

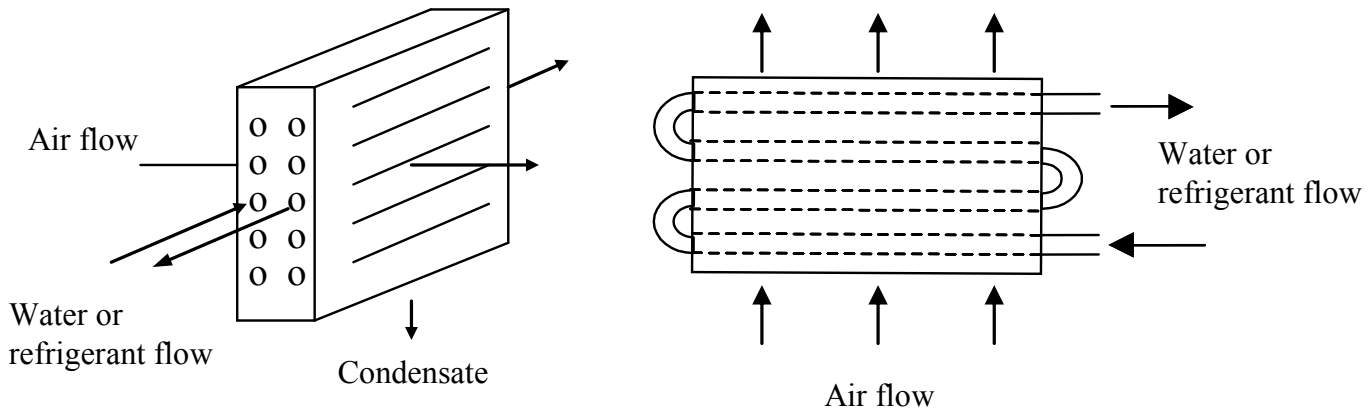
ME 418

Lecture 9 - Heat Exchanger Analysis & Design

- Heat exchanger overview
- Overall heat exchanger conductance
- Heat transfer analysis – LMTD method
- Heat transfer analysis – effectiveness-NTU method
- Fin efficiencies
- Heat transfer coefficient
- Flow pressure drop

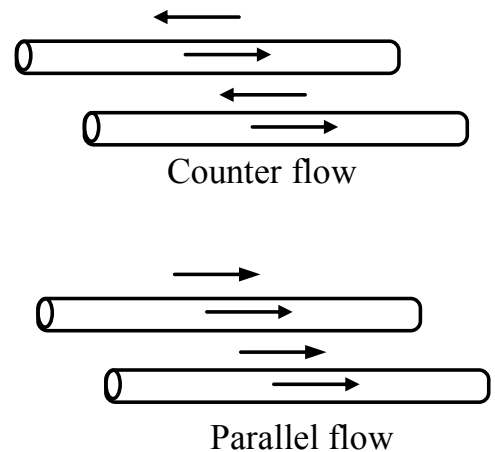
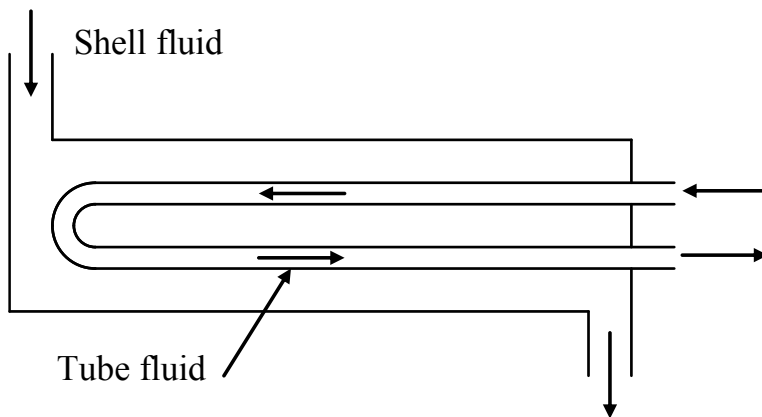
Heat Exchanger Types

Air-to-liquid Coil used for heating or cooling air



Liquid-to-liquid

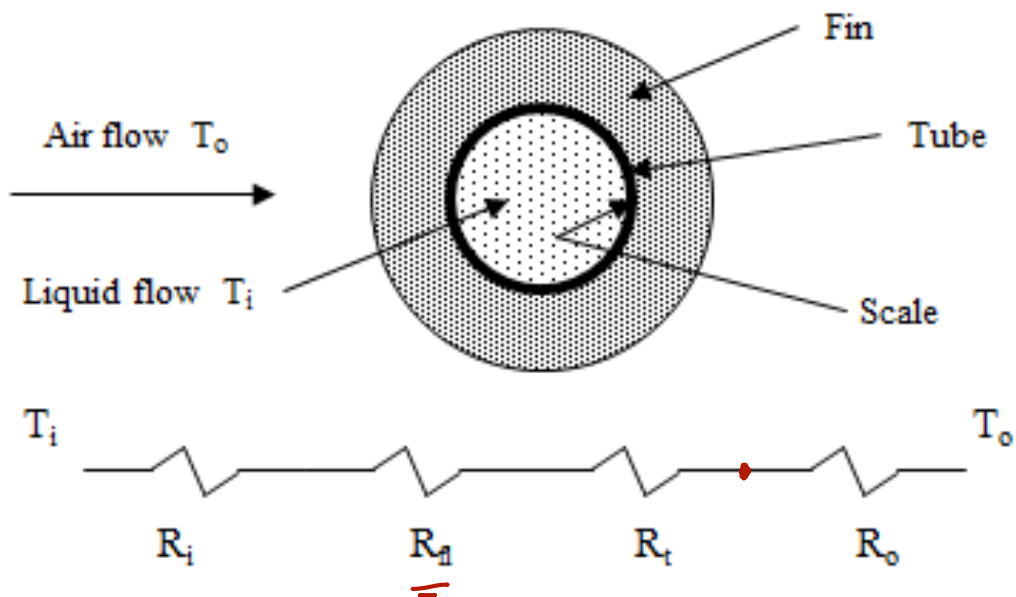
Shell and tube exchanger used for liquids



