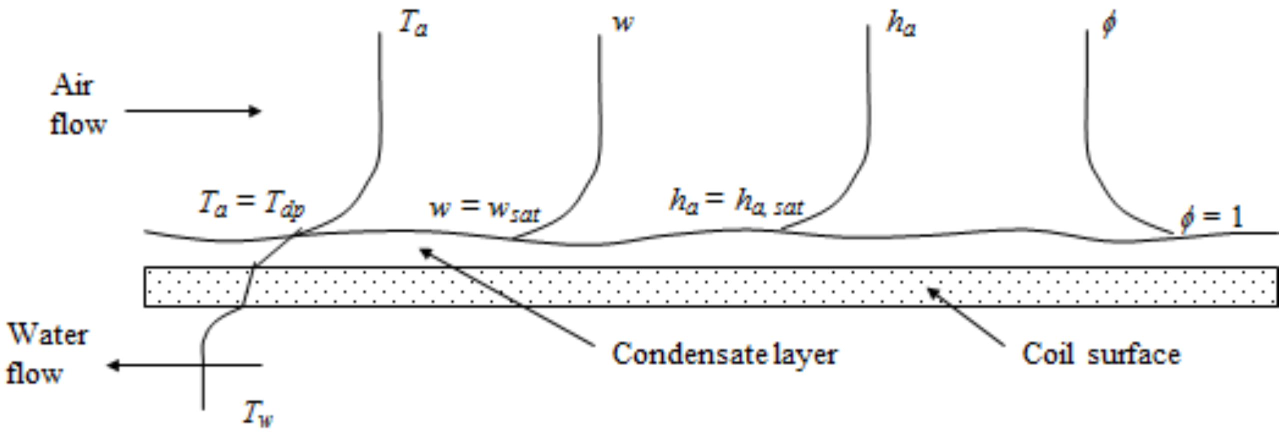
ME 418

Lecture 10 - Cooling Coil Analysis & Design

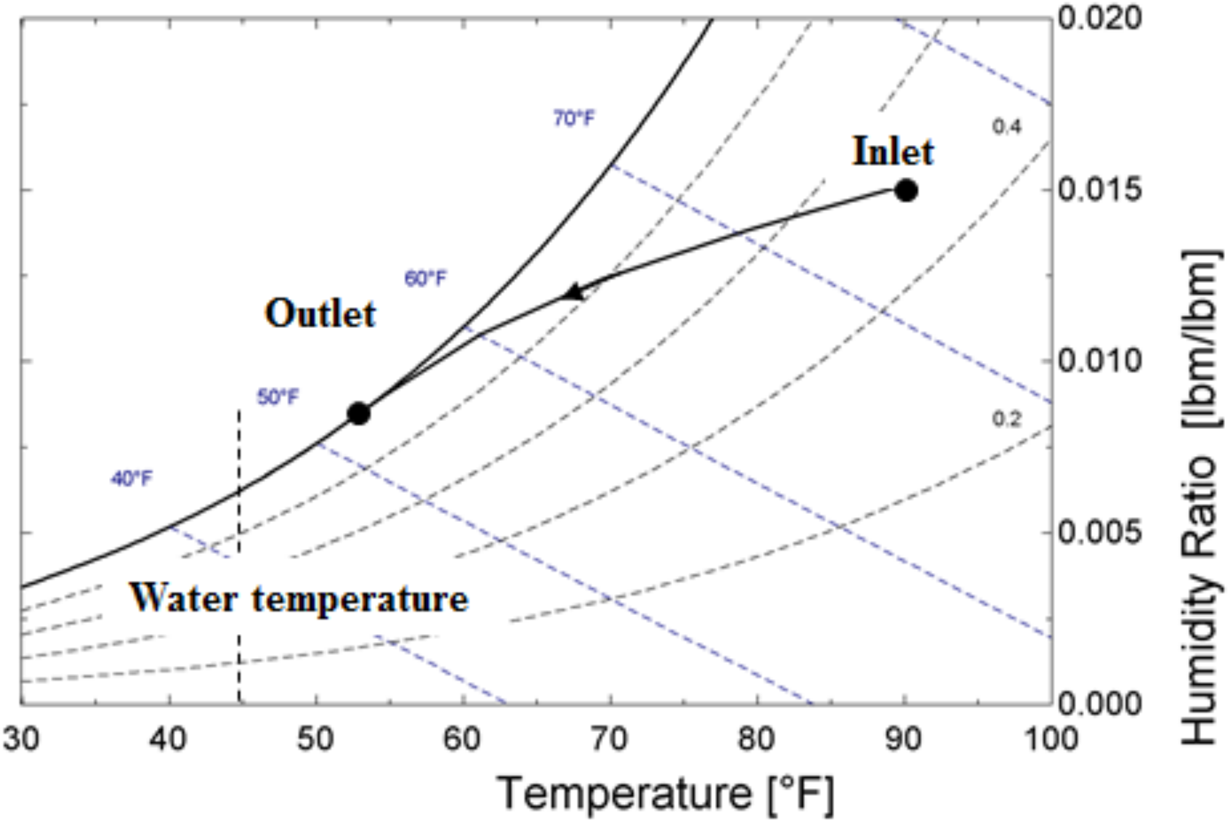
In-Class Notes for Fall 2024

- Cooling coil processes
- Heat and mass transfer analogy
- Effectiveness-NTU method
- Determining outlet state and SHR
- Wet surface fin efficiency

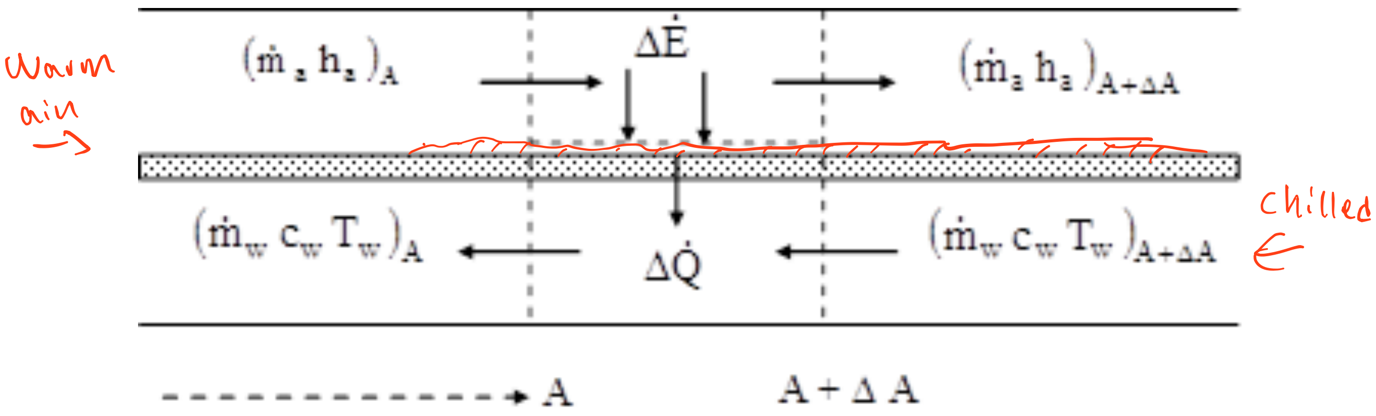


Cooling Coil Processes

- A cooling coil both cools (sensible) and **dehumidifies (latent)** an air stream.
- Latent load can make up to 40% of total load
- Due to imperfect air mixing, cooling process does not follow the saturation line.



Energy flows in the wet section of a cooling coil



Overall energy balance

