ME 418
Lecture 6 and 7 – Heat Transfer
In-Class Notes for Fall 2024

- Conduction, Convection, Radiation
- Combined-Mode Heat Transfer
- Heat and Mass Transfer for Air-Water Vapor Mixtures

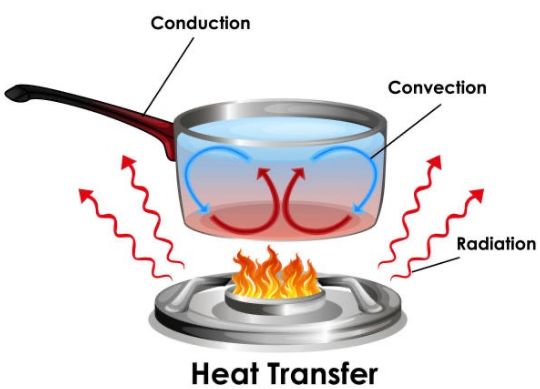
Heat Transfer

- energy transfer due to temperature difference

Conduction: heat transfer by molecular interactions

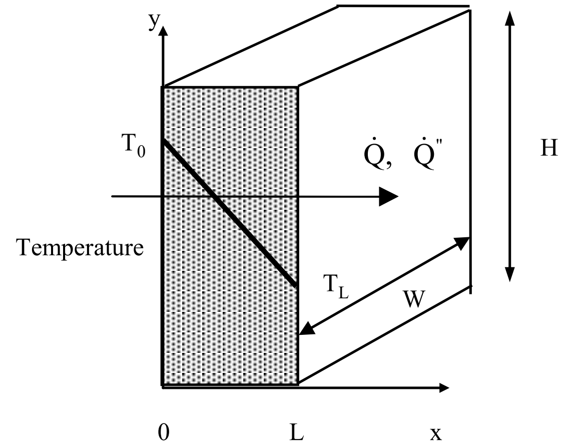
Convection: heat transfer by fluid motion

Radiation: heat transfer by electromagnetic waves



Conduction Heat Transfer

Consider 1-dimensional heat transfer in a wall ⇒ pretty much all we need in this class



Heat flux is represented with Fourier's law

$$\dot{Q}'' = -k\,\frac{dT}{dx}$$

k is thermal conductivity

Then, the total heat transfer rate for any wall cross-sectional area (A=LxH) is

$$\dot{Q} = -k\,A\,\frac{dT}{dx}$$

we'll consider transient later for building load analysis and thermal storage

At **steady state**, the heat transfer is the same at any cross-section in the wall. Assuming a constant k, then

$$\dot{Q} = k\,A\,\frac{T_0 - T_L}{L}$$

Typical Thermal Conductivity Ranges

Type of Material	Thermal Conductivity (Btu/hr-ft-F)	Thermal Conductivity (W/m-K)
Gases	0.01 - 0.10	0.02 - 0.20
Insulation materials	0.015 - 0.050	0.02 - 0.10
Building materials	0.1 - 1.0	0.2 - 2.0
Liquids	0.1 - 1.0	0.2 - 2.0
Metals	20 - 250	40 - 500

use for heat exchangers

** EES contains k values for many substances ***

Thermal Resistance

Thermal resistance is helpful in analyzing conduction problems with series and parallel heat transfer paths. It is defined as the ratio of the temperature difference to the heat flow or

$$R_W = \frac{T_0 - T_L}{\dot{Q}} = \frac{L}{kA}$$

Then $$\dot{Q} = \frac{T_0 - T_L}{R_W}$$

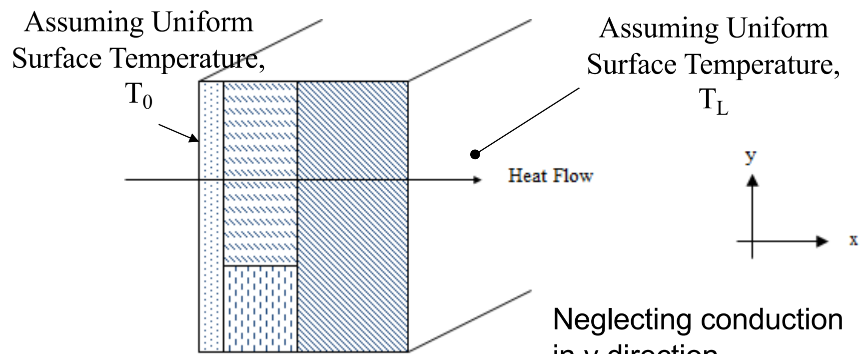
analagous to $i = \frac{V}{R}$ — voltage across resistor

Often use unit thermal resistance (resistance per unit area), **termed "R-value"** and defined as

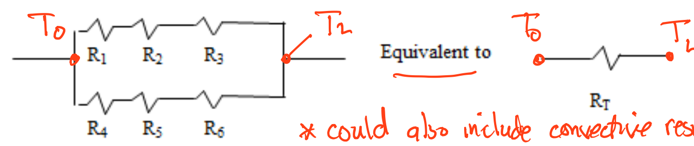
$$R''_W = \frac{L}{k} = A \cdot R_W$$

↳ how insulation is sold

1-D SS Conduction through Multiple Materials



Neglecting conduction in y direction



* could also include convective resistance *

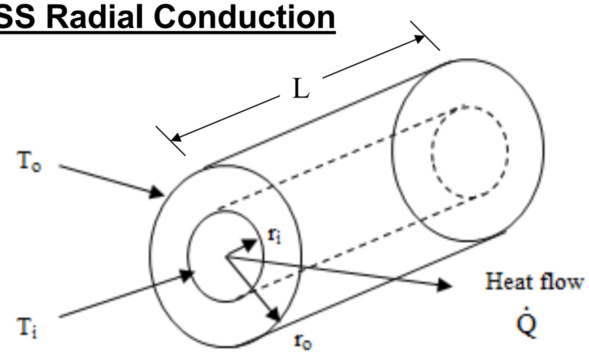
Add resistances (R's) that are in series and add conductances (1/R) that are in parallel to determine overall resistance to heat transfer

$$R_T = \frac{1}{\left[\left(\frac{1}{R_1 + R_2 + R_3}\right) + \left(\frac{1}{R_4 + R_5 + R_6}\right)\right]}$$