Overall Heat Exchanger Conductance

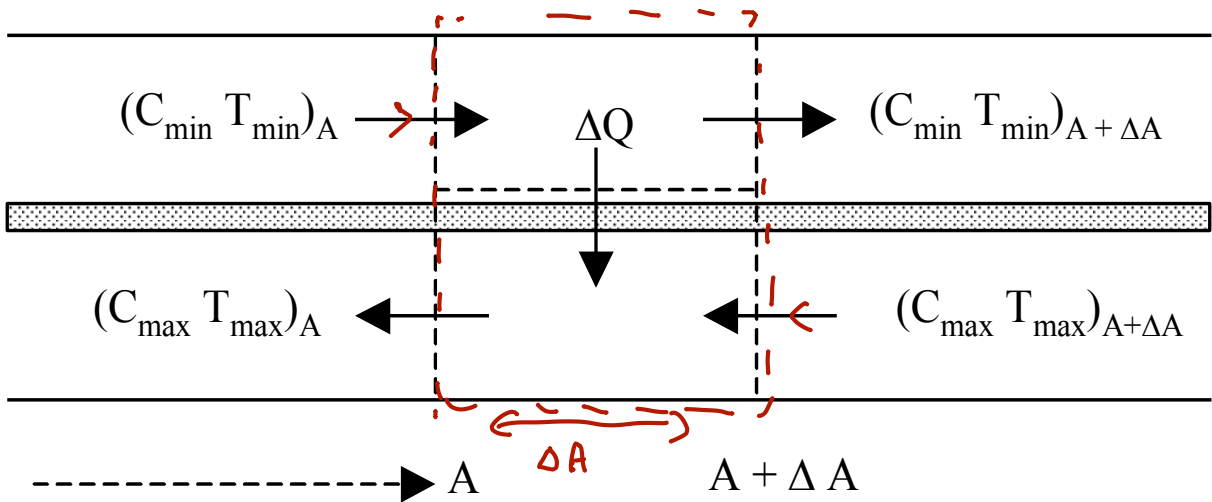
Total heat transfer resistance between two fluids in a finned tube heat exchanger

$$R_T = R_i + R_{fi} + R_t + R_o$$



- R_i: tube inside convection resistance
- R_{fi}: tube inside fouling resistance
- R_t: tube wall conduction resistance
- R_o: outside surface (fin+tube) convection resistance

Energy flows for a counter-flow heat exchanger



We define

$$C_{\min} = \dot{m}_{\min} c_{p,\min} \quad \text{and} \quad C_{\max} = \dot{m}_{\max} c_{p,\max}$$

Energy balance for the infinitesimal CV

$$\underbrace{(C_{\min} T_{\min})_A}_{\text{Steady state}} + \underbrace{(C_{\max} T_{\max})_{A+\Delta A}}_{\text{Steady state}} - (C_{\min} T_{\min})_{A+\Delta A} - (C_{\max} T_{\max})_A = 0$$

$$C_{\min} \frac{dT_{\min, A}}{dA} = C_{\max} \frac{dT_{\max}}{dA}$$

Heat transfer between two small CVs

$$\Delta Q = U \Delta A (T_{\min} - T_{\max})$$

Energy balance for one side

$$C_{\min} (T_{\min, A} - T_{\min, A + \Delta A})$$



$$= -U \Delta A (T_{\min} - T_{\max})$$

$$C_{\min} \frac{dT_{\min}}{dA} = -U (T_{\min} - T_{\max})$$

$$C_{\max} \frac{dT_{\max}}{dA} = -U (T_{\min} - T_{\max})$$

*Integrate above equations to find total heat transfer

Log-Mean-Temperature-Difference Method

Log-Mean-Temperature-Difference (LMTD) is defined as

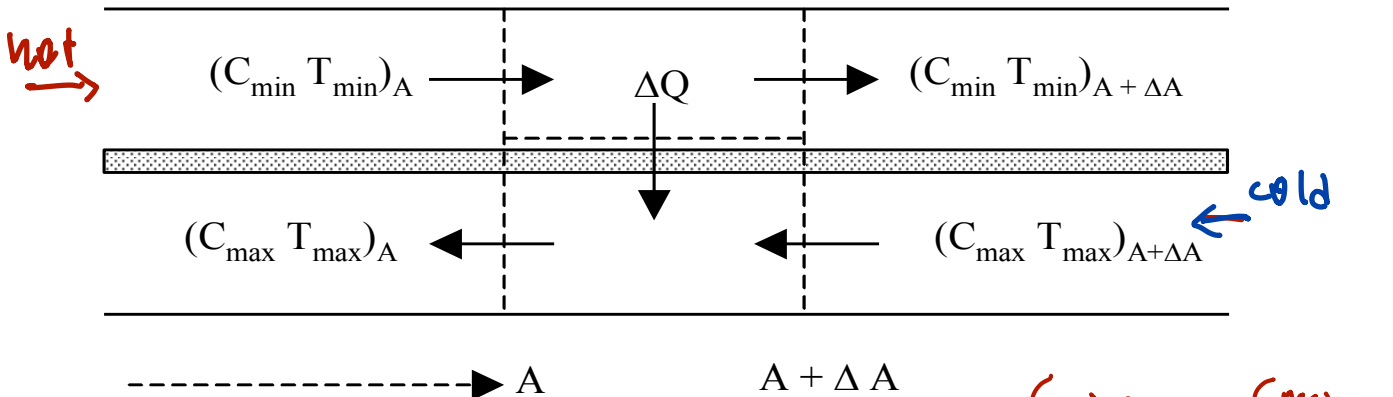
$$LMTD = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{\ln \left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}} \right)}$$

Heat transfer rate is calculated by

$$\dot{Q} = UA (\text{LMTD}) F$$

where F is a correction factor for flow types other than counterflow.