$$(\dot m_a h_a)_A + (\dot m_w c_{p,w} T_w)_{A+\Delta A} - (\dot m_a h_a)_{A+\Delta A} - (\dot m_w c_{p,w} T_w)_A - \underbrace{\Delta \dot m_{cond} h_f}_{\text{negligible.}} = 0$$

Neglecting energy flow of condensate draining

$$\dot m_a \frac{dh_a}{dA_a} = \dot m_w c_{p,w} \frac{dT_w}{\underbrace{dA_a}_{\text{air-side HX area}}} \qquad (\#)$$

Air-side energy balance

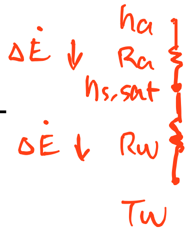
$$(\dot m_a h_a)_A - (\dot m_a h_a)_{A+\Delta A} - \underbrace{\Delta \dot E}_{\text{total energy loss}} = 0 \qquad (*)$$

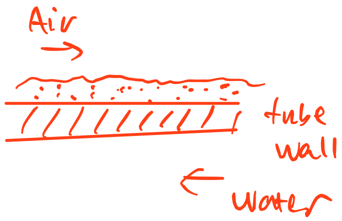
Air-side energy transfer rate

$$\Delta\dot{E} = \left[\underbrace{h_c\,(T_a - \overset{\text{surface temp}}{\boxed{T_s}})}_{\text{sensible}} + \underbrace{h_m\,(w_a - \overset{= h(T_s,\,\phi=1)}{\boxed{w_{s,sat}}})}_{\text{latent}}\; h_{fg,s}\right]\Delta A$$