Then, the steady-state heat transfer rate is

$$\dot{Q} = \frac{T_0 - T_L}{R_T}$$

1-D SS Radial Conduction



insulation on a pipe

heat exchanger tube

From Fourier's law, the heat transfer rate at any cross-section in the radial direction is

$$\dot{Q} = -kA\frac{dT}{dr} = -k\overbrace{2\pi r}^{A(r)}L\frac{dT}{dr}$$

At steady state, the heat transfer rate is same for all cross-sections. For constant k, integrating the above equation between inner and outer surfaces leads to

$$\dot{Q} = -k(2\pi L)\frac{T_o - T_i}{\ln(r_o / r_i)} = \frac{T_i - T_o}{R_{cyl}}$$

Then, the overall heat transfer resistance is

$$R_{cyl} = \frac{\ln(r_o / r_i)}{2\pi k L}$$

Convection Heat Transfer

- Two types of convective flow

  heat exchanger { Forced convection: flow resulting from external source – pump, fan, wind,

  walls, window gaps, { Natural convection: buoyancy-driven flow due to temperature difference between surface and fluid

- Two types of flow geometries

  External Flow: fluid flows over a single surface (e.g., wall, outside of a pipe, …)

  Internal Flow: fluid flows through confined space (inside pipe or duct, between parallel plates, ..)

- Two types of flow regimes

  Laminar: streamline flow with no mixing, heat transfer by molecular conduction between layers

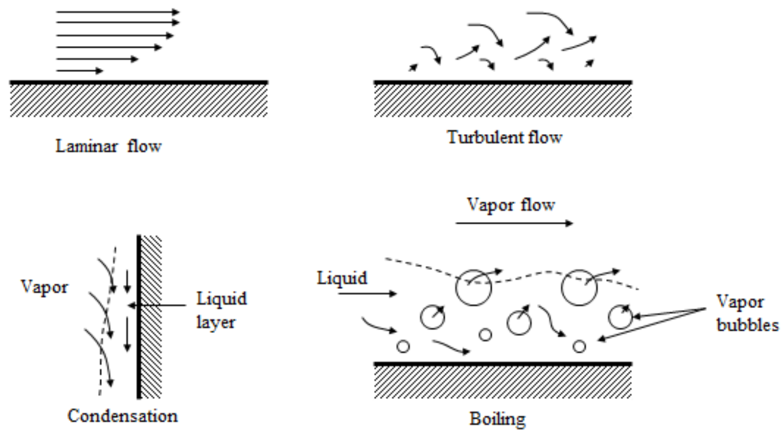
  Turbulent: fluid mixes across the flow stream to enhance heat transfer

- Sensible versus latent heat transfer

  Sensible: results in only a temperature change of the fluid

  Latent: associated with a change in phase such as for evaporation and condensation

* all of these are important for environmental systems *



- Heat transfer is enhanced through turbulence and two-phase heat transfer

Newton's Law of Cooling

Regardless of flow type, can represent heat transfer rate as

$$\boxed{\dot{Q} = h_c A (T_s - T_f)}$$

just need to get h_c

h_c convection coefficient
A surface area
T_s surface temperature
T_f fluid temperature (undisturbed fluid temperature for external flow, average fluid temperature for internal flow, other surface temperature for natural convection between parallel plates)

Typical Ranges for Convection Coefficients

Mode	h_c (Btu/hr-ft²-F)	h_c (W/m²-C)
Natural Convection		
Gases	0.3 - 5	1 - 25
Liquids	10 - 200	50 – 1000
Forced Convection		
Gases	5 - 50	25 – 250
Liquids	10 - 4000	50 – 20,000
Condensation/Boiling	200 - 20,000	1000 – 100,000

Liquids have larger thermal conductivities and specific heats than gases resulting in significantly higher heat transfer coefficients

Can also represent convective heat transfer using a thermal resistance

$$\dot{Q} = \frac{(T_s - T_f)}{R_c}$$

where R_c is

$$\boxed{R_c = \frac{1}{h_c A}}$$

This allows conduction and convective resistances to be combined.

