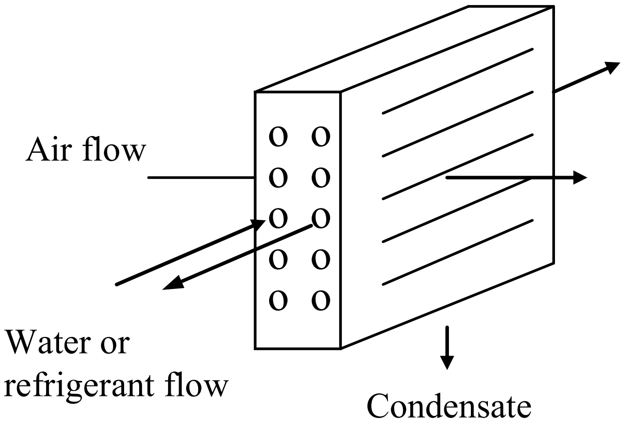
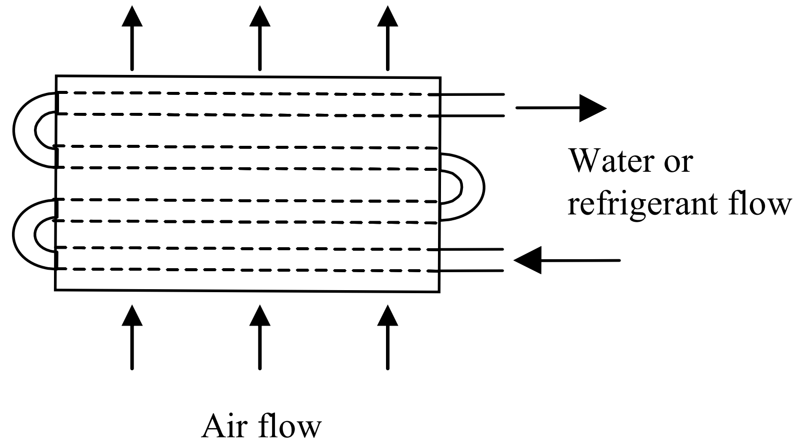
ME 418
Lecture 9 - Heat Exchanger Analysis & Design
In-Class Notes for Fall 2024

- Heat exchanger overview
- Overall heat exchanger conductance
- Heat transfer analysis – LMTD method
- Heat transfer analysis – effectiveness-NTU method
- Fin efficiencies
- Heat transfer coefficient
- Flow pressure drop

