Chapter SM 9: Combustion Heating Equipment

SM 9.1 Introduction

The equipment used for heating most commercial and residential buildings involves combustion of hydrocarbon fuels. Natural gas, oil, and coal are common fuels, with propane and wood used in some rural areas. The main piece of equipment in a heating system is the furnace or boiler. The term **furnace** is used for devices that heat air, while the term **boiler** is used for those that heat water, even though the water may not boil. The temperature of the air leaving a furnace is typically about 100 to 110 °F (40 to 45 °C) for condensing furnaces and 130 to 170 °F (55 to 75 °C) for noncondensing furnaces (ASHRAE, 2008). Low-pressure boilers that deliver water temperatures up to 250 °F (120 °C) at a maximum pressure of 15 psig (100 kPa gage) for steam and 160 psig (1100 kPa gage) for water are common in residences. The medium- and high-pressure boilers used in commercial buildings operate at higher pressures and temperatures and are constructed differently from the low-pressure units. Regardless of the specific type, all combustion heating devices adhere to the same fundamental relations. The basic principles underlying these commonly used heating devices are covered in this chapter.

An overall energy balance on a heating device relates the fuel input to the useful heating energy supplied to the building. Not all of the fuel energy is converted to useful heating because of losses associated with the combustion process. The overall energy balance on a device is

$$\textit{Fuel Energy input} = \textit{Useful heat output} + \textit{Losses} \tag{9.1}$$

The efficiency of a device is a measure of its performance and is defined as the ratio of the useful heat output to the fuel energy input. Using the energy balance equation 9.1, the efficiency can also be written in terms of the losses:

$$\eta_{fr} = \frac{\textit{Useful heat output}}{\textit{Fuel energy input}} = 1 - \frac{\textit{Losses}}{\textit{Fuel energy input}} \tag{9.2}$$

Two efficiency measures are commonly used to rate combustion equipment. The **combustion efficiency** is based on steady-state operation, where the losses are due to the exhaust of energy contained in the water vapor formed during combustion and in the high-temperature products. The **overall efficiency,** or **utilization efficiency,** or **annual fuel utilization efficiency (AFUE)** is the ratio of the total useful heat produced to the total fuel input supplied over a time period such as the heating season, and it includes losses incurred in both steady and cyclic operation.

The efficiency of a given furnace or boiler depends upon a number of factors such as the fuel used, the combustion chamber design, and the age of the unit. For furnaces and modern boilers, in which the water in the exhaust is not condensed and leaves as vapor, combustion efficiencies are in the range of 75 to 85%. Units in which the water is condensed and the heat of condensation is converted to useful heat have efficiencies in the range of 85 to 95%. Older units have considerably lower efficiencies than modern units, and it is not uncommon for the efficiency of an old residential furnace or boiler to be 50% or lower. Older oil-fired boilers tend to have significantly lower efficiencies than natural gas-fired units of similar age.

The combustion and heat transfer processes inside a heating device have a major effect on its performance. Air and fuel need to be supplied and mixed in a chamber to ensure complete combustion, a heat exchanger is needed to transfer heat from the products of combustion to the circulating fluid, and the products of combustion need to be exhausted to the ambient. These major components of a typical combustion heating unit are shown schematically in Figure 9.1.

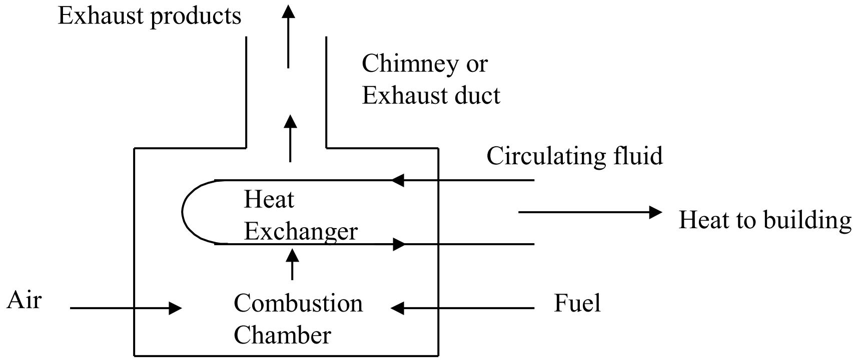

Figure 9.1 Schematic of a combustion furnace of boiler

There are several constraints on the design and operation of a heating device. The primary requirements are to provide a safe and reliable source of heat, and these considerations take precedence over efficiency. The impetus in the 1970s and ’80s to reduce energy consumption and the more recent concerns over environmental quality have led to significant improvements in furnace design. However, a large number of heating devices still in use predate the more modern improvements.