A unit thermal resistance is then defined as

$$R_c'' = R_c A = \frac{1}{h_c}$$

Heat Transfer Coefficient Correlations

- Convective heat transfer coefficients (h_c) usually cannot be calculated from theory
- Typically correlate Nusselt Number (Nu) with other dimensionless numbers, where

$1/k_f$

$$\mathrm{Nu} = \frac{h_c L}{k_f} = \frac{conductive\ resistance}{convective\ resistance}$$

$1/h_c$

$\Rightarrow$ then $h_c = Nu \cdot \frac{k_f}{L}$

k_f - fluid thermal conductivity
L - characteristic length of surface in contact with fluid

- circular pipe - pipe diameter (D)
- non-circular pipe or duct – hydraulic diameter (D_H)
- flow between parallel plates – plate spacing
- flow parallel to flat plate – plate length along flow
- flow across a cylinder - cylinder diameter (D)
- flow over a sphere - sphere radius (D)

Functional relationships for Nu depend on nature of the flow:

- forced or natural convection
- internal or external flow
- laminar or turbulent flow
- single or two-phase flow

For **forced convection** of a **single-phase fluid**, average Nusselt numbers typically are correlated as a function of Reynolds (Re) and Prandtl (Pr) numbers

$$Nu_L = f(Re_L, \mathrm{Pr})$$

where

free stream for external flow, average for internal flow

$$\mathrm{Re}_L = \frac{V_f\, \rho_f\, L}{\mu_f} = \frac{inertial\ forces}{viscous\ forces}$$

characteristic length

$$\mathrm{Pr} = \frac{c_{p,f}\, \mu_f}{k_f} = \frac{\nu_f}{\alpha_f} = \frac{momentum\ diffusivity}{thermal\ diffusivity}$$

Nu$_L$ increases with Re and Pr

$$\nu_f = \frac{\mu_f}{\rho_f} = momentum\ diffusivity$$

$$\alpha_f = \frac{k_f}{\rho_f c_{p,f}} = thermal\ diffusivity$$

importance of momentum to thermal conductance transport

and μ_f is dynamic viscosity of the fluid.

Depending on the flow geometry, L is length, diameter, or hydraulic diameter (DH). Recall that

$$D_H = \frac{4A_c}{WP}$$

where WP is wetted perimeter.

Nusselt correlations for **forced convection** of a **single-phase fluid**

- see Table 4.3 for different flow geometries and regimes
- correlations for different flow geometries and regimes included as functions in EES

For **natural convection**, average Nusselt numbers are generally correlated in terms of Grashof or Rayleigh numbers along with Prandtl number

$$\mathrm{Nu} = f(\mathrm{Gr}, \mathrm{Pr}) \text{ or } f(\mathrm{Ra}_L, \mathrm{Pr})$$

Where

Nu increases with Gr and Pr ⇒ Ra

$$\mathrm{Gr}_L = \frac{g\beta_f |T_s - T_\infty| L^3}{\nu_f^2} = \frac{bouyancy\ forces}{viscous\ forces}$$

$$\mathrm{Ra}_L = \mathrm{Gr}_L \mathrm{Pr} = \frac{g\beta_f |T_s - T_\infty| L^3}{\nu_f \alpha_f}$$

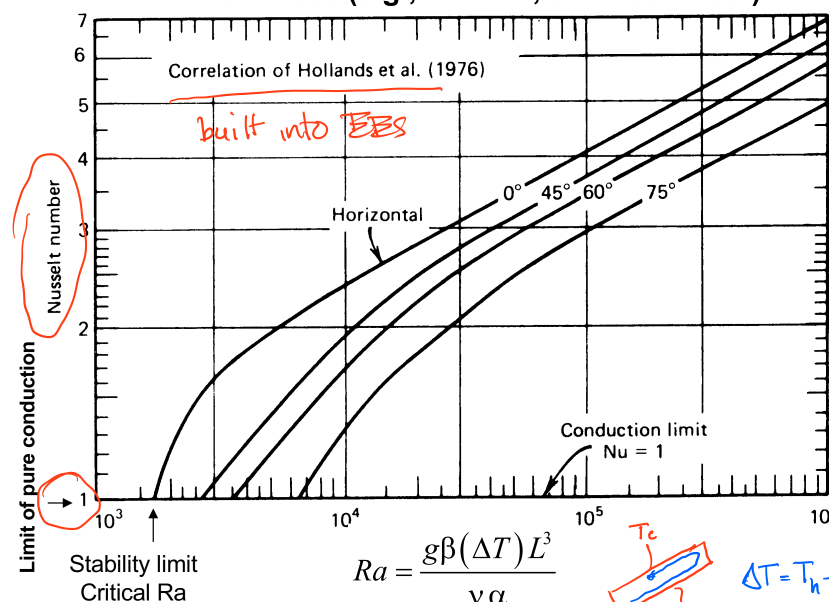
β_f is the expansion coefficient for the fluid, g is gravitational acceleration, T_∞ is temperature of the fluid far from the surface (or other surface for parallel plates)

Unlike forced convection, heat transfer coefficients for natural convection are driven by temperature differences.

Nusselt correlations for **natural convection** of a **single-phase fluid**

- see Table 4.4 for different flow geometries and regimes
- correlations for different flow geometries and regimes included as functions in EES

Example Correlation for Natural Convection between Parallel Plates (e.g., window, solar collector)



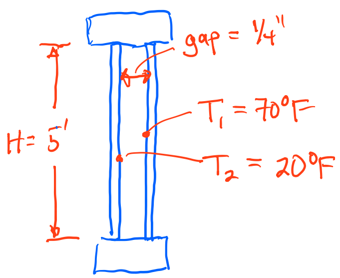
$$Ra = \frac{g\beta(\Delta T)L^3}{\nu\alpha}$$

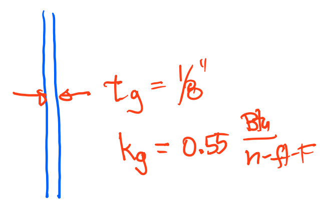
T_c T_h $\Delta T = T_h - T_c$

L = plate spacing; $\beta = 1/T_{avg}$ (for air); ΔT is temperature difference between surfaces of parallel places; use 75° curve for vertical plates