$F=1$ for counterflow



Assume $C_{\min} = C_h$, $C_{\max} = C_c$

$$C_h \cdot \Delta T_h = C_c \cdot \Delta T_c$$

$$\Delta T_c < \Delta T_h$$

$$\Delta Q = U \cdot \Delta A (T_h - T_c)$$

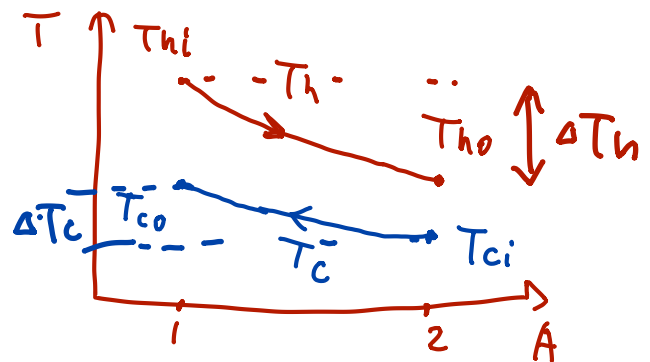
$$= -C_h \cdot \Delta T_h = -C_c \cdot \Delta T_c$$

negative in A-direction

$$\Rightarrow dT_h = -\frac{dQ}{C_h} \quad dT_c = -\frac{dQ}{C_c}$$

$$(-) \Rightarrow d(T_h - T_c) = dQ \left(\frac{1}{C_c} - \frac{1}{C_h} \right)$$

$$= U \Delta A (T_h - T_c) \left(\frac{1}{C_c} - \frac{1}{C_h} \right)$$



divide $T_h - T_c$
 $\Rightarrow \frac{d(T_h - T_c)}{T_h - T_c} = U dA \left(\frac{1}{C_c} - \frac{1}{C_h} \right)$ $()' = \frac{1}{()}$

integrate $\int_1^2$
 $\Rightarrow \ln(T_h - T_c)|_2 - \ln(T_h - T_c)|_1 = UA \left(\frac{1}{C_c} - \frac{1}{C_h} \right)$ (4)

$\ln\left(\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}}\right) = \frac{UA}{Q} (T_{c,o} - T_{c,i} - T_{h,i} + T_{h,o})$ ↑

$\Rightarrow Q = UA \cdot \frac{(T_{h,o} - T_{c,i}) - (T_{h,i} - T_{c,o})}{\ln\left(\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}}\right)}$ ⎧ $C_h = \frac{Q}{T_{h,i} - T_{h,o}}$
 $C_c = \frac{Q}{T_{c,o} - T_{c,i}}$

LMTD

Effectiveness-NTU Method

Overall energy balance

$$\dot{Q} = \underbrace{C_h (T_{h,i} - T_{h,o})}_{\text{hot side}} = \underbrace{C_c (T_{c,o} - T_{c,i})}_{\text{cold side}}$$

We define heat transfer effectiveness

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}}$$

If $C_h = C_{\min}$, $\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$

If $C_c = C_{\min}$, $\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}}$

where $\dot{Q}_{\max} = C_{\min} (T_{h,i} - T_{c,i})$ is the max heat transfer.

Then the heat transfer rate is

$$\dot{Q} = \varepsilon C_{\min} (T_{h,i} - T_{c,i})$$

$\dot{Q}_{\max}$

We further define capacitance rate ratio

$$\underline{C^*} = \frac{C_{\min}}{C_{\max}}$$

and number of transfer units (NTUs)

$$\underline{Ntu} = \frac{UA}{C_{\min}}$$

Then effectiveness can be calculated as

$$\varepsilon = f(C^*, Ntu) \quad C_h = C_{\min}$$

Exp (*) $\Rightarrow$

$$\begin{aligned} \frac{T_{ho} - T_{ci}}{T_{hi} - T_{co}} &= \exp\left(UA \left(\frac{1}{C_c} - \frac{1}{C_h}\right)\right) \\ &= \exp\left(\frac{UA}{C_{\min}} \left(\frac{C_{\min}}{C_{\max}} - 1\right)\right) \\ &= \exp(Ntu \cdot C^* - 1) \quad \text{NTU} \\ &= \frac{T_{ho} - T_{hi} + T_{hi} - T_{ci}}{T_{hi} - T_{ci} + T_{ci} - T_{co}} \end{aligned}$$

$$= \frac{T_{h0} - T_{hi} + \boxed{T_{hi} - T_{ci}}}{\boxed{T_{hi} - T_{ci}} + T_{ci} - T_{co}}$$