Assuming Lewis number of unity gives

$$h_m = \frac{h_c}{c_{p,m}}$$





where the moist air specific heat is

$$c_{p,m} = c_{p,a} + w\,c_{p,v}$$

combined heat & mass transfer

Assuming $\underline{\eta_o^*}$ is the overall surface efficiency accounting for finned area, overall energy transfer is

$$\Delta\dot{E} = \frac{\eta_o^* h_c\ \Delta A}{c_{p,m}}\underbrace{(h_a - h_{s,sat})}$$

(*) divided by ΔA

$$\Downarrow$$

$$\dot{m}_a \frac{dh_a}{\underbrace{dA_a}_{\text{air-side HX area}}} = \underbrace{-\frac{\eta_o^* h_c}{c_{p,m}}(h_a - h_{s,sat})}_{\text{energy transfer from air to condensate surface}}$$

between ha and hs,sat

Energy balance on water side

$$\dot{m}_w\, c_{p,w} \frac{dT_w}{d\underbrace{A_w}_{\text{water-side HX area}}} = \underbrace{-U_w\left(T_{s,sat} - T_w\right)}_{\text{heat transfer from condensate surface to water}} \quad (**)$$

<u>Note</u>: conductance U_w includes heat resistance of condensate layer, conduction resistance of tube wall, and convection resistance of water stream

<u>Challenge</u>: air- and water-side energy balances involve enthalpy and temperature, respectively.

We define effective specific heat

$$f(T_w) = c_s = \left(\frac{dh_{w,sat}}{dT_w}\right)_{T_w}$$

$= h(\phi = 1,\ T = T_w)$

$= f(T_w)$

⇩

$$\frac{dh_{w,sat}}{d\cancel{T_w}} \cdot \frac{d\cancel{T_w}}{dA} = c_s \frac{dT_w}{dA} = \frac{dh_{w,sat}}{dA}$$

Then overall energy balance eq (#) becomes

$$\frac{dh_a}{dA_a} = \left(\frac{\dot{m}_w \, c_{p,w}}{\dot{m}_a \; c_s} \right) \frac{dh_{w,sat}}{dA_a}$$

replace $\frac{dT_w}{dA}$ with $\frac{1}{c_s} \cdot \frac{dh_{w,sat}}{dA}$ in (#)

Water-side energy balance can be re-written

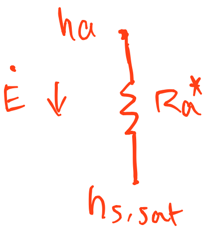
$$\dot{m}_w \, c_{p,w} \frac{dh_{w,sat}}{dA_w} = -\, U_w \left(h_{s,sat} - h_{w,sat} \right)$$

Multiply (**) by c_s on both sides

Effective specific heat can be numerically calculated using water inlet and outlet temperatures

$$c_s = \left(\frac{h_{w,sat,in} - h_{w,sat,out}}{T_{w,in} - T_{w,out}} \right)$$

Thermal resistance between the air enthalpy and the saturated air enthalpy at condensate layer temperature

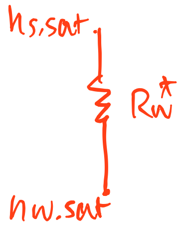


$$R_a^* = \frac{c_{p,m}}{\eta_o^* h_c \, A_a}$$

$$\dot{m}_a \Delta h_a = \dot{E} = \frac{\eta_o^* \cdot h_c \cdot A_a}{c_{pm}} \left[h_a - h_{s,sat} \right]$$

$$= \frac{h_a - h_{s.sat}}{R_a^*}$$

Thermal resistance between the saturated air enthalpy at the condensate layer temperature and the saturated air enthalpy at the water temperature

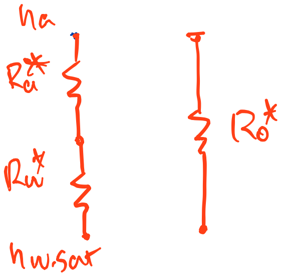


$= h(\phi=1, T_w)$

$$R_w^* = \frac{c_s}{U_w A_w}$$

$$\dot{E} = U_w \cdot A_w \cdot (T_{s,sat} - T_w)$$
$$= \frac{\boxed{U_w \cdot A_w} (h_{s,sat} - h_{w,sat})}{\boxed{c_s}}$$
$$= \frac{h_{s.sat} - h_{w.sat}}{R_w^*}$$