Natural Convection Example

Consider a 5 ft high window having 2 panes of glass with an air gap of 1/4 inch. The interior and exterior of the window are at 70 F and 20 F, respectively. Estimate the heat transfer coefficient for natural convection in the air gap. How does unit thermal resistance due to convective heat transfer in the gap compare to the unit thermal resistance for conduction through a glass pane (thickness of 1/8 inch and thermal conductivity of 0.55 Btu/h-ft-F)? **Note: use the EES function "FC_Tilted_Rect_Enclosure_ND" for natural convection in an enclosure.**





$$\boxed{R''_w = \frac{t_g}{k_g}}$$

use air properties and conditions to compute Ra

use ~~EES~~ function to get Nu

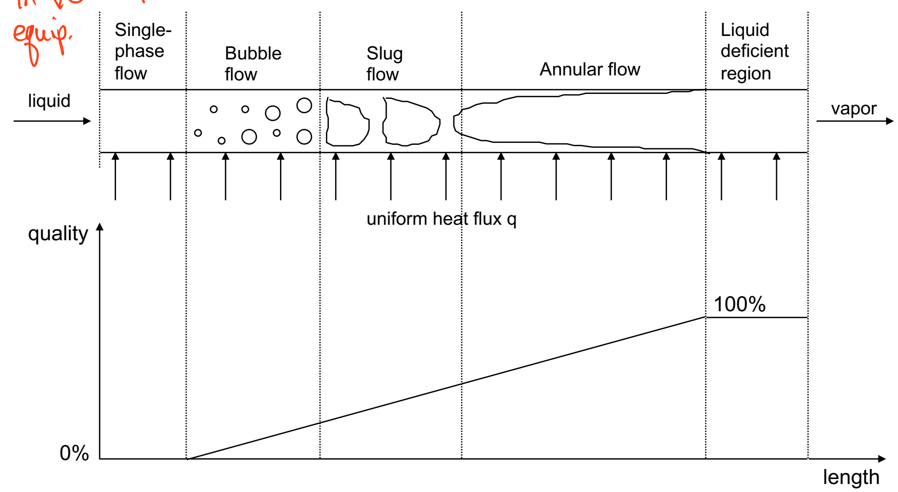
$\Rightarrow h_c = Nu \cdot \frac{k_a}{gap}$ ← thermal cond. of air

$$\boxed{R''_c = \frac{1}{h_c}}$$

For **two-phase (boiling or condensing) heat transfer**, heat transfer coefficient can vary significantly throughout the flow and determining an average value is more complex.

Need for evaporators in VC equip.

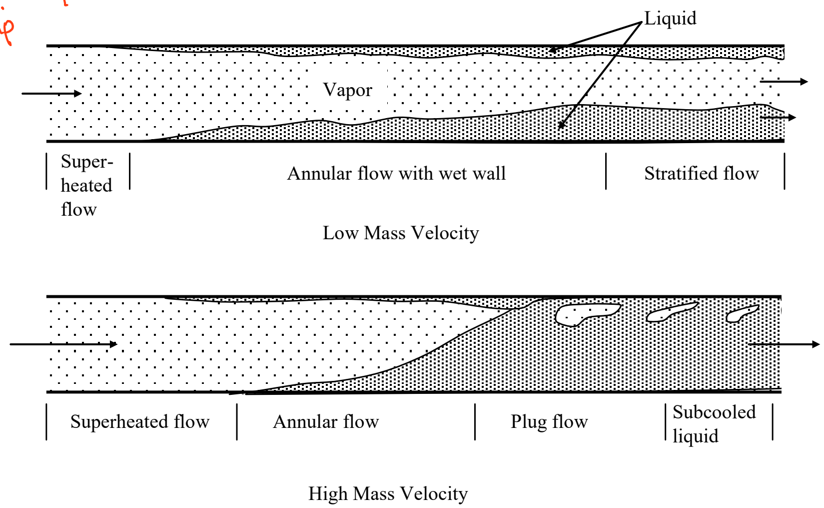
Flow Regimes for Boiling Heat Transfer in a Horizontal Tube



- Typically divide heat exchanger into single and two-phase sections for analysis
- Two-phase heat transfer coefficient depends strongly on quality --> necessary to average across qualities to determine average heat transfer coefficient

Condens used in re. equip

Flow Regimes for Condensing Heat Transfer in a Horizontal Tube



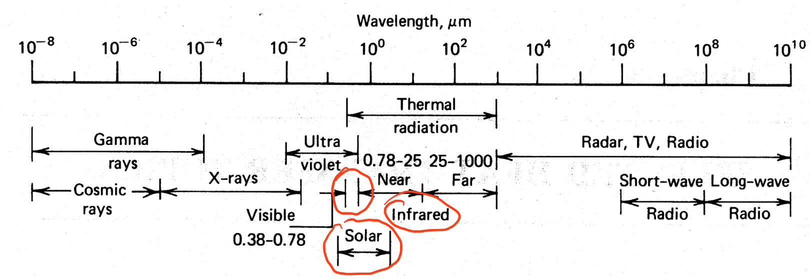
Correlations for **boiling (evaporating) and condensing heat transfer coefficients**

- see Supplemental Material Section 7 (SM7)
- correlations for different flow geometries included as functions in EES

**

Radiation Heat Transfer

- Radiation is the transfer of energy via electromagnetic waves.
- All objects emit radiation to an extent dependent on their temperature and surface properties.



