^{(nc)} \quad \Rightarrow$$

$$\frac{1}{2} m \left[\dot{R}_2^2 + R_2^2 \dot{\theta}_2^2 \right] = \frac{1}{2} m v_{P1}^2 + F\left(R_1 - R_2\right) \quad \Rightarrow$$

$$\dot{R}_2 = \sqrt{v_{P1}^2 + 2\frac{F}{m}\left(R_1 - R_2\right) - R_2^2 \dot{\theta}_2^2} = \sqrt{v_{P1}^2 + 2\frac{F}{m}\left(R_1 - R_2\right) - R_2^2 \left(\frac{R_1 v_{P1} sin\phi_1}{R_2^2} \right)^2}$$