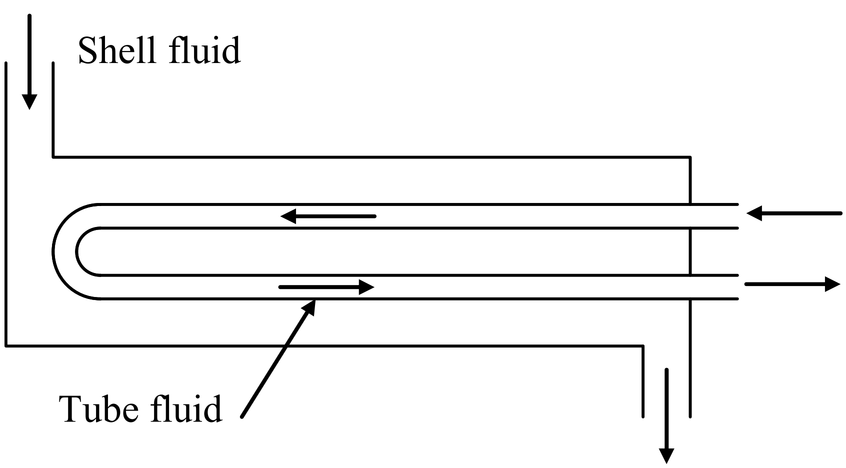
Heat Exchanger Types

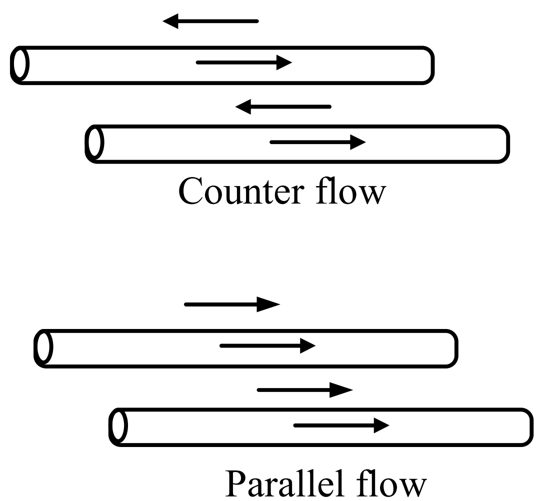
Coil used for heating or cooling air





Shell and tube exchanger used for liquids

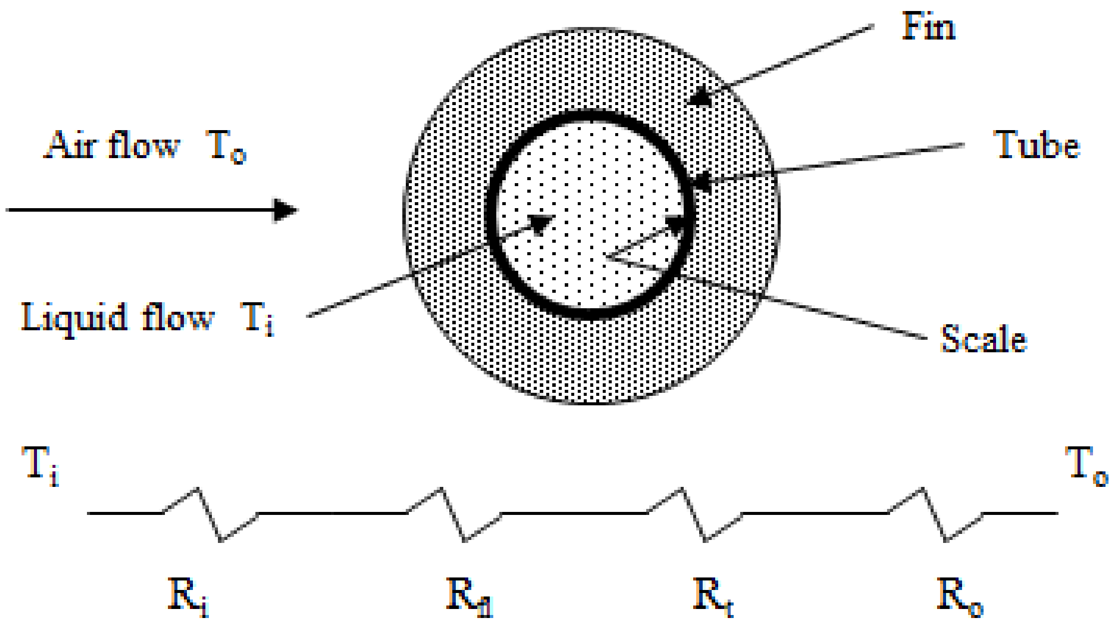




Overall Heat Exchanger Conductance

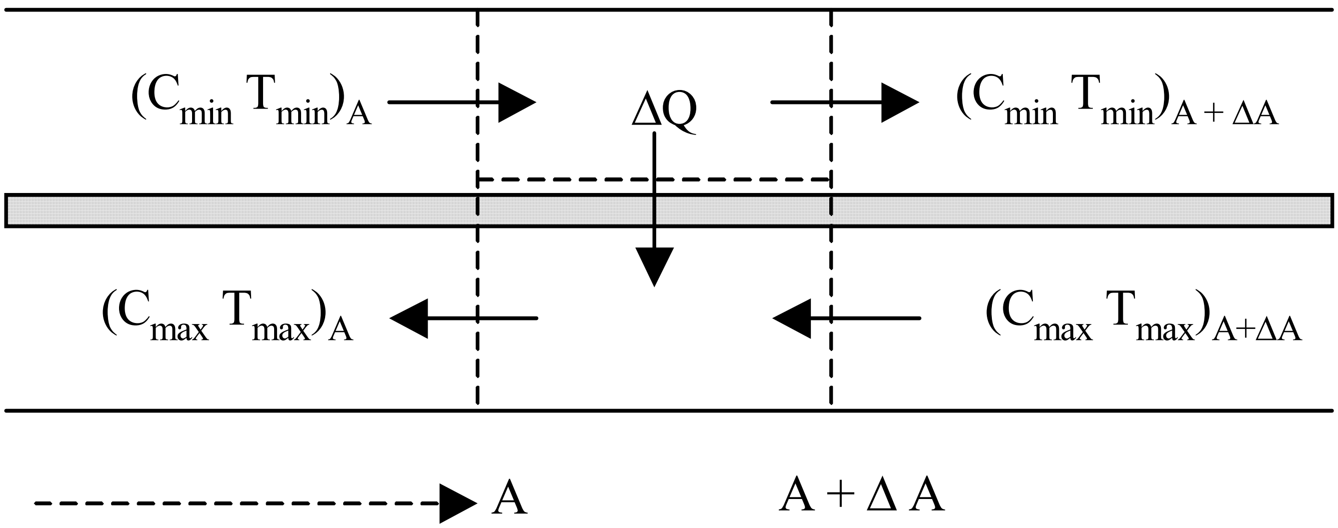
Total heat transfer resistance between two fluids in a finned tube heat exchanger