- The waves propagate through a medium of refractive index n at speed C, where

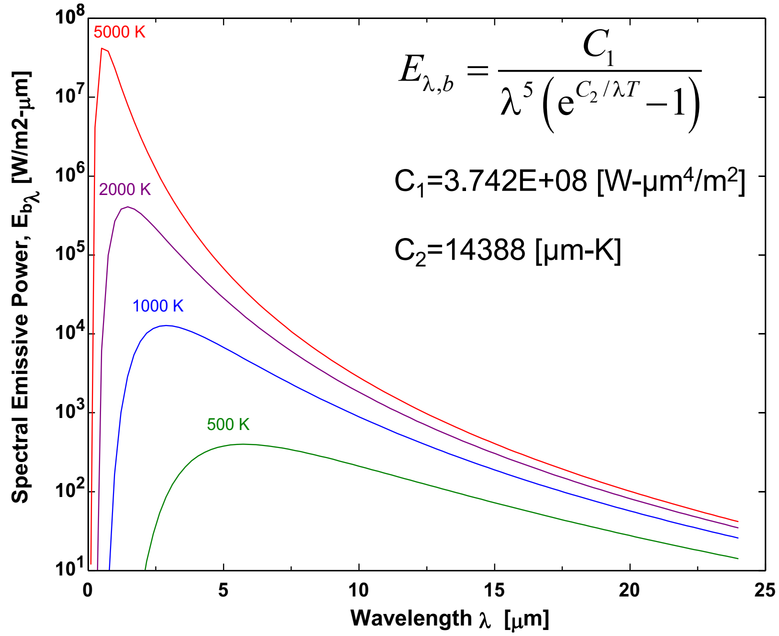
$$C = C_o / n = \lambda \nu \qquad C_o = 299{,}796 \text{ km/s}$$

where is λ wavelength and ν is frequency

Radiation Emission

- Thermal radiation energy is distributed over a range of wave wavelengths with overall intensity that depends temperature
- A **blackbody** is an "idealized" surface that emits the maximum possible thermal radiation

Blackbody Emissive Power (Planck's Law)



- At higher surface temperatures: greater total energy emission and a larger fraction of emissions at shorter wavelengths (higher frequencies)
- Two types of radiation emission to consider for engineered environments

  Short-wave radiation: solar radiation (high temperature source (~5777 K), high frequency)

  Long-wave radiation: infrared radiation (low temperature source (< 500 K), low frequency)

- Determine **total blackbody emission** (per unit area) at given T using Stefan-Boltzman equation

$$\dot{E}_b = \int_0^\infty \dot{E}_{b\lambda} d\lambda = \sigma T_s^4$$

where σ is the Stefan-Boltzmann constant (0.1714 x 10^{-8} Btu/hr-ft^2-R^4 or 5.67x10^{-8} W/m^2-K^4) and T_s is the absolute temperature of the surface (R or K)

- Total **radiation emission from an actual surface (E_s)** can be related to blackbody emission using an **overall emissivity (ε),** so that

$$\boxed{\dot{E}_s = \varepsilon \cdot \dot{E}_b = \varepsilon \cdot \sigma T_s^4}$$

per unit area

where ε depends on surface temperature (i.e., wavelength distribution of blackbody emission) and properties (wavelength dependence of emissivity, ε_λ)

Radiation Absorption

- Ideal black surface absorbs all incident radiation
- **Absorbed radiation for an actual surface** is

$$\dot{E}_{abs} = \alpha \cdot \dot{G}$$

where $\dot{G}$ is incident radiation (per unit area) on the surface and α is the surface absorptivity (fraction of incident radiation absorbed)

- Overall $\boldsymbol{\alpha}$ **depends on the source temperature** (i.e., wavelength distribution of the incoming radiation) **and properties of the surface** (i.e., wavelength dependence of absorptance, α_λ)

therefore, not a surface property

Emissivity vs Absorptivity

If the **radiation source and receiving surface are close in temperature** (e.g., radiation exchange between surfaces in a room), then the wavelength distributions of incident and emitted radiation are similar for each surface. In this case, for each surface a **"gray" surface assumption** applies and

$$\alpha = \varepsilon$$

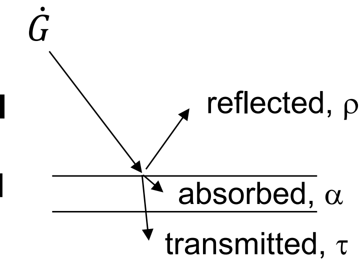
Not true if radiation source is at a much higher temperature than the surface. For instance, a surface's solar absorptance (α_s) is not typically very similar to its long-wave emissivity (ε)

$$\alpha_s \neq \varepsilon$$

Material	ε Long-wave emissivity	α_s Solar absorptivity
Building materials	0.90 - 0.96	0.6 - 0.7
Wood	0.9	0.9 - 0.96
Dark colored paints	0.91 - 0.95	0.98
Light colored paints	0.8	0.2
Galvanized metal	0.28	0.8
Aluminum, polished	0.03	0.09
Window glass	0.9 – 0.95	0.02 – 0.04
Water	0.96	
Ice	0.95	0.3 - 0.4

Transmissivity

For a **transparent material** (e.g., glass), radiation can be reflected, absorbed, and transmitted.



$$\alpha + \rho + \tau = 1$$

where τ is transmissivity and ρ is reflectivity. For an **opaque surface** (e.g., a wall),

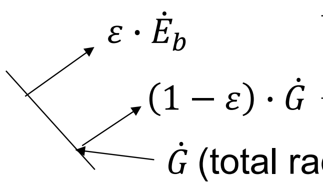
$$\alpha + \rho = 1 \rightarrow \rho = 1 - \alpha$$