Divide $T_{hi} - T_{ci}$

$$= \frac{(-\xi) + 1}{1 + (-\xi \cdot C^*)}$$

$$\frac{T_{h0} - T_{hi}}{T_{hi} - T_{ci}} = -\xi$$

$$\frac{T_{ci} - T_{co}}{T_{hi} - T_{ci}} = -\xi \cdot C^*$$

$$C_h(T_{h0} - T_{hi}) = C_c(T_{ci} - T_{co})$$

Div C_h
 $\Rightarrow T_{h0} - T_{hi} = \frac{1}{C^*}(T_{ci} - T_{co})$

$$\Rightarrow -\xi + 1 = \exp(\dots) - \xi \cdot C^* \cdot \exp(\dots)$$

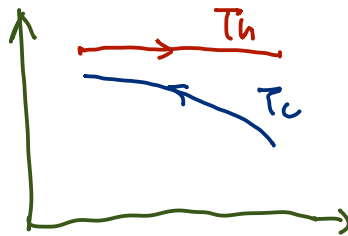
$$\Rightarrow \xi \cdot (1 - C^* \cdot \exp(\dots)) = 1 - \exp(\dots)$$

$$\xi = -\frac{(T_{h0} - T_{hi})}{T_{hi} - T_{ci}} = -\frac{1}{C^*} \frac{T_{ci} - T_{co}}{T_{hi} - T_{ci}}$$

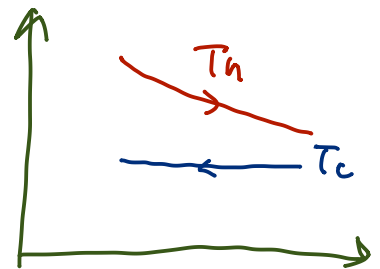
$$\Rightarrow \xi = \frac{\exp(NTU(C^*+1)) - 1}{C^* \cdot \exp(NTU(C^*+1)) - 1} \quad \checkmark$$

Scenarios for C_h, C_c .

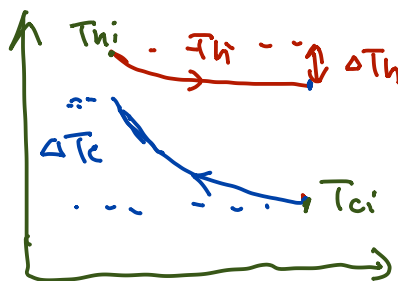
$C_h \gg C_c \rightarrow$ condensation



$C_h < C_c \rightarrow$ evaporation



$C_h > C_c \Rightarrow \Delta T_h < \Delta T_c$



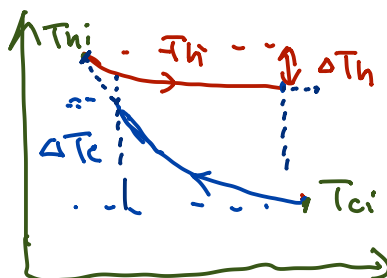
$$T_{hi} - T_{ci} = \Delta \bar{T} \rightarrow$$

driving potential

max ΔT achievable only on

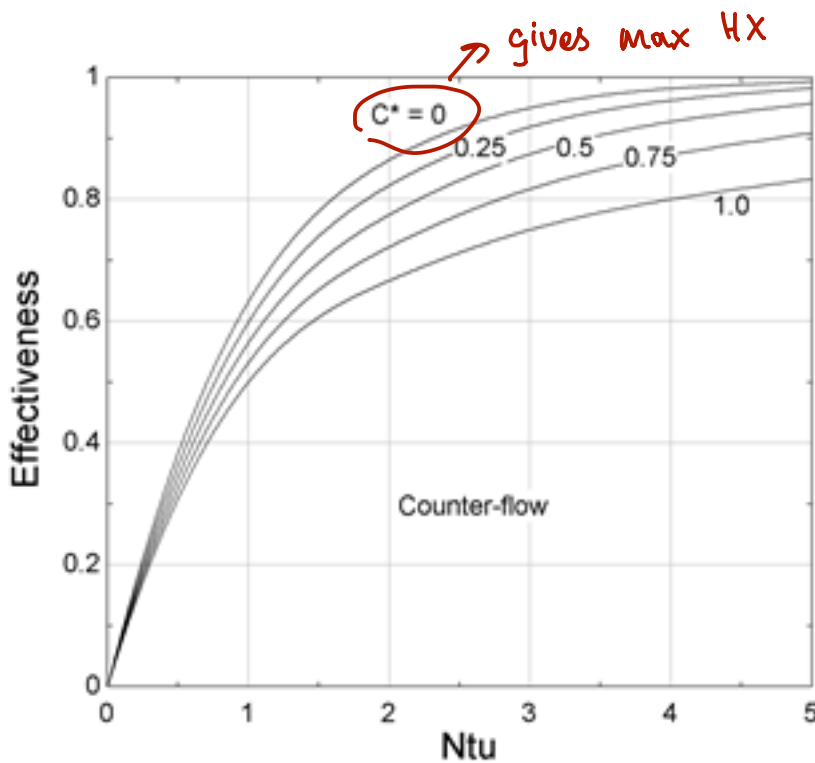
C_{min} side, not on

C_{max} side.