$$R_T = R_i + R_{fl} + R_t + R_o$$



- R_i: tube inside convection resistance
- R_{fl}: tube inside fouling resistance
- R_t: tube wall conduction resistance
- R_o: outside surface (fin+tube) convection resistance

Energy flows for a counter-flow heat exchanger



We define

$$C_{min} = \dot{m}_{min}\, c_{p,min} \quad \text{and} \quad C_{max} = \dot{m}_{max}\, c_{p,max}$$

Energy balance for the infinitesimal CV

$$\left(C_{min}\ T_{min}\right)_A + \left(C_{max}\ T_{max}\right)_{A+\Delta A} - \left(C_{min}\ T_{min}\right)_{A+\Delta A} - \left(C_{max}\ T_{max}\right)_A = 0$$

$$\Updownarrow$$

$$C_{min}\frac{dT_{min}}{dA} = C_{max}\frac{dT_{max}}{dA}$$

Heat transfer between two small CVs

$$\Delta Q = U\Delta A\left(T_{min} - T_{max}\right)$$

Energy balance for one side

$$C_{min}\frac{dT_{min}}{dA} = -U\left(T_{min} - T_{max}\right)$$

$$C_{max}\frac{dT_{max}}{dA} = -U\left(T_{min} - T_{max}\right)$$

*Integrate above equations to find total heat transfer

Log-Mean-Temperature-Difference Method

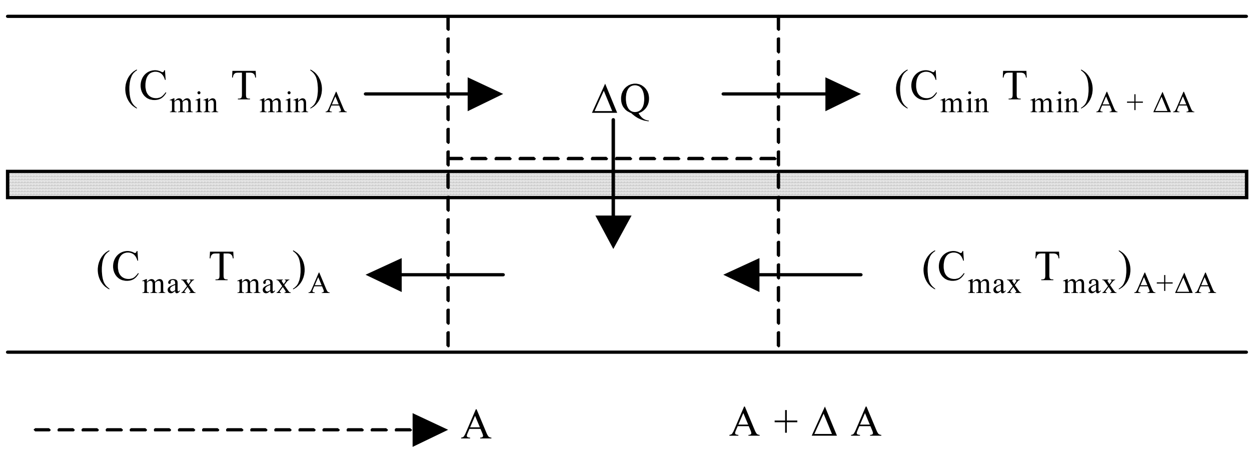
Log-Mean-Temperature-Difference (LMTD) is defined as

$$LMTD = \frac{\left(T_{h,in} - T_{c,out}\right) - \left(T_{h,out} - T_{c,in}\right)}{\ln\left(\dfrac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

Heat transfer rate is calculated by

$$\dot{Q} = UA\ (LMTD)\ F$$

where F is a correction factor for flow types other than counterflow.



Effectiveness-NTU Method

Overall energy balance

$$\dot{Q} = C_h \left(T_{h,i} - T_{h,o} \right) = C_c \left(T_{c,o} - T_{c,i} \right)$$

We define heat transfer effectiveness

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}}$$

where $\dot{Q}_{max} = C_{min} \left(T_{h,i} - T_{c,i} \right)$ is the max heat transfer.