Historically, furnaces were mechanically simple devices and were, consequently, relatively inexpensive. Residential furnaces were generally natural draft units in which the flow of hot combustion products up the chimney induced combustion air into the furnace and over the heat exchanger. Heat exchanger designs ensured that the temperature of the exhaust products leaving the exchanger was high enough to guarantee enough natural airflow under all conditions, which meant that the thermal loss due to the exhaust flow was high. The amount of excess air was usually considerably more than necessary to ensure that combustion was complete. Heating excess air and then exhausting it to the outdoors is a loss. In these older furnaces the chimney was directly open to the environment, and as a result warm air from the building went up the chimney when the furnace was off. Most older furnaces used a pilot light that burned continuously to ignite the fuel when heat was required, but did not provide useful heat when the furnace was off. These older units were very reliable and safe and were made during times in which the cost of fuel was not a major concern.

In a modern high-efficiency furnace using natural gas as the fuel, a fan is used to meter the amount of the combustion air flow and keep the amount of excess air to the minimum required for complete combustion. The heat exchanger is designed to reduce the temperature of the products as much a possible. In high-efficiency units the products are cooled below the dew point, which allows the heat of condensation to be used, minimizing the loss due to water vapor in the exhaust. Modern furnaces and boilers using fuel oil have many of the same features. However, they are rarely condensing units because some of the products of combustion, especially sulfur, dissolve in the condensate and corrode the heat exchanger and duct surfaces. In modern furnaces and boilers a combination of forced air combustion fans, electric ignition, low exhaust temperatures, and, for furnaces, condensation heat exchangers are used to minimize the losses and achieve high fuel efficiency.

In this chapter the basic combustion relations will be formulated and used to illustrate the process inside a furnace or boiler. The contribution of the different mechanisms to furnace efficiency and environmental impact will be established. The effect of the design on performance will be assessed.

SM 9.2 Combustion Processes

The relations necessary to determine the thermal performance of a heating device are conservation relations for mass and energy and rate equations for heat transfer. A device that could be either a furnace or boiler is shown schematically in Figure 9.2 along with the relevant mass flows and enthalpy terms. Fuel and air, termed the reactants, enter at low temperatures T_f and T_A, respectively, and combust, producing a flow of products at a high temperature T_c. These products flow over the heat exchanger surfaces, heating the circulating fluid from the temperature returning from the building zone T_Z to the supply temperature T_s. The products are cooled due to this heat transfer and leave the furnace at the products temperature T_p.

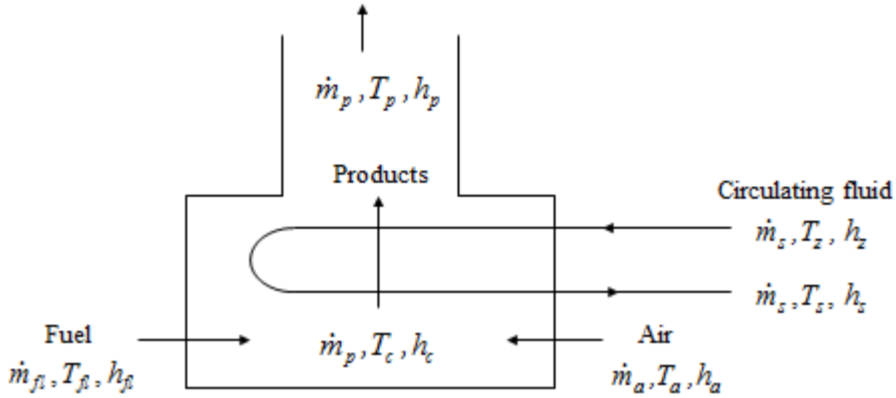

Figure 9.2 Temperature, mass flow, and energy terms for a furnace or boiler

Conservation of mass for the heating device (Section 3.2) relates the incoming air and fuel flow rates to the outgoing flow rate of the products:

$$\dot{m}_a + \dot{m}_{fl} - \dot{m}_p = 0 \tag{9.3}$$

Conservation of energy (Section 3.3) relates the energy flows of the air, fuel, products, and circulating fluid:

$$\dot{m}_a h_a + \dot{m}_{fl} h_{fl} + \dot{m}_s h_z - \dot{m}_p h_p - \dot{m}_s h_s = 0 \tag{9.4}$$