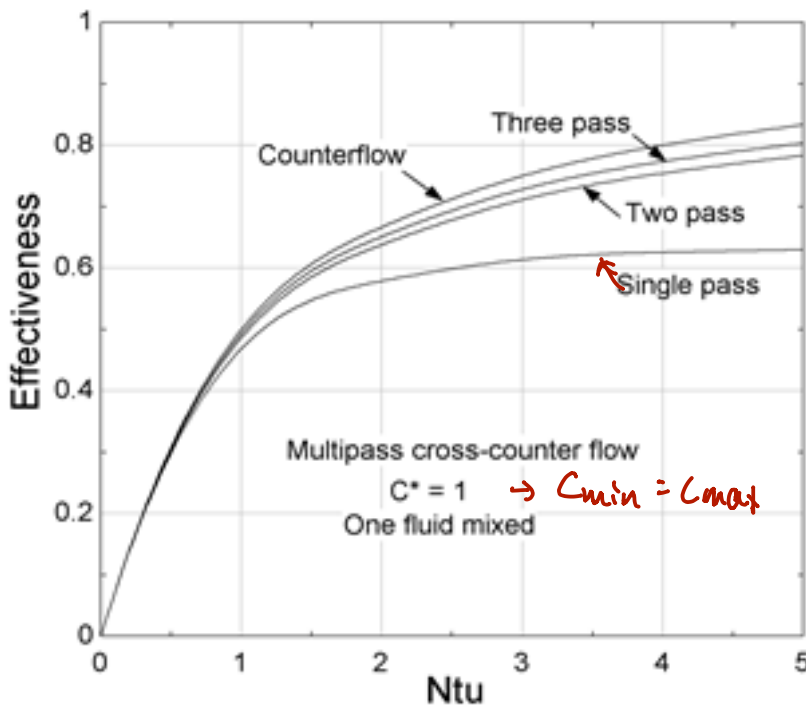
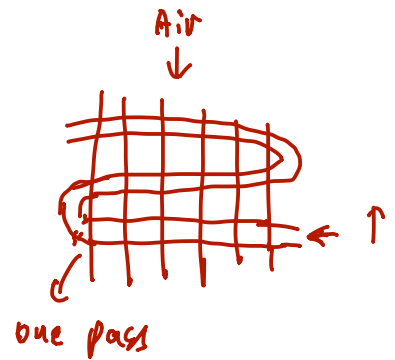


$$\xi = 1$$

Flow arrangement	Conditions	Effectiveness
All	$C^* = 0$	$\varepsilon = 1 - e^{-Ntu}$ → HW
<u>Counterflow</u>	$C^* \neq 1$	$\varepsilon = \frac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}}$ Derived above
	$C^* = 1$	$\varepsilon = \frac{Ntu}{1 + Ntu}$
Crossflow, one fluid mixed	Single pass	$\varepsilon = \frac{1 - e^{-C^*(1 - e^{-Ntu})}}{C^*}$
	<u>Two pass</u>	$\varepsilon = [1 - 0.0643 C^* (1 - e^{-0.548 Ntu})] \left[\frac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}} \right]$
	Three pass	$\varepsilon = [1 - 0.0411 C^* (1 - e^{-0.414 Ntu})] \left[\frac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}} \right]$
Shell and tube: even number of passes (2, 4, 6, ...)		$\varepsilon = \frac{2}{(1 + C) + \sqrt{1 + C^2} \left(\frac{1 + e^{-Ntu\sqrt{1+C^2}}}{1 - e^{-Ntu\sqrt{1+C^2}}} \right)}$
Crossflow	Both fluids unmixed	$\varepsilon = 1 - \exp \left[\frac{Ntu^{0.22}}{C^*} (\exp(-C^* Ntu^{0.78}) - 1) \right]$



Counterflow



Multi-pass
cross-flow with
one fluid mixed

tube internal fluid
is mixed

air side { mixed, no fins
unmixed, w/ fins

If more than 3 passes,
counter flow can be
good approximate

Notes:

- LMTD and ϵ -NTU methods are equivalent
- LMTD is easier to use when desired fluid outlet conditions are given
- ϵ -NTU is better when heat exchanger size and performance (i.e., UA) is given

Heat Exchanger Example: Determine the outlet temperatures and the heat transfer rate for a heating coil in which air is heated using hot water with two passes. The overall heat transfer conductance is 4 kW/C. The air stream enters at 24 C with a flow rate of 3 kg/s, and the water stream enters at 60 C with a flow rate of 1.0 kg/s.

$$C_{air} = \dot{m}_a \cdot c_{p,a}$$

$$C_w = \dot{m}_w \cdot c_{p,w}$$

$$C_{min} = \min(C_{air}, C_w)$$

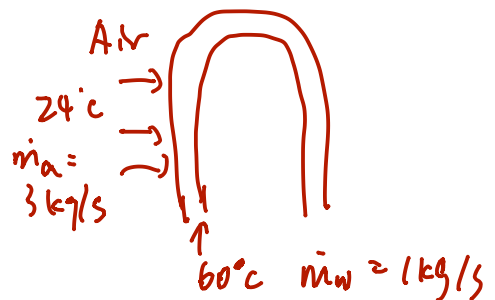
$$NTU = \frac{UA}{C_{min}} \quad C^* = \frac{C_{min}}{C_{max}}$$

$$\epsilon = f(NTU, C^*)$$

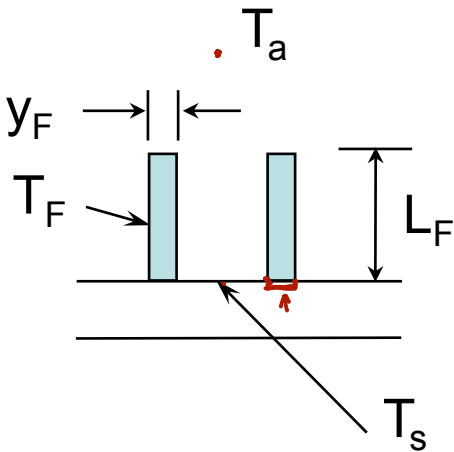
$$\dot{Q} = \epsilon \cdot C_{min} \cdot (T_{wi} - T_{ai})$$

$$T_{ao} = T_{ai} + \frac{\dot{Q}}{C_{air}}$$

$$T_{wo} = T_{wi} - \frac{\dot{Q}}{C_w}$$



Fin Efficiencies



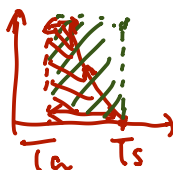
- fin heat transfer depends on local temperature difference ($T_F - T_a$)
- characterize finned surface heat transfer using a fin efficiency