Overall thermal resistance between air and water



$$R_o^* = \frac{c_{p,m}}{\eta_o^* h_c A_a} + \frac{c_s}{U_w A_w}$$

$$\dot{E} = \frac{h_a - h_{w.sat}}{R_o^*}$$

Overall energy conductance between the air and water streams

$$U_o^* A_a = \frac{1}{R_o^*} = \frac{\dfrac{\eta_o^* h_c A_a}{c_{p,m}}}{1 + \dfrac{c_s \eta_o^* h_c A_a}{c_{p,m} U_w A_w}}$$

Energy balance on air side

$$\dot{m}_a \frac{dh_a}{dA_a} = -U_o^* (h_a - h_{w,sat}) = \frac{\partial \dot{E}}{dA_a}$$

Cooling Coil Performance using an Analogy to Heat Transfer

Overall energy balance

$$\frac{dh_a}{dA_a} = \underbrace{\left(\frac{\dot{m}_w\, c_{p,w}}{\dot{m}_a\, c_s}\right)}_{\frac{1}{m^*}} \frac{dh_{w,sat}}{dA_a}$$

Equivalent capacitance rate ratio $\frac{1}{m^*}$ $\sim c^* = \frac{C_{min}}{C_{max}}$

⇓

$$\frac{dh_a}{dA_a} = \frac{1}{m^*}\frac{dh_{w,sat}}{dA_a}$$

$$\frac{dT_{min}}{dA} = \frac{1}{C^*} \cdot \frac{dT_{max}}{dA}$$

Energy balance on air side

$$\dot{m}_a \frac{dh_a}{dA_a} = -U_o^*\left(h_a - h_{w,sat}\right)$$

$$C_{min} \cdot \frac{dT_{min}}{dA} = U \cdot (T_{min} - T_{max})$$

Define heat & mass transfer NTU

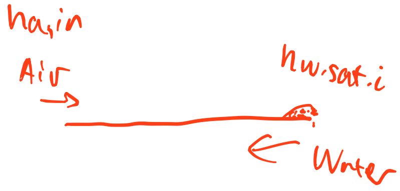
$$\mathrm{Ntu}^{*} = \frac{U_o^{*} A_a}{\dot{m}_a}$$

Energy balance on air side can be re-written as

$$\frac{dh_a}{dA_a} = -\frac{\mathrm{Ntu}^{*}}{A_a}\left(h_a - h_{w,\,sat}\right)$$

Define heat & mass transfer effectiveness

$$\varepsilon^{*} = \frac{\overbrace{\left(h_{a,in} - h_{a,out}\right)}^{\text{Actual enthalpy drop}}}{\underbrace{\left(h_{a,in} - h_{w,sat,in}\right)}_{\text{max enthalpy drop on air side}}}$$



Overall heat & mass transfer rate

$$\dot{Q} = \varepsilon^{*}\, \dot{m}_a \left(h_{a,in} - h_{w,sat,in}\right)$$

Analogous parameters for sensible heat exchangers and cooling coils

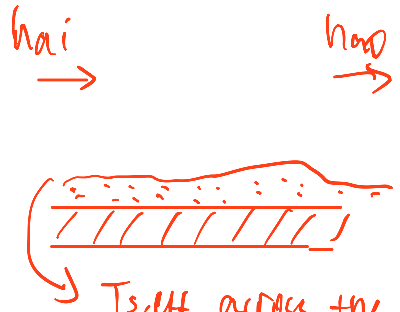
Parameter	Sensible heat exchanger	Cooling coil
Capacitance rate ratio	C^*	m^*
Number of transfer units	Ntu	Ntu^*
Effectiveness	$\varepsilon = f\left(C^*, Ntu\right)$	$\varepsilon^* = f\left(m^*, Ntu^*\right)$
Maximum heat flow	$C_{\min}\left(T_{h,i} - T_{c,i}\right)$	$\dot{m}_a\left(h_{a,in} - h_{w,sat,in}\right)$
Heat flow	$\varepsilon C_{\min}\left(T_{h,i} - T_{c,i}\right)$	$\varepsilon^* \dot{m}_a\left(h_{a,in} - h_{w,sat,in}\right)$

$= m_a \cdot (h_{ai} - \underline{h_{ao}})$

Air Outlet Temperature Calculation

Airside NTU for **energy** transfer with wetted surface

$$\frac{1}{R_a^* \cdot \dot{m}_a} = Ntu_a^* = \frac{\eta_o^* h_c A_a}{\dot{m}_a c_{p,m}}$$



Ts,eff across the whole HX surface (averagetemp).