The energy balance may be rearranged to equate the energy change of the fuel and air to that of the circulating fluid:

$$\dot{m}_a h_a + \dot{m}_{fl} h_{fl} - \dot{m}_p h_p = \dot{m}_s \left(h_s - h_z\right) \tag{9.5}$$

The energy change of the circulating flow is the useful heat transfer from the furnace $\dot{Q}_h$. By rearranging equation 9.5, this heat flow can be equated to the energy difference between the air and fuel that enter and the products that leave:

$$\dot{Q}_h = \dot{m}_a h_a + \dot{m}_{fl} h_{fl} - \dot{m}_p h_p \tag{9.6}$$

The thermodynamic process inside the furnace can be represented on a plot of the total flow enthalpy (mass flow rate times the enthalpy per unit mass) as a function of temperature. Pressure has a small effect on the enthalpy of these flows, which are either liquids or gases, and so the total enthalpies of the reactants and products are functions of temperature only. The flow enthalpy of the reactants is the sum of the products of flow rate and enthalpy for the air and fuel flows, and both increase with temperature. At any temperature the enthalpy of the reactants is much greater than that of the products due to the chemical energy bound up in the fuel. The specific heats of the components increase with temperature and so the relationships for the reactants and products curve upward, as shown in Figure 9.3.

The inlet state for the reactants is at a low temperature and high flow enthalpy. The enthalpy of the products immediately after combustion is essentially the same as that of the reactants since the combustion process is basically adiabatic. However, the temperature is very high because the chemical energy has been converted to sensible energy. The heat transfer to the circulating fluid cools the products down to the leaving products temperature. The heat transfer to the circulating fluid is the difference between the enthalpy in the reactants and products flow, as shown in equation 9.6.

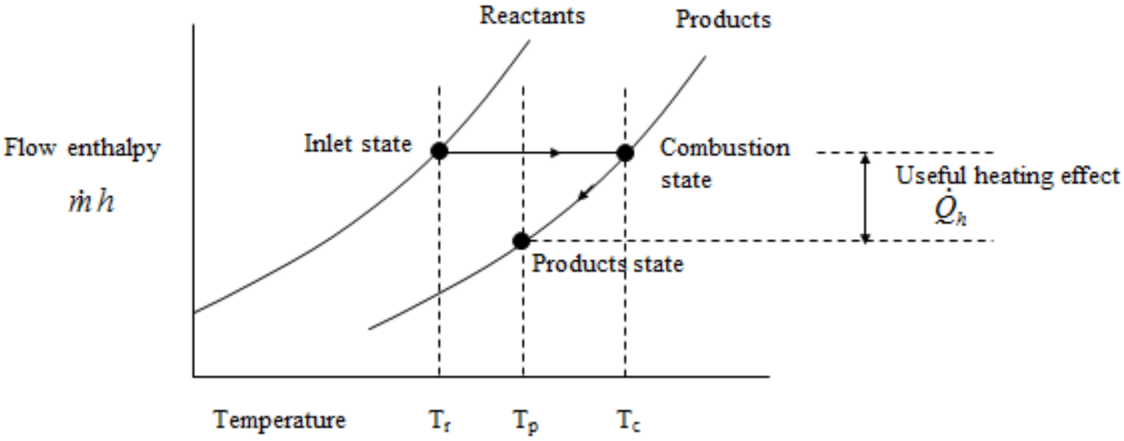

Figure 9.3 Thermodynamic representation for the combustion process

Equation 9.6 relates the heat flow to the difference in enthalpy between the reactants and products at their respective temperatures T_r and T_p. It is convenient to express the enthalpy difference between reactants and products as two sets of differences. One is the total enthalpy difference between reactants and products at the reactants temperature T_r, and the second is the total enthalpy difference between products at the reactants temperature T_r and at the products temperature T_p. The heat flow given by equation 9.6 can then be expressed as

$$\dot{Q}_h = \left(\dot{m}_a h_a + \dot{m}_{fl} h_{fl} - \dot{m}_p h_p\right)_{T_r} - \dot{m}_p\left(h_{p,T_P} - h_{p,T_r}\right) \tag{9.7}$$

The first term on the right-hand side of the equation, which is the change in total enthalpy from reactants to products at the same temperature, is directly related to the heating value of the fuel. For reactants and products at reference conditions of 1 atm pressure and a temperature of 77 °F (25 °C), this enthalpy difference between reactants and products is the product of the **fuel heating value** HV and the fuel flow rate:

$$\dot{m}_f HV = \left(\dot{m}_a h_a + \dot{m}_{fl} h_{fl} - \dot{m}_p h_p\right)_{T_o} \tag{9.8}$$

where T_o is the reference temperature.