Effective overall surface area

$$\underbrace{\eta_o}_{\text{overall fin efficiency}} \underbrace{A_o}_{\text{fin surface area \& tube area}} = \underbrace{(A_o - A_F)}_{\text{exposed tube area}} + \underbrace{\eta_F}_{\text{individual fin efficiency}} \underbrace{A_F}_{\text{fin surface area}}$$

Individual Fin Efficiency

$$\eta_F = \frac{\text{heat transfer at base of fin}}{\text{maximum possible heat transfer if entire fin at } T_s}$$



$$u \int \Delta T \cdot dA$$

$$u \cdot A (T_s - T_a)$$

green area.

$$u \int \Delta T dA$$

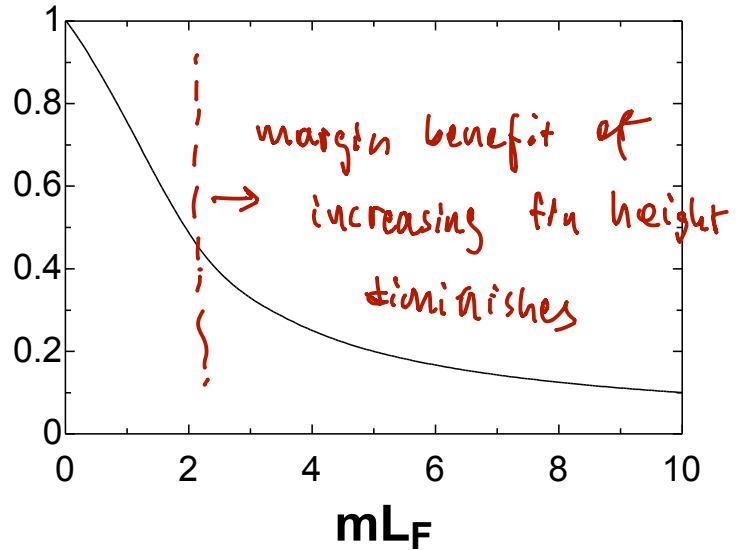
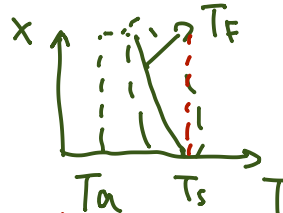
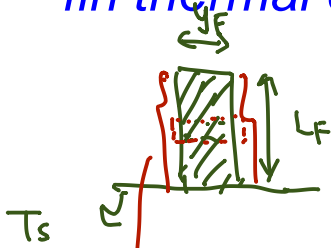
red area

For straight fins,

$$\eta_F = \frac{\tanh(mL_F)}{mL_F}$$

$$m = \left(\frac{2h_o}{k_F y_F} \right)^{1/2}$$

fin thermal conductivity



$$\eta_F = \frac{\int_0^{L_F} h_o dx (T_F - T_a)}{h_o \cdot L_F (T_s - T_a)}$$

$$Q_o = -k_F \cdot y_F \cdot \frac{dT_F}{dx} \Big|_{x+L_F}$$

$$Q_i = -k_F \cdot y_F \cdot \frac{dT_F}{dx} \Big|_x$$

$$Q_c = 2\Delta x \cdot h_o (T_F - T_a)$$

Energy balance: $2 \cdot \Delta x \cdot h_o (T_F - T_a) = k_F \cdot y_F \left(-\frac{dT_F}{dx} \Big|_x + \frac{dT_F}{dx} \Big|_{x+\Delta x} \right)$

divide Δx

$$2h_o (T_F - T_a) = k_F \cdot y_F \cdot \frac{d^2 T_F}{dx^2}$$

$$T_F = T_s \cdot \frac{\cosh(m(L_F - x))}{\cosh(mL_F)}$$

Define $z = T_F - T_a \Rightarrow 2h_o z = k_F \cdot y_F \cdot z''$

$$\Rightarrow \left[\frac{2h_o}{k_F \cdot y_F} \right] z = z'' \Leftrightarrow m^2 z = z''$$

$$m^2 \cdot z = z''$$

$$\parallel$$

$$z_1 = e^{mx}$$

$$z_2 = e^{-mx}$$

General sol'n

$$z = C_1 \cdot e^{mx} + C_2 \cdot e^{-mx}$$

$$T_F(x=0) = T_s \Rightarrow C_1 + C_2 = T_s$$

$$h_o \cdot y_F \cdot (T_F(x=L_F) - T_a) = -k_F \cdot y_F \cdot \frac{dT_F}{dx} (x=L_F)$$

For circular fins,

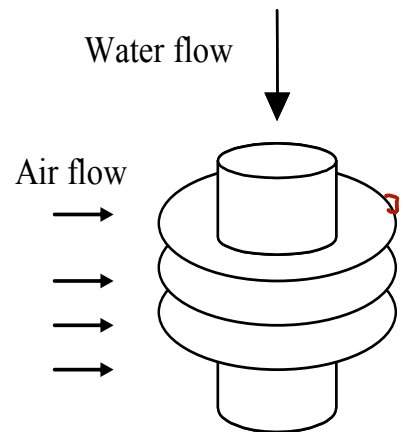
$$\eta_f = \left[\frac{2r_i}{m_f (r_{o,c}^2 - r_i^2)} \right] \left[\frac{K_1(m_f r_i) I_1(m_f r_{o,c}) - I_1(m_f r_i) K_1(m_f r_{o,c})}{I_0(m_f r_i) K_1(m_f r_{o,c}) + K_0(m_f r_i) I_1(m_f r_{o,c})} \right]$$