Then the heat transfer rate is

$$\dot{Q} = \varepsilon\, C_{min}\left(T_{h,i} - T_{c,i}\right)$$

We further define capacitance rate ratio

$$C^{*} = \frac{C_{min}}{C_{max}}$$

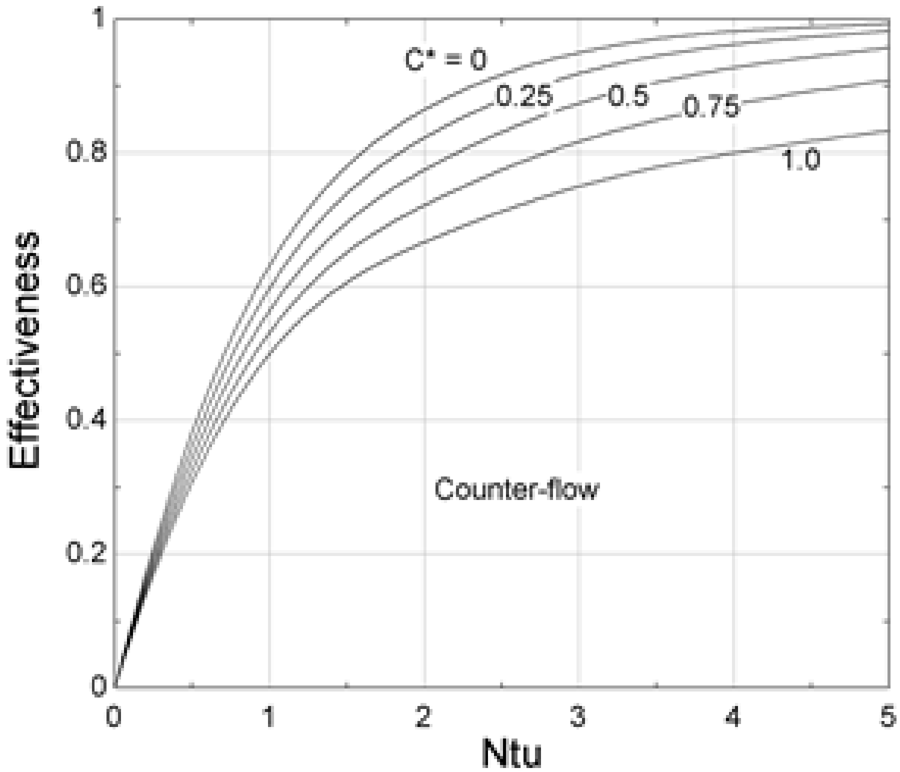
and number of transfer units (NTUs)