The temperatures at which fuel and air enter a combustion chamber are generally very close to the reference temperature, and the energy associated with the difference between energy at these two temperatures is small relative to the heating value. It is common to assume that the fuel and air enter at the reference temperature and to neglect the difference in energy if the fuel enters at a different temperature. However, if the inlet temperatures were significantly different from the reference temperature, the enthalpy difference between reactants at the two temperatures would need to be included in equation 9.8.

The useful heat transfer can then be expressed in terms of the heating value and the difference in energy between the products at the products and reference temperatures:

$$\dot{Q}_h = \dot{m}_{fl} HV - \dot{m}_p \left(h_{p,T_p} - h_{p,T_o}\right) \tag{9.9}$$

There are two heating values that are relevant for fuels that contain hydrogen. The **higher heating value** (*HHV*) is the enthalpy difference between the reactants and the products per unit mass of fuel when the water that is produced leaves as liquid. The higher heating value thus includes the latent heat of condensation of the water vapor as a useful heat transfer. The **lower heating value** (*LHV*) is the enthalpy difference between reactants and products per unit mass of fuel when the water formed by the combustion process leaves as a vapor. Since the enthalpy of water vapor is higher than that of liquid, there is more energy in the exhaust flow when the water leaves as vapor, and the lower heating value is then smaller than the higher heating value. The higher and lower heating values differ by product of the heat of condensation at the reference temperature and flow rate of water formed during combustion, divided by the mass flow rate of the fuel:

$$HHV = LHV + \left(\frac{m_w}{m_{fl}}\right) h_{fg} \tag{9.10}$$

where the ratio (m_w/m_f) is the ratio of the mass of the water formed by combusting the hydrogen in the fuel to the mass of the fuel.

The heating value of a hydrocarbon fuel depends on the relative amounts of carbon and hydrogen, since each has a different chemical energy. Fuels with a high ratio of hydrogen to carbon have higher heating values than those with a lower ratio. Higher heating values for some common fuels are given in Table 9.1 both in terms of the energy per unit mass and also in the common units in which these fuels are sold. Natural gas is a mixture of methane, propane, and small amounts of other gases, and the heating value depends on the composition of the mixture. The density value is for fuels at the reference temperature and pressure. The range in the values of substances such as natural gas and gasoline reflect the fact that they are a mixture of fuel types.

Table 9.1a Higher Heating Values for common fuels, English units

Fuel	*HHV*		Density
Gaseous	(Btu/lb)	(Btu/ft^3)	(lbm/ft^3)
Methane	23,857	1,000	0.042
Natural gas	23,800 to 25,000	1000 to 1050	0.042
Propane	21,699	2,595	0.120
Coal gas	13,000	600	0.046
Liquid		(Btu/gal)	(lbm/gal)
Gasoline	19,700	132,000 to 137,000	6.7 to 7.0
Diesel oil	19,500	137,000 to 141,800	7.0 to 7.3
Fuel oil	19,200	143,100 to 148,100	7.4 to 7.8
Solid		(Btu/ton)	(lbm/ft^3)
Anthracite coal	12,700	26 x 10^6	90
Bituminous coal	14,000	13 x 10^6	90
Lignite	6,000	5.6 x 10^6	90

Table 9.1b Higher Heating Values for common fuels, SI units

Fuel	*HHV*		Density
Gaseous	(kJ/kg)	(kJ/m^3)	(kg/m^3)
Methane	55,492	37,200	0.67
Natural gas	55,400	37,200	0.67
Propane	50,500	96,700	1.92
Coal gas	30,200	22,400	0.74
Liquid		(kJ/L)	
Gasoline	45,800	37,000 to 38,000	0.80
Diesel oil	45,400	38,000 to 39,500	0.84
Fuel oil	44,700	39,900 to 41,000	0.89
Solid		(kJ/1000 kg)	
Anthracite coal	29,500	30 x 10^6	1400
Bituminous coal	32,600	15 x 10^6	1400
Lignite	14,000	6.5 x 10^6	1400

(Chapter 28, *ASHRAE Handbook*, 2009; *North American Combustion Handbook*, 1986)

The heat flow rate in equation 9.9 can be written in terms of the higher heating value and the energy associated with the water vapor in the products using equation 9.10. For combustion with dry air, all of the water in the products is the result of the combustion process, and if the water is condensed then all of the energy (*HHV*) goes into heat. (Combustion with humid air is covered in Section 9.3.)