For radiation exchange with the gray approximation, $\alpha = \varepsilon$ and

$$\rho = 1 - \varepsilon$$

Radiation Exchange

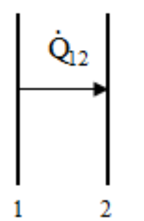
Consider radiation exchange within an enclosure. For any surface, the net radiation gain is the difference between total incident radiation emitted from all other surfaces and radiation leaving due to reflection and emission

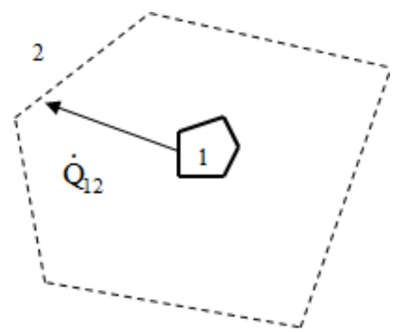


$$\dot{J} = \underbrace{\varepsilon \cdot \dot{E}_b + (1-\varepsilon) \cdot \dot{G}}$$

$\dot{G}$ (total radiation incident on surface)

where $\dot{J}$ is radiosity. $\dot{G}$ for each surface should account for all reflections and emissions from other surfaces (i.e., radiosities for other surfaces). Let's consider exchange between 2 surfaces





The net radiation heat transfer from surface 1 to surface 2 is

$$\dot{Q}_{12} = \underbrace{A_1 \dot{J}_1}_{\text{everything leaving 1}} \underbrace{F_{12}}_{\text{fraction striking 2}} - \underbrace{A_2 \dot{J}_2}_{\text{everything leaving 2}} \underbrace{F_{21}}_{\text{fraction striking 1}}$$

where F_{12} and F_{21} are view factors from 1 to 2 and 2 to 1, respectively. A view factor only depends on geometry and represents the fraction of energy leaving one surface that goes directly to the other surface.

The two view factors are related through the areas by an expression termed reciprocity.

$$A_1 F_{12} = A_2 F_{21}$$

So that

$$\dot{Q}_{12} = A_1 F_{12} \left(\dot{J}_1 - \dot{J}_2\right) = A_2 F_{21} \left(\dot{J}_1 - \dot{J}_2\right)$$

We can formulate similar equations for all surfaces in an enclosure, requiring solution of a system of equations (for given areas and view factors).

Let's consider enclosures with only 2 surfaces (large parallel plates, surface radiating to all other surfaces in an enclosure)

In this case, $F_{12} \sim 1$ and

$$\dot{Q}_{12} = A_1(\dot{J}_1 - \dot{J}_2)$$

$$\dot{J}_1 = \varepsilon_1 \cdot \sigma T_1^4 + (1-\varepsilon_1) \cdot \frac{\dot{J}_2 A_2}{A_1}$$

$$\dot{J}_2 = \varepsilon_2 \cdot \sigma T_2^4 + (1-\varepsilon_2) \cdot \frac{\dot{J}_1 A_1}{A_2}$$

Solving these equations results in

$$\dot{Q}_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\frac{A_1}{A_2}}$$

For the **parallel surfaces** case: $A_1 = A_2$ and

$$\boxed{\dot{Q}_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}}$$

For a small surface radiating to a large enclosure, $A_1 << A_2$ and

$$\boxed{\dot{Q}_{12} = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4)}$$

* need to use absolute T's *

surface 2 acts like a black body so no ε_2

Linearized Radiation Coefficient

When considering combined mode heat transfer (conduction, convection, radiation), it is convenient to define a linearized radiation coefficient as

$$h_r = \frac{\dot{Q}_{12}}{A_1(T_1 - T_2)}$$

For the **parallel surfaces** case with $A_1 = A_2$,

$$h_r = \frac{\sigma(T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)}{(1/\varepsilon_1 + 1/\varepsilon_2 - 1)(T_1 - T_2)}$$

or

$$\boxed{h_r = \frac{\sigma(T_1^2 + T_2^2)(T_1 + T_2)}{(1/\varepsilon_1 + 1/\varepsilon_2 - 1)}} \sim \frac{4\sigma\bar{T}^3}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

For **small surface radiating to large enclosure** $(A_1 << A_2)$

$$\boxed{h_r = \varepsilon_1 \sigma (T_1^2 + T_2^2)(T_1 + T_2)} \sim \varepsilon_1 4\sigma\bar{T}^3$$

For either case,

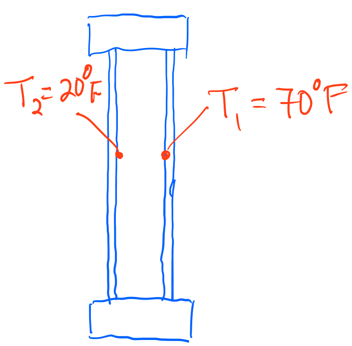
$$(T_1^2 + T_2^2)(T_1 + T_2) = 4\frac{(T_1^2 + T_2^2)}{2}\frac{(T_1 + T_2)}{2} \sim 4\bar{T}^3$$

where

$$\boxed{\bar{T} = \frac{(T_1 + T_2)}{2}}$$

Example

Consider radiation exchange between two glass panes within a window where the glass emissivity for longwave radiation is 0.9. The interior of the window is at a temperature of 70 F and the exterior of the window is at 20 F. Estimate the linearized heat transfer coefficient for radiation exchange across the air gap and compare with the natural convection coefficient previously determined. Now study the effect of exterior window temperature on linearized radiation heat transfer coefficient between 20 F and 90 F for the exact and approximate approaches. Is the approximate approach reasonable? Is it reasonable to assume constant radiation heat transfer coefficient?