where the effective fin radius and area are

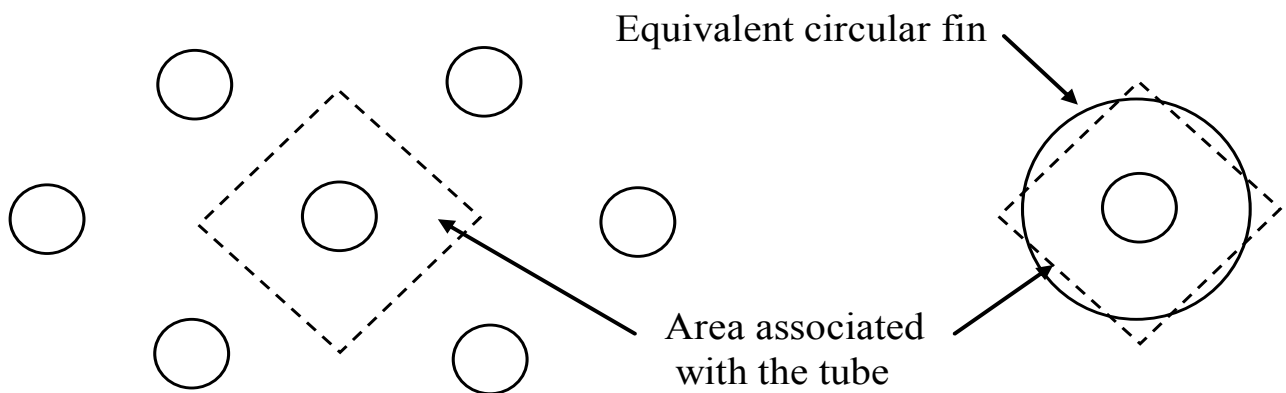
$$r_{o,c} = r_o + \frac{t_f}{2}$$

t_f = fin thickness

$$A_f = 2\pi (r_{o,c}^2 - r_i^2)$$



Equivalent fin radius for plate-fin geometries



Fin Efficiency Example: Determine the fin efficiency, overall surface efficiency, and thermal resistance per foot of tube length for a cross-flow heat exchanger using finned tubes. The tube diameter is 0.774 inch. The fins are steel with a thickness of 0.012 inch, a diameter of 1.463 inch, and a pitch of 9.05 fins per inch. The conductivity of steel is 35 Btu/hr-ft-F. The heat transfer coefficient is 14.4 Btu/hr-ft²-F."

h_o

$$r_i = \frac{0.774}{2} \text{ in} \quad t_F = y_F = 0.012 \text{ in}$$

$$r_o = \frac{1.463}{2} \text{ in} \quad \text{--- } k_{o,c}$$

$$k_F = 35 \text{ Btu/hr-ft-F}$$

$$h_o = 14.4 \text{ Btu/hr-ft}^2\text{-F}$$

$$R = \frac{1}{\eta_o \cdot h_o \cdot A_o}$$

Tube Internal Single Phase Heat Transfer Coefficients and Pressure Drop

- Forced convection is generally involved in heat exchanger flow
- Correlations covered in Lectures 3 and 5 can be used for calculating heat transfer coefficient and pressure drop

Reynolds number is defined as

$$Re_{D_H} = \frac{\rho_f V D_H}{\mu_f} \quad \text{or} \quad Re_{D_H} = \frac{4 \dot{m}}{\underbrace{WP}_{\mu_f}}$$

where hydraulic diameter is

$$D_H = \frac{4 A_c}{WP}$$

Heat transfer coefficient is related to Nusselt number defined as

$$Nu_{D_H} = \frac{h_c D_H}{k_f}$$

where k_f is the fluid conductivity.

From Lecture 3, we can calculate **pressure drop** by

$$\Delta p = f \frac{L}{D_H} \left(\frac{\rho_f V^2}{2} \right) \quad \text{or} \quad \Delta p = f \frac{L}{D_H} \frac{G^2}{2\rho}$$

where G is mass velocity

$$G = \frac{\dot{m}}{A_c}$$

Heat transfer and friction factor relations for internal turbulent flow for $Re > 2500$

Smooth tubes	$Nu_{D_H} = 0.023 Re_{D_H}^{0.8} Pr^n$ $n = 0.4 \text{ heating or } n = 0.3 \text{ cooling}$
Or (larger Re_D range)	$Nu_{D_H} = \frac{(f/2)(Re_{D_H} - 1000)Pr}{1 + 12.7(f/2)^{1/2}(Pr^{2/3} - 1)}$
Rough tubes	$Nu_{D_H} = \frac{(Re_{D_H} Pr (f/2))}{1 + (f/2)^{1/2}(4.5 Re_{D_H}^{0.2} Pr^{0.5} - 8.48)}$

Smooth tubes	$f = \frac{0.3164}{Re_{D_H}^{0.25}}$
Or (larger Re_D range)	$f = 0.0032 + \frac{0.221}{Re_{D_H}^{0.237}}$
Rough tubes	$\underline{Re_{D_H} < 10^6}$ $f^{-0.5} = 1.14 + 2 \log \left(\frac{D_H}{\varepsilon} \right) - 2 \log \left(1 + \frac{9.3}{Re \left(\frac{\varepsilon}{D_H} \right) f^{0.5}} \right)$ $\underline{Re_{D_H} > 10^6}$ $f^{-0.5} = 1.14 + 2 \log \left(\frac{D_H}{\varepsilon} \right)$