Although the enthalpy of the products at the products and reference temperatures can be determined using the number of moles and the enthalpy of each component, it is more instructive to express the difference in the enthalpy in terms of an average specific heat of the mixture of products $c_{p,p}$ over the temperature range and the temperature difference. Further, the term containing the enthalpy difference for the products at the two temperatures is on the order of 10% or less of the heating value term, and thus extreme accuracy is not warranted. Using an average specific heat of the products allows the heat flow in equation 9.9 to be expressed as

$$\dot{Q}_h = \dot{m}_{fl}\, HHV - \dot{m}_w h_{fg} - \dot{m}_p c_{p,p}\left(T_p - T_r\right) \tag{9.11}$$

where $\dot{m}_w$ is the mass flow rate of water vapor in the products.

In this form, the major sources of furnace inefficiency are readily identified. The maximum possible heat transfer is given by the first term on the right-hand side of the equation, which is the product of the fuel higher heating value and fuel flow rate. The losses are due to the formation of water that leaves as vapor and due to the flow rate of products that leave at a higher temperature than that of the entering reactants.

The furnace or boiler efficiency as expressed by equation 9.2 can be written in terms of the heat flow, the fuel flow rate, and the higher heating value as

$$\eta_{fr} = \frac{\dot{Q}_h}{\dot{m}_{fl}\, HHV} \tag{9.12}$$

Combining equation 9.11 with 9.12 allows the efficiency to be expressed in terms of the losses as

$$\eta_{fr} = 1 - \frac{\dot{m}_w\, h_{fg}}{\dot{m}_{fl}\, HHV} - \frac{\dot{m}_p\, c_{p,p}\left(T_p - T_r\right)}{\dot{m}_{fl}\, HHV} \tag{9.13}$$

The second term on the right-hand side is the **latent loss** and the third term is the **stack loss**. Equation 9.13 shows that the highest efficiency is obtained by condensing as much water as possible and cooling the products down as close to the temperature at which the reactants enter the combustion chamber as feasible. In addition, the last term on the right shows that the stack loss is lowest when the mass flow rate of the products is lowest, which means that the amount of excess air should be maintained at the minimum feasible level.

SM 9.3 Combustion Reactions

The hydrocarbon fuels used in furnaces and boilers consist of carbon, hydrogen, and sometimes oxygen molecules bonded together. In the combustion process, the bonds between the carbon and hydrogen are broken, and these elements then bind with the oxygen in the air stream or in the fuel, releasing energy that increases the temperature of the products. The combustion process is termed **complete** if all of the carbon and hydrogen form water and carbon dioxide. If the amount of oxygen is insufficient for complete combustion of all of the carbon, some carbon monoxide will be formed and there will be unburned fuel. In this case, the combustion is termed **incomplete**. The furnace efficiency is lowered when incomplete combustion occurs because not all of the fuel energy is released.

All hydrocarbon fuels include the same reactions of carbon and hydrogen with oxygen. The differences are in the relative amounts of carbon and hydrogen in the fuel, which affects the relative amounts of carbon dioxide and water in the products. The relations for these reactions allow the energy release, the temperature after combustion, and the components of the products to be determined. The chemical reactions will be illustrated with methane (CH_4), which is the major component of natural gas, as the fuel and with pure oxygen as the oxidizer. The chemical balance equation for the reaction of methane and oxygen (reactants) to form carbon dioxide and water (products) is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \tag{9.14}$$

The equation is **balanced** in that the number of atoms of each of the species (C, H, and O) is the same for both reactants and products. One molecule of methane contains one atom of carbon and four atoms of hydrogen. Four atoms, or two molecules, of oxygen are required to produce one molecule of carbon dioxide and two molecules of water. The chemical equation is also balanced in terms of mass in that the mass of the reactants (methane and oxygen) equals the mass of the products (carbon dioxide and water). It is also conventional to describe the reactions in terms of the number of moles of the components. A **mole** is a specific number of molecules, and the reaction equation states that one mole of methane reacts with two moles of oxygen to produce one mole of carbon dioxide and two moles of water. One mole of a compound has a mass value numerically equal to its molecular weight. For example, one lbm-mole (1 mol) of methane has a mass of 16 lb and one kg-mole has a mass of 16 kg.