Air outlet enthalpy

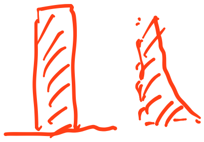
$$\underbrace{h_{s,eff}}_{T_{s,eff}} = h_{a,in} + \frac{h_{a,out} - h_{a,in}}{\underbrace{1 - e^{-Ntu_a^*}}_{\varepsilon \text{ equation for } C^* = 0}}$$

$\hat{\eta}_o$ η_o η_o^*

Airside NTU for **heat** transfer with wetted surface

$$Ntu_a = \frac{\hat{\eta}_o h_c A_a}{\dot{m}_a c_{p,m}}$$

overall fin efficiency for sensible transfer only, but with wet surface



Air outlet temperature

$$T_{a,out} = T_{s,eff} + \left(T_{a,in} - T_{s,eff}\right)e^{-Ntu_a}$$

Coil Heat Transfer

Totally Dry Coil

- Air-side surface temperature is above entering air dew point
- Condensation does not occur in the coil.
- Conventional sensible heat exchanger analysis using ε-Ntu

Totally Wet Coil

- Air-side surface temperature is below entering air dew-point
- Condensation occurs throughout coil.
- Analogy relations used to determine heat transfer

Partially Dry and Partially Wet Coil

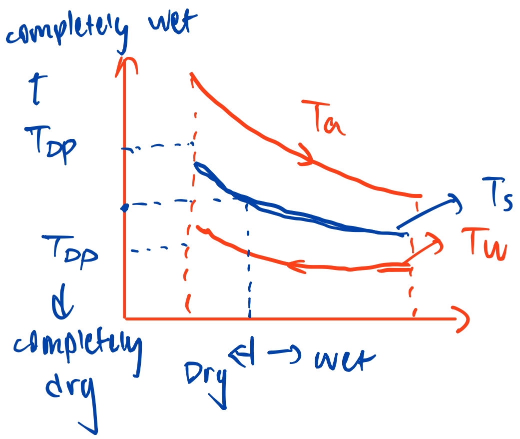
- Air-side surface is initially above entering air dewpoint and then drops.
- Dry section near the air entrance
- Wet section for the remainder of the coil.
- Determine location where condensation first occurs.

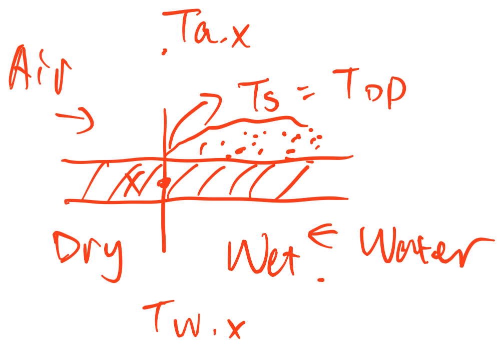
Determine the dry-wet interface location with a surface temperature of T_s

$$\frac{\left(T_{a,x} - T_s\right)}{R_a} = \frac{\left(T_s - T_{w,x}\right)}{R_w}$$

where $R_a = \frac{1}{\eta_o h_c A_a}$ $R_w = \frac{1}{U_w A_w}$

- $T_{a,x}$ is the mean air temperature at the interface
- $T_{w,x}$ is the water temperature at the interface





Cooling Coil Example: Determine the energy transfer, moisture condensate rate, and outlet air and water conditions for a chilled water coil. The air enters the coil at a dry bulb temperature of 80 F and a wet bulb temperature of 64 F with a flow rate of 21,000 lb/hr. The water enters at 42 F with a flow rate of 30,000 lb/hr. For this example the overall surface efficiencies for heat and energy transfer are taken to be the same value and for the airside the heat transfer conductance is 50 Btu/hr-ft^2 U_a and the A_a surface area is 360 ft^2. On the waterside the conductance is 1000 Btu/hr-ft^2 U_w and the area is 18 ft^2 A_w

Assume a counterflow

Dry analysis:

$C_a = m_a \cdot C_{pa}$ $\quad C_w = m_w \cdot C_{pw}$

$C_{min} = \min(C_a, C_w)$

$R_a = \frac{1}{U_a A_a}$ $\quad R_w = \frac{1}{U_w A_w}$ $\quad \frac{1}{UA} = \frac{1}{U_a A_a} + \frac{1}{U_w A_w}$