$$\varepsilon_1 = \varepsilon_2 = 0.9$$

$$h_r = \frac{\sigma (T_1^2 + T_2^2)(T_1 + T_2)}{\left(\frac{2}{\varepsilon} - 1\right)}$$

$$h_{r,alt} = \frac{4\sigma \bar{T}^3}{\left(\frac{2}{\varepsilon} - 1\right)}$$

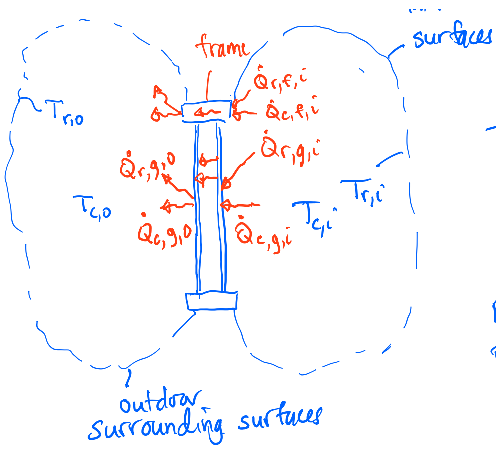
$$\bar{T} = \frac{T_1 + T_2}{2} + 459.67$$

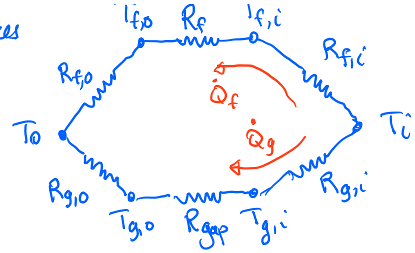
Combined Mode Heat Transfer

- Many applications involve all 3 modes of heat transfer occurring simultaneously
- Thermal circuits can be used to depict heat transfer paths, resistances, etc
- Energy balances applied at nodes for heat flows
- Parallel and series resistances can be combined to simplify problem solutions
- Algebraic equations can be solved for SS analysis

Example

Consider a 2-pane window with characteristics from earlier examples and having overall dimensions of 3' wide and 5' high with a 3" wide and 1.5" thick solid wood (pine) window frame. The window is located on a building that is maintained at an indoor temperature of 70 F with an outdoor temperature 20 F. Assume the interior window surface has radiation exchange with interior surfaces at the interior air temperature and the exterior window surface radiates to surfaces at the outdoor temperature. The convection coefficients for inside and outside surfaces are ~~3~~ 0.5 Btu/hr-ft^2-F and 2 ~~5~~ Btu/hr-ft^2-F, respectively. Draw a thermal circuit. For the window system, determine: a) overall heat transfer rate; b) overall unit thermal resistance, c) overall unit thermal conductance.

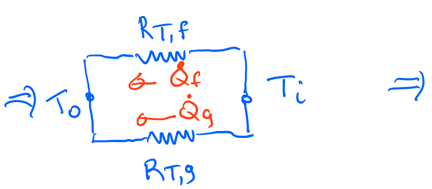




$R_{f,i} = 1/(h_{c,i} \cdot A_f + h_{r,i} \cdot A_f)$
$R_{g,i} = 1/(h_{c,i} \cdot A_g + h_{r,i} \cdot A_g)$
$R_f = t_f/k_f/A_f$
$R_{gap} = 1/(h_{c,gap} \cdot A_g + h_{r,gap} \cdot A_g)$
$R_{f,o} = 1/(h_{c,o} \cdot A_f + h_{r,o} \cdot A_f)$
$R_{g,o} = 1/(h_{c,o} \cdot A_g + h_{r,o} \cdot A_g)$

Assume $T_{r,o} = T_{c,o} = T_o$
$T_{r,i} = T_{c,i} = T_i$
$\varepsilon_g = \varepsilon_f = 0.9 = \varepsilon$
negligible glass conduction resistance

$\dot{Q} = \dot{Q}_f + \dot{Q}_g$



$$R_T = \frac{1}{1/R_{T,f} + 1/R_{T,g}}$$

$$\left.\begin{aligned} R_{T,f} &= R_{f,i} + R_f + R_{f,o} \\ R_{T,g} &= R_{g,i} + R_{gap} + R_{g,o} \end{aligned}\right\}$$

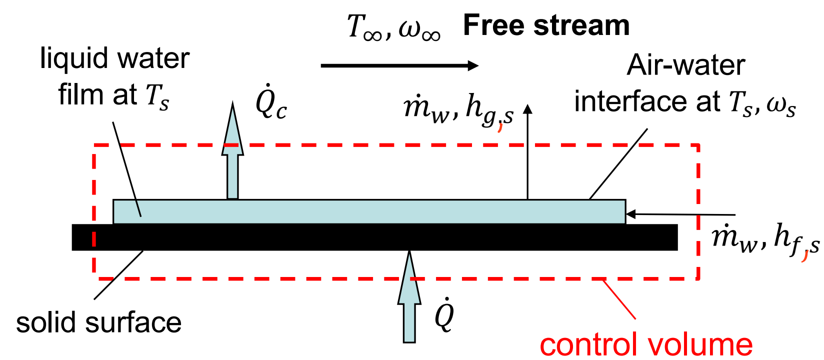
R-value $= (A_g + A_f) \cdot R_T$
U-value $= 1/$R-value

Also $\left.\begin{aligned} h_{r,i} &\cong \varepsilon \cdot \sigma \cdot 4 T_i^3 \\ h_{r,o} &\cong \varepsilon \cdot \sigma \cdot 4 T_o^3 \end{aligned}\right\}$ radiation to large enclosure, $\bar{T}_i \sim T_i$, $\bar{T}_o \sim T_o$