The heating value of the fuel can be determined from the reaction equation and the definition of the heating value. Equation 9.8, rewritten in terms of reactants and products, gives the heating value as

$$HV = \frac{\left(\dot{m}_r h_r - \dot{m}_p h_p\right)_{T_o}}{\dot{m}_{fl}} \tag{9.15}$$

The ratio of a mass flow rate of any component within the reactants or products to that of the fuel can be written in terms of the ratio of the number of moles and the molecular weight as

$$\frac{\dot{m}_i}{\dot{m}_{fl}} = \frac{N_i M_i}{N_{fl} M_{fl}} \tag{9.16}$$

where i indicates one of the components.

Using equation 9.16, the heating value can be written in terms of the number of moles, molecular weights, and component enthalpies as

$$HV = \sum_{reactants} \frac{\left(N_i M_i h_i\right)_{T_0}}{N_{fl} M_{fl}} - \sum_{products} \frac{\left(N_i M_i h_i\right)_{T_0}}{N_{fl} M_{fl}} \tag{9.17}$$

where the enthalpy of each of the components h_i is evaluated at the reference conditions.

The molecular weight, mass, and enthalpy of each of the compounds involved in the reaction of methane and oxygen are given in Table 9.2 for the standard reference conditions. The reference base for evaluating the enthalpy is that of a pure element. The enthalpy of an element such as oxygen or nitrogen at standard conditions is taken to be 0 Btu/lb or 0 kJ/kg. The enthalpy of a compound is the energy required to produce the compound from the elements of which it is composed. For example, energy is required to synthesize methane out of carbon and hydrogen. The energy required is termed the **enthalpy of formation** and is a negative value for most compounds, since energy is usually required to produce the compound. The enthalpy difference between the reactants and products at the same temperature is the heating value of the fuel. The heating value is a combination of the energy of the fuel and of the products since the products may not have zero enthalpy values at the reference state. The use of the enthalpies of elements and compounds to determine the heating value of a fuel is illustrated in Example 9.1.

Table 9.2 Enthalpy values for the compounds of the reaction of methane with oxygen at reference conditions (77 °F and one atmosphere pressure)

Compound	Molecular Weight (lb/lb-mol) (kg/kg-mol)	Mass (lb, kg)	Enthalpy (Btu/lb)	Enthalpy (kJ/lb)
CH_4	16.04	16	2,006	4,667
O_2	32.00	64	0	0
CO_2	44.01	44	3,844	8,942
H_2O (vapor)	18.02	36	5,771	13,423
H_2O (liquid)	18.02	36	6,820	15,864

Data from EES (2005) and NIST (1998)

Example 9.1 Determine the heating value for methane. For methane, the reactants are methane and oxygen and the products are carbon dioxide and water. The heating value can be evaluated using equation 9.17 with the specific components of the reaction.

$$HV_{CH_4} = \left[\left(\frac{N_{CH_4} M_{CH_4}}{N_{CH_4} M_{CH_4}} \right) h_{CH_4} + \left(\frac{N_{O_2} M_{O_2}}{N_{CH_4} M_{CH_4}} \right) h_{O_2} - \left(\frac{N_{CO_2} M_{CO_2}}{N_{CH_4} M_{CH_4}} \right) h_{CO_2} - \left(\frac{N_{H_2O} M_{H_2O}}{N_{CH_4} M_{CH_4}} \right) h_{H_2O} \right]_{T_o}$$

The mass ratios, which are the ratios of the products of the number of moles and molecular weights for each of the reactants and products to that of the fuel, are computed first. For the first term, which is for the fuel itself, the ratio is unity. For the second term, which is for the reactant oxygen, we have:

$$\frac{N_{O_2}M_{O_2}}{N_{CH_4}M_{CH_4}} = \frac{2(mole)*32\ (lb_{O_2}\ /\ mole)}{1\ (mole)*16.04\ (lb_{CH_4}\ /\ mole)} = 3.99\ \frac{lb_{O_2}}{lb_{CH_4}} = 3.99\ \frac{kg_{O_2}}{kg_{CH_4}}$$