$$Ntu = \frac{UA}{C_{min}}$$

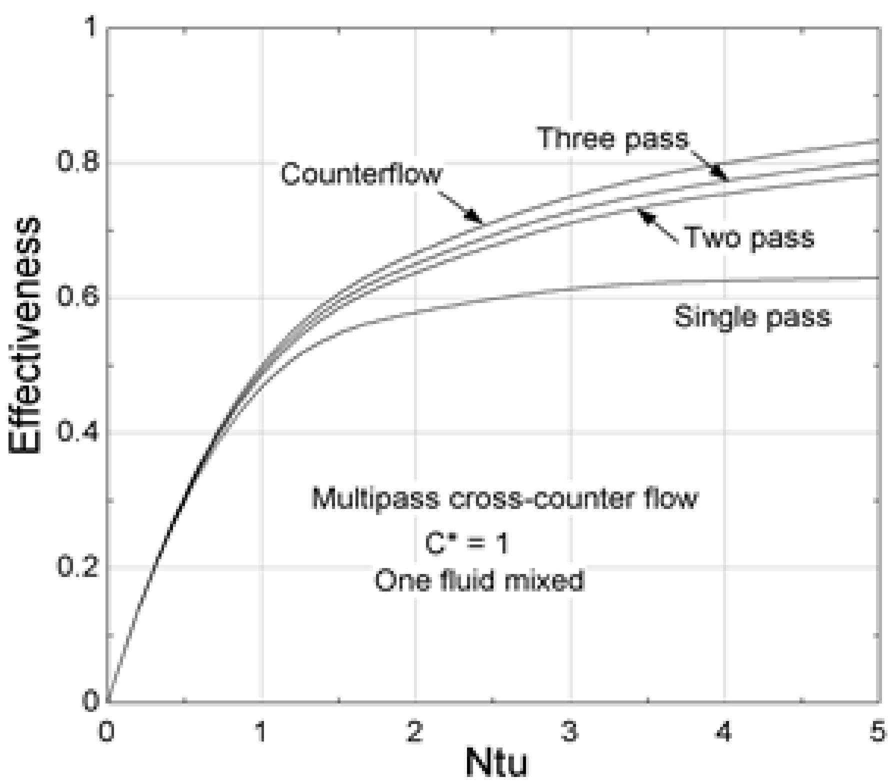
Then effectiveness can be calculated as

$$\varepsilon = f\left(C^{*}, Ntu\right)$$

Flow arrangement	Conditions	Effectiveness
All	$C^* = 0$	$\varepsilon = 1 - e^{-Ntu}$
Counterflow	$C^* \neq 1$	$\varepsilon = \dfrac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}}$
	$C^* = 1$	$\varepsilon = \dfrac{Ntu}{1 + Ntu}$
Crossflow, one fluid mixed	Single pass	$\varepsilon = \dfrac{1 - e^{-C^*(1-e^{-Ntu})}}{C^*}$
	Two pass	$\varepsilon = \left[1 - 0.0643\, C^* \left(1 - e^{-0.548\, Ntu}\right)\right] \left[\dfrac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}}\right]$
	Three pass	$\varepsilon = \left[1 - 0.0411\, C^* \left(1 - e^{-0.414\, Ntu}\right)\right] \left[\dfrac{1 - e^{-Ntu(1-C^*)}}{1 - C^* e^{-Ntu(1-C^*)}}\right]$
Shell and tube: even number of passes (2, 4, 6, ...)		$\varepsilon = \dfrac{2}{(1 + C) + \sqrt{1 + C^{*2}} \left(\dfrac{1 + e^{-Ntu\sqrt{1+C^{*2}}}}{1 - e^{-Ntu\sqrt{1+C^{*2}}}}\right)}$
Crossflow	Both fluids unmixed	$\varepsilon = 1 - \exp\left[\dfrac{Ntu^{0.22}}{C^*}\left(\exp\left(-C^* Ntu^{0.78}\right) - 1\right)\right]$



Counterflow



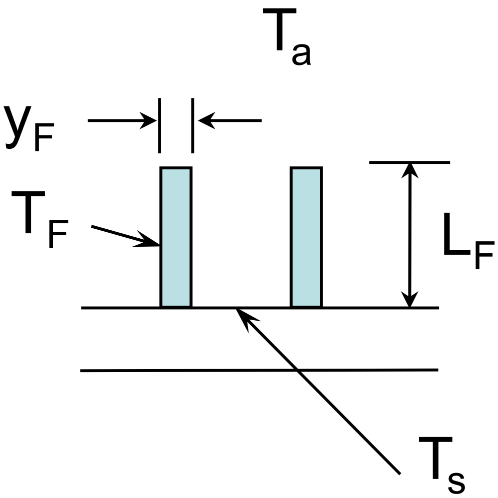
Multi-pass cross-flow with one fluid mixed

Notes:

- LMTD and ε-NTU methods are equivalent
- LMTD is easier to use when desired fluid outlet conditions are given
- ε-NTU is better when heat exchanger size and performance (i.e., UA) is given

<u>Heat Exchanger Example</u>: Determine the outlet temperatures and the heat transfer rate for a heating coil in which air is heated using hot water with two passes. The overall heat transfer conductance is 4 kW/C. The air stream enters at 24 C with a flow rate of 3 kg/s, and the water stream enters at 60 C with a flow rate of 1.0 kg/s.

Fin Efficiencies



- fin heat transfer depends on local temperature difference (T_F-T_a)
- characterize finned surface heat transfer using a fin efficiency

Effective overall surface area

$$\underbrace{\eta_o}_{\text{overall fin efficiency}} A_o = \underbrace{(A_o - A_F)}_{\text{exposed tube area}} + \underbrace{\eta_F}_{\text{individual fin efficiency}} \overbrace{A_F}^{\text{fin surface area}}$$