$Ntu = \frac{UA}{C_{min}} \Rightarrow \varepsilon$

$Q = \varepsilon \cdot C_{min}(T_{ai} - T_{wi})$

Wet analysis

$C_s = \frac{h_{wo} - h_{wi}}{T_{wo} - T_{wi}}$ $\quad m^* = \frac{m_a \cdot C_s}{m_w \cdot C_{pw}}$

$U_a^* = U_a \Rightarrow \frac{1}{U^* A} = \frac{C_{pa}}{U_a^* A_a} + \frac{C_{pw}}{U_w A_w}$

$Ntu^* = \frac{U^* A}{m_a} \Rightarrow \varepsilon^*$

$Q = \varepsilon^* \cdot m_a \cdot (h_{ai} - h_{w,sat,i})$

$= m_a(h_{ai} - h_{aout})$

$NTU_a = Ntu_a^* = \frac{U_a A_a}{m_a C_{pw}} \rightarrow h_{s,eff}, T_{s,eff}, T_{ao}$

Partially dry - partially wet analysis

$X \cdot A_a \rightarrow$ dry surface area

$(1-X) \cdot A_a \rightarrow$ wet surface area.

$Ntu_{dry} = Ntu \cdot X$

$Ntu^*_{wet} = Ntu^* \cdot (1-X)$ $\qquad T_s = T_{DP}$

$\frac{T_{ax} - T_s}{R_a} = \frac{T_s - T_{wx}}{R_w}$

Heat and Mass Transfer Fin and Surface Efficiencies

Fin efficiency for a wet coil can be calculated with an analogy to the dry coil fin efficiency.

	Sensible heat transfer only	Heat and Mass Transfer
Fin parameter	$m_f L_c = \sqrt{\frac{2h_c}{k_f t_f}} L_c$	$m_f^* L_c = \sqrt{\frac{2h_c}{k_f t_f} \frac{c_s}{c_{p,m}}} L_c$
Overall efficiency	$\eta_0 = 1 - \frac{A_f}{A_0}(1 - \eta_f)$	$\eta_o^* = 1 - \frac{A_f}{A_o}(1 - \eta_f^*)$

A heat transfer fin efficiency, need to determine outlet air temperature, can be estimated with

$$\hat{\eta}_{\cancel{o}f} = 1 - C_f \left(1 - \eta_f^*\right)$$

where C_f is a correction factor

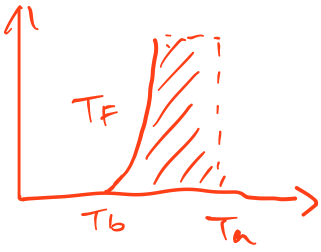
$$C_f = \frac{1}{\underbrace{c_s}_{\text{evaluated at } T_b}} \frac{h_{s,b} - \overset{\text{air enthalpy}}{\boxed{h_a}}}{\boxed{T_b} \underbrace{- T_a}_{\rightarrow \text{fin base temp}}}$$

Fin Efficiency Example: Determine the fin and overall surface efficiencies for a cooling coil. The tube diameter is 0.774 inch and the fins are steel with a thickness of 0.012 inch, a diameter of 1.463 inch, and a pitch of 9.05 fins per inch. The conductivity of steel is 35 Btu/hr-ft-F and the heat transfer coefficient is 14.4 Btu/hr-ft2-F. The air temperature is 70 F and saturated and the surface temperature is 50 F. $= T_b$

RH = 1

$$C_s \text{ evaluated at } T_b := \frac{H(\phi=1, T_b+1°F) - H(\phi=1, T_b)}{1°F}$$

$$m_f^* = \sqrt{\frac{2h_c}{k_f \cdot t_F} \cdot \frac{c_s}{c_{pm}}}$$



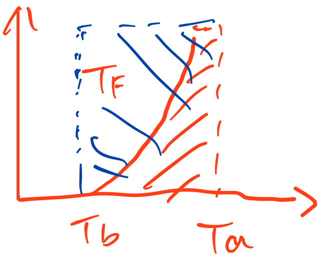
→ HT only.

η_f

$\eta_f = 0.76$

$\eta_f^* = 0.599$

$\hat{\eta}_f = 0.5$



→ H & M T.

$\eta_f^* < \eta_f$

$= \frac{\text{Red}}{\text{Blue}}$.