$h_{r,gap} \cong 4 \cdot \sigma \bar{T}^3/(2/\varepsilon - 1)$ $\quad \bar{T} = (T_i + T_o)/2$

Heat and Mass Transfer for Air-Water Mixtures

- Heat transfer is accompanied by evaporation ~~of~~ or condensation of water vapor in many applications (e.g., cooling towers/coils, …)
- Convective mass and heat transfer mechanisms are similar (often called "analogy between heat and mass transfer")
- Consider air flow over a wetted surface with mass transfer & evaporation (e.g., cooling tower)



where $\dot{Q}_c$ is convective (sensible) heat transfer rate from surface to air, $\dot{m}_w$ is the water mass flow rate (water vapor from surface due to mass transfer and liquid water make up to surface), $\dot{Q}$ is the total heat transfer rate to the surface to balance energy flows

From a steady-state energy balance for the control volume

$$\dot{Q} = \dot{Q}_c + \dot{m}_w\left(h_{g,s} - h_{f,s}\right) = \dot{Q}_c + \dot{m}_w\, h_{fg,s}$$

but applying Newton's Law of Cooling, the convective heat transfer is

$$\dot{Q}_c = h_C A(T_s - T_\infty)$$

An analogous relationship for mass transfer of water vapor from the air-water interface can be developed from Fick's law

$$\dot{m}_w = h_m A(\omega_s - \omega_\infty)$$

where h_m is a mass transfer coefficient and ω_s is the humidity ratio of saturated air at the air-water interface ($\omega_s = \omega(T_s, \phi = 1)$)

But the mass transfer coefficient can be related to the heat transfer coefficient

$$h_m \cong \frac{h_c}{c_{p,m}} \mathrm{Le}^{-2/3}$$

where Le is the Lewis number (ratio of thermal to mass transfer diffusivities) and $c_{p,m}$ is the mixture specific heat ($c_{p,m} \sim c_{p,a} + \omega_\infty c_{p,v}$)

So

$$\dot{Q} = h_C A(T_s - T_\infty) + \overbrace{\frac{h_c A}{c_{p,m}} \mathrm{Le}^{-2/3}(\omega_s - \omega_\infty)}^{\dot{m}_w} h_{fg,s}$$

For air-water mixtures $\mathrm{Le} \sim 1$, so

$$\dot{Q} = \frac{h_c A}{c_{p,m}} \left\{c_{p,m}(T_s - T_\infty) + (\omega_s - \omega_\infty) h_{fg,s}\right\}$$

But recall that the mixture enthalpy is

$$h = c_{p,m} T + \omega h_{fg,0}$$

So

$$\dot{Q} = \frac{h_c A}{c_{p,m}} \left\{(h_s - h_\infty) + (\omega_s - \omega_\infty)\left(h_{fg,s} - h_{fg,0}\right)\right\}$$

where h_s is saturated air enthalpy at the air-water interface ($h_s = h(T_s, \phi = 1)$). Note that we have assumed that $c_{p,m}$ is the same in the free stream and near the interface (a very reasonable assumption). Also, note that

$$(\omega_s - \omega_\infty)\left(h_{fg,s} - h_{fg,0}\right) \ll (h_s - h_\infty)$$

also a very good assumption

So

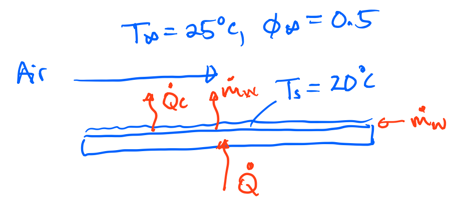
$$\boxed{\dot{Q} = \frac{h_c A}{c_{p,m}}(h_s - h_\infty)} \quad **$$

- air-water vapor mixture specific enthalpy is the driving force for total energy transfer for heat and mass exchangers that involve evaporation and condensation (can apply to cooling coils, cooling towers, evaporative coolings, …)
- can be extended to adsorption (desiccant) and membrane devices

Also $$\boxed{\dot{m}_w = \frac{h_c A}{c_{p,m}}(\omega_s - \omega_\infty)} \quad *$$

Example

Air at a temperature of 25 C and 50% relative humidity flows over a heated surface covered with a film of water at 20 C. The convection coefficient is 70 W/m^2-C and the surface area is 2 m^2. Determine the evaporation mass transfer rate, the convection heat transfer rate, and the rate of energy that must be supplied to maintain the surface at 20 C. What would be the temperature of the surface if the surface were not heated?



$\dot{Q}_c = h_c \cdot A (T_s - T_\infty)$ < 0 since $T_s < T_\infty$

$\dot{m}_w = \dfrac{h_c \cdot A}{c_{p,m}} (\omega_s - \omega_\infty)$ > 0 if there is evap.

$\dot{Q} = \dfrac{h_c A}{c_{p,m}} (h_s - h_\infty)$ could be > 0 or < 0 depending on specified T_s

where $h_s = h(T = T_s, \phi = 1)$
$h_\infty = h(T = T_\infty, \phi = \phi_\infty)$
$\omega_s = \omega(T = T_s, \phi = 1)$
$\omega_\infty = \omega(T = T_\infty, \phi = \phi_\infty)$

If $\dot{Q} = 0$, then

$h_s = h_\infty$ $\Rightarrow$ surface is evap. cooled, so $T_s \sim T_{WB,\infty}$ (can check)

$T_s = T(h = h_\infty, \phi = 1)$