Individual Fin Efficiency

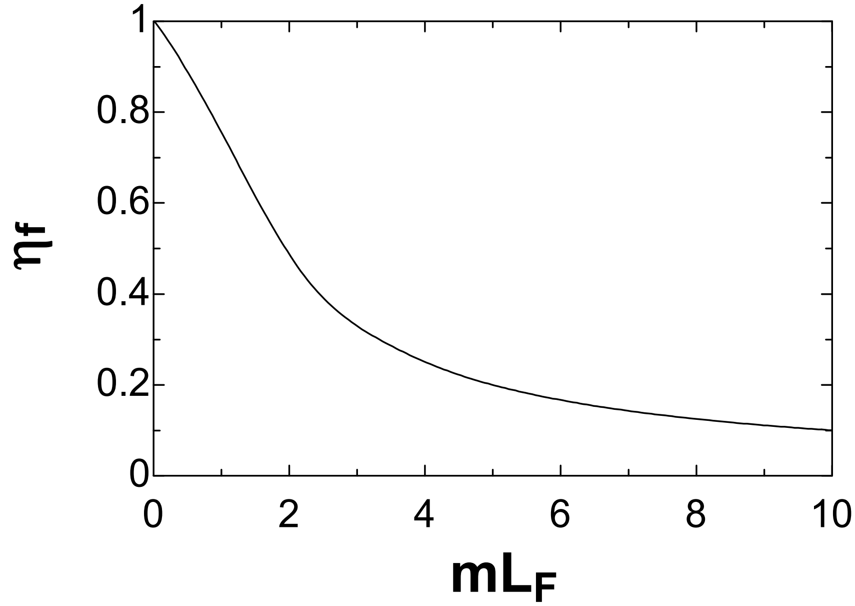
$$\eta_F = \frac{\text{heat transfer at base of fin}}{\text{maximum possible heat transfer if entire fin at } T_s}$$

For straight fins,

$$\eta_F = \frac{\tanh(mL_F)}{mL_F}$$

$$m = \left(\frac{2h_o}{k_F y_F} \right)^{1/2}$$

fin thermal conductivity



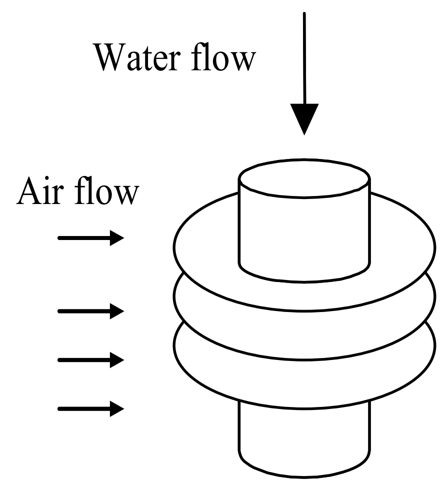
For circular fins,

$$\eta_f = \left[\frac{2r_i}{m_f\left(r_{o,c}^2 - r_i^2\right)}\right]\left[\frac{K_1(m_f r_i)I_1(m_f r_{o,c}) - I_1(m_f r_i)K_1(m_f r_{o,c})}{I_0(m_f r_i)K_1(m_f r_{o,c}) + K_0(m_f r_i)I_1(m_f r_{o,c})}\right]$$

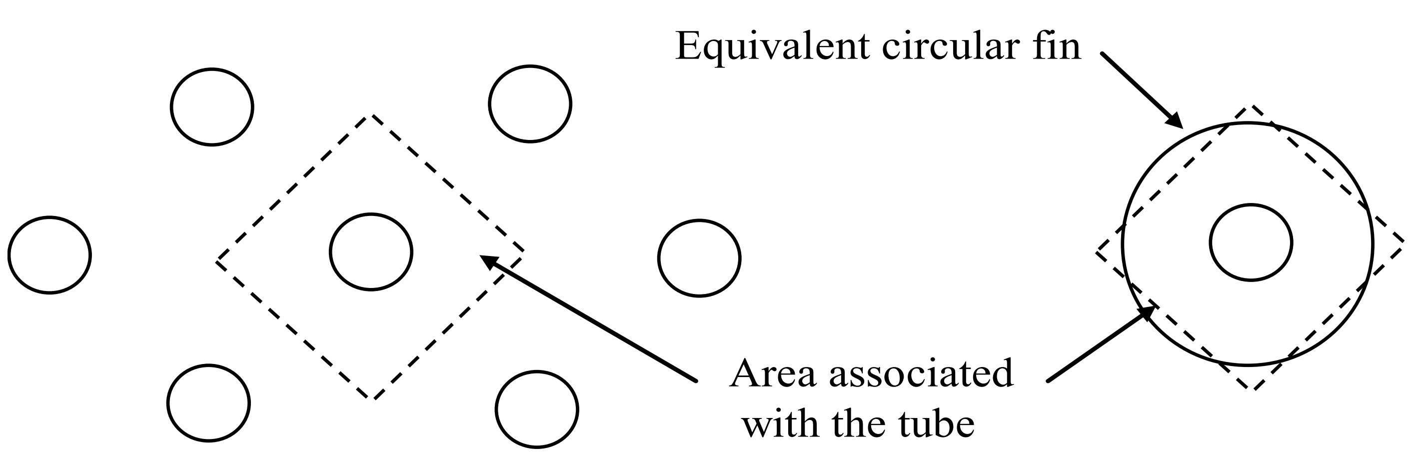
where the effective fin radius and area are