Finned Surface Heat Transfer Coefficients and Pressure Drop

h_o

ΔP

We define the ratio of free flow to frontal area

$$\sigma = \frac{A_c}{A_{fr}} \rightarrow L \times H$$

Reynolds number $N_R = Re_{D_H} = \frac{D_H G}{\mu}$

The heat transfer coefficient is related to Stanton number

$$St = \frac{h_c}{G c_p}$$

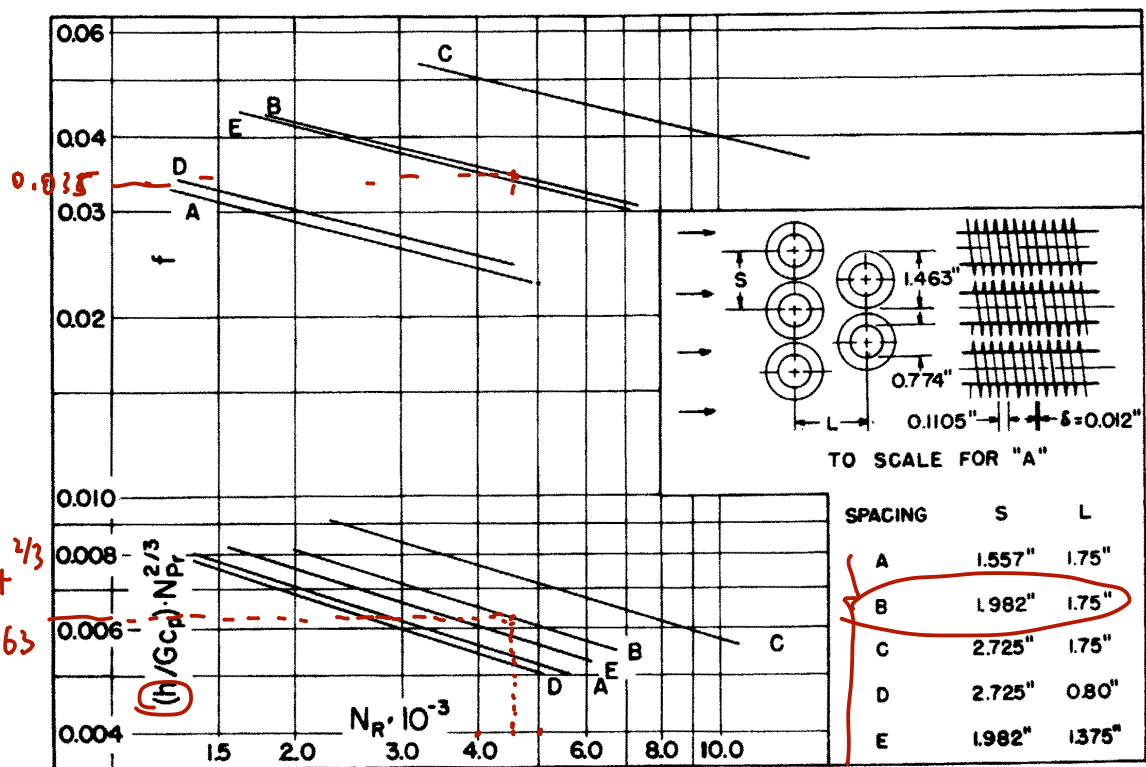
Friction factor $f = \frac{\rho \tau_0}{G^2 / 2}$

Pressure drop can be calculated by

$$\Delta p = f \frac{A}{A_c} \frac{G^2}{2 \rho}$$

$\rightarrow$ heat transfer area

$\downarrow$ free flow area



Tube outside diameter = 0.774 in.

Fin pitch = 9.05 per in.

Fin thickness = 0.012 in.

Fin area/total area = 0.835

Flow passage hydraulic diameter, $4r_h =$

	A	B	C	D	E
	0.01681	0.02685	0.0445	0.01587	0.02108 ft

Free-flow area/frontal area, $\sigma =$

	A	B	C	D	E
	0.455	0.572	0.688	0.537	0.572

Heat transfer area/total volume, $\alpha =$

	A	B	C	D	E
	108	85.1	61.9	135	108 ft ² /ft ³

Note: Minimum free-flow area in all cases occurs in the spaces transverse to the flow, except for D, in which the minimum area is in the diagonals.

* Prandtl number raised to the two-thirds power is used to correlate the properties of other fluids.

Finned tube example: Determine the air-side convective heat transfer coefficient ^{h_o} , thermal resistance ^{R} , and pressure drop ^{ΔP} for a coil made of finned tubes with configuration B of figure above. The coil frontal area is 4 ft², there are four rows of coils, and the fins are made of aluminum. The airflow is 4000 cfm at a temperature of 75 F and 50 % relative humidity. ^{$\dot{V}$}

$$\dot{m} = \dot{V} \cdot \rho$$

$$V = A_{fr} \cdot L$$

$$L = 1.75' \times 4$$

$$V \cdot d = A_o$$

$$A_{ft} \cdot \sigma = A_c \rightarrow \text{free flow}$$

$$\frac{\dot{m}}{A_c} = \dot{G} \Rightarrow Re = \frac{D_H \cdot \dot{G}}{\mu} \quad D_H = 0.02685'$$

$$h_o \leftarrow \frac{h_o}{G \cdot C_p} \cdot Pr^{2/3} = \# \text{ from chart.}$$

$$Ra = \frac{L}{\eta_o \cdot h_o \cdot A_o} \quad \eta_o = 0.8 \text{ from prev example}$$

Find f from chart

$$\Delta P = f \cdot \frac{A_o}{A_c} \cdot \frac{\dot{G}^2}{2\rho}$$