$$r_{o,c} = r_o + \frac{t_f}{2}$$

$$A_f = 2\pi\left(r_{o,c}^2 - r_i^2\right)$$



Equivalent fin radius for plate-fin geometries



<u>Fin Efficiency Example</u>: Determine the fin efficiency, overall surface efficiency, and thermal resistance per foot of tube length for a cross-flow heat exchanger using finned tubes. The tube diameter is 0.774 inch. The fins are steel with a thickness of 0.012 inch, a diameter of 1.463 inch, and a pitch of 9.05 fins per inch. The conductivity of steel is 35 Btu/hr-ft-F. The heat transfer coefficient is 14. 4 Btu/hr-ft2-F."

Tube Internal Single Phase Heat Transfer Coefficients and Pressure Drop

- Forced convection is generally involved in heat exchanger flow
- Correlations covered in Lectures 3 and 5 can be used for calculating heat transfer coefficient and pressure drop

Reynolds number is defined as

$$\mathrm{Re}_{D_H} = \frac{\rho_f \ V \ D_H}{\mu_f} \quad \text{or} \quad \mathrm{Re}_{D_H} = \frac{4 \ \dot{m}}{WP \ \mu_f}$$

where hydraulic diameter is

$$D_H = \frac{4 \ A_c}{WP}$$

Heat transfer coefficient is related to Nusselt number defined as

$$Nu_{D_H} = \frac{h_c D_H}{k_f}$$

where k_f is the fluid conductivity.

From Lecture 3, we can calculate **pressure drop** by

$$\Delta p = f \frac{L}{D_H} \frac{\rho_f V^2}{2} \quad \text{or} \quad \Delta p = f \frac{L}{D_H} \frac{G^2}{2\rho}$$

where G is mass velocity

$$G = \frac{\dot{m}}{A_c}$$

Heat transfer and friction factor relations for internal turbulent flow for Re > 2500

Smooth tubes	$Nu_{D_H} = 0.023\ Re_{D_H}^{0.8}\ Pr^{n}$ n = 0.4 heating or n = 0.3 cooling
Or (larger Re_D range)	$Nu_{D_H} = \dfrac{(f/2)\left(Re_{D_H} - 1000\right)Pr}{1 + 12.7(f/2)^{1/2}\left(Pr^{2/3} - 1\right)}$
Rough tubes	$Nu_{D_H} = \dfrac{\left(Re_{D_H}\ Pr(f/2)\right)}{1 + (f/2)^{1/2}\left(4.5\,Re_{D_H}^{0.2}\ Pr^{0.5} - 8.48\right)}$

Smooth tubes	$f = \dfrac{0.3164}{Re_{D_H}^{0.25}}$
Or (larger Re_D range)	$f = 0.0032 + \dfrac{0.221}{Re_{D_H}^{0.237}}$
Rough tubes	$\underline{Re_{D_H} < 10^{6}}$ $f^{-0.5} = 1.14 + 2\log\left(\dfrac{D_H}{\varepsilon}\right) - 2\log\left(1 + \dfrac{9.3}{Re\left(\dfrac{\varepsilon}{D_H}\right)f^{0.5}}\right)$ $\underline{Re_{D_H} > 10^{6}}$ $f^{-0.5} = 1.14 + 2\log\left(\dfrac{D_H}{\varepsilon}\right)$

Finned Surface Heat Transfer Coefficients and Pressure Drop

We define the ratio of free flow to frontal area

$$\sigma = \frac{A_c}{A_{fr}}$$

Reynolds number $N_R = Re_{D_H} = \frac{D_H G}{\mu}$

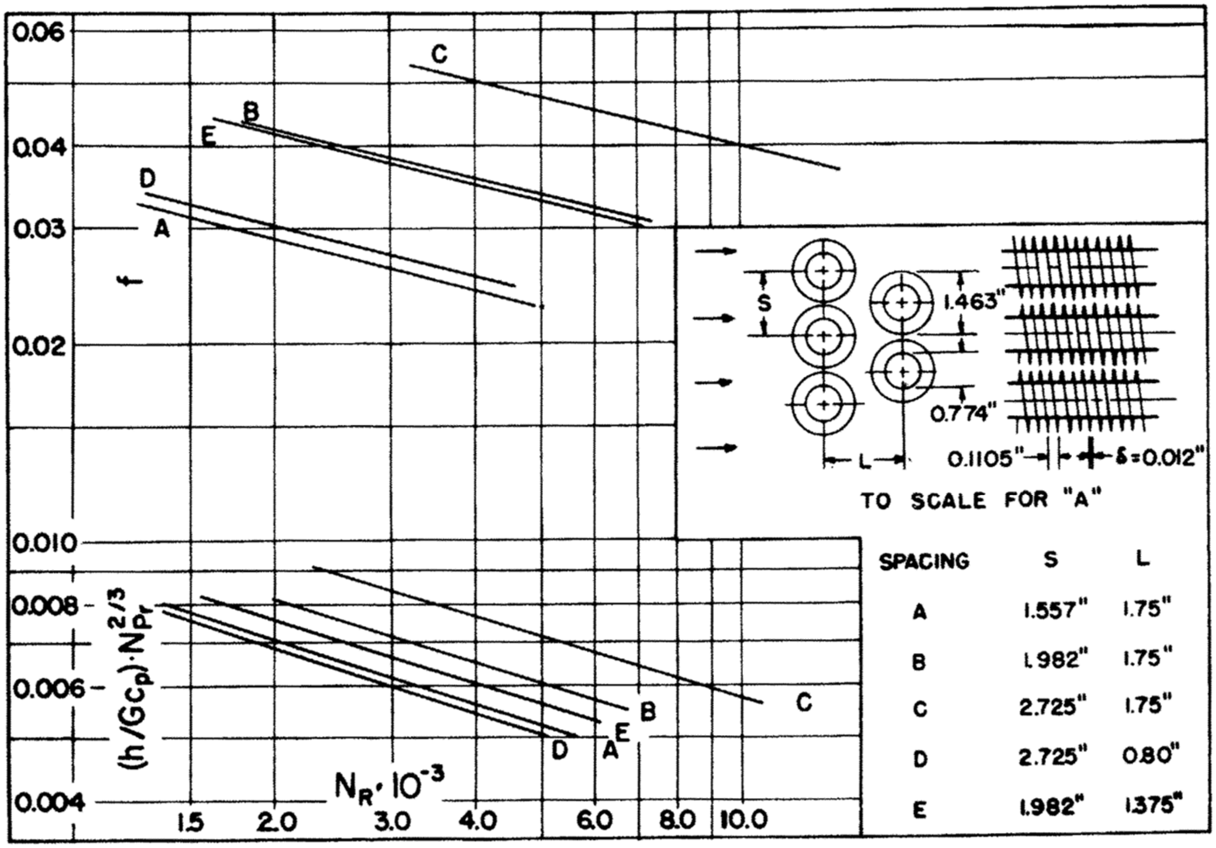
The heat transfer coefficient is related to Standon number

$$St = \frac{h_c}{Gc_p}$$

Friction factor $f = \frac{\rho \, \tau_0}{G^2 / 2}$

Pressure drop can be calculated by

$$\Delta p = f \frac{A}{A_c} \frac{G^2}{2 \rho}$$



Tube outside diameter = 0.774 in.

Fin pitch = 9.05 per in.

Fin thickness = 0.012 in.

Fin area/total area = 0.835

	A	*B*	*C*	*D*	*E*
Flow passage hydraulic diameter, $4r_h$ =	0.01681	0.02685	0.0445	0.01587	0.02108 ft
Free-flow area/frontal area, σ =	0.455	0.572	0.688	0.537	0.572
Heat transfer area/tota total volume, α =	108	85.1	61.9	135	108 ft^2/ft^3

Note: Minimum free-flow area in all cases occurs in the spaces transverse to the flow, except for *D*, in which the minimum area is in the diagonals.

* Prandtl number raised to the two-thirds power is used to correlate the properties of other fluids.

Finned tube example: Determine the air-side convective heat transfer coefficient, thermal resistance, and pressure drop for a coil made of finned tubes with configuration B of figure above. The coil frontal area is 4 ft^2, there are four rows of coils, and the fins are made of aluminum. The airflow is 4000 cfm at a temperature of 75 F and 50 % relative humidity.