In a similar manner, the mass flow rate ratios for the products are determined

$$\frac{N_{CO_2}M_{CO_2}}{N_{CH_4}M_{CH_4}} = 2.74\ \frac{lb_{CO_2}}{lb_{CH_4}} = 2.74\ \frac{kg_{CO_2}}{kg_{CH_4}}$$

and

$$\frac{N_{H_2O}N_{H_2O}}{N_{CH_4}M_{CH_4}} = 2.25\ \frac{lb_{H_2O}}{lb_{CH_4}} = 2.25\ \frac{kg_{H_2O}}{kg_{CH_4}}$$

The higher heating value of the fuel is computed using the enthalpy of the water as a liquid from Table 9.2. In English units:

$$\begin{aligned} HHV &= 1\frac{lb_{CH_4}}{lb_{CH_4}}*(-2006)\frac{Btu}{lb_{CH_4}} + 3.99\frac{lb_{O_2}}{lb_{CH_4}}*\left(0\right)\frac{Btu}{lb_{O_2}} - 2.74\frac{lb_{CO_2}}{lb_{CH_4}}*\left(-3844\right)\frac{Btu}{lb_{CO_2}} \\ &\quad -2.25\frac{lb_{H_2O}}{lb_{CH_4}}*\left(-6820\right)\frac{Btu}{lb_{H_2O}} \quad = 23{,}857\ \ Btu\ /\ lb_{CH_4} \end{aligned}$$

This is the value given in Table 9.1. In similar fashion, the lower heating value is computed using the enthalpy of water as a vapor as

$$LHV = 21{,}500\ \ Btu\ /\ lb_{CH_4}$$

The difference between the two values equals the product of the heat of vaporization of water, which is 1050 Btu/lb$_{H2O}$ at the reference condition, and the ratio of the mass of water formed to that of the fuel.

In SI units, the heating value is

$$\begin{aligned} HHV &= 1\frac{kg_{CH_4}}{kg_{CH_4}}*(-4667)\frac{kJ}{kg_{CH_4}} + 3.99\frac{kg_{O_2}}{kg_{CH_4}}*\left(0\right)\frac{kJ}{kg_{O_2}} - 2.74\frac{kg_{CO_2}}{kg_{CH_4}}*\left(-8942\right)\frac{kJ}{kg_{CO_2}} \\ &\quad -2.25\frac{kg_{H_2O}}{kg_{CH_4}}*\left(-13{,}423\right)\frac{kJ}{kg_{H_2O}} \quad = 55{,}492\ \ kJ\ /\ kg_{CH_4} \end{aligned}$$

The lower heating value is then

$$LHV = 50{,}009 \ kJ / kg_{CH_4}$$

SM 9.4 Use of EES in Combustion Reactions

Example 9.1 uses the values for molecular weight and enthalpy at reference conditions given in Table 9.2. EES includes the properties of many chemical compounds and can be used to evaluate the properties of the components directly. For methane, for example, the statement

```
MW_CH4=MolarMass(CH4)
```

is used to determine the molecular weight. The units of molecular weight (lbm/lb-mol or kg/kg-mol) are set by the units of the equation window. Similarly, the enthalpy of methane is determined using the statement

```
h_CH4 = Enthalpy(CH4,T=T)
```

In EES the proper reference states are used only for combustion problems (i.e., zero enthalpies for stable elements at standard conditions), with properties for ideal gases specified by their chemical formulas. For example, in EES the properties for “air” do not have the proper reference state, whereas the separate components of the air such as O2 and N2 do have the proper reference state. For combustion problems the enthalpy of water as vapor needs to be specified as “H2O” rather than “water” or “steam.” The set of EES statements used to determine the heating value of methane, Example 9.1, follow.

```
"The chemical reaction relation is CH4 + 2*O2 = CO2 + 2*H2O.  The enthalpies of the
components are determined at the reference temperature."
T_o=77  "F"                                    "Reference temperature"

"Number of moles of compounds"
N_CH4 = 1 “llbmole”                            “No of mole CH4”
N_O2 = 2  "lbmole"                             “No of mole O2”
N_CO2 = 1  "lbmole"                            “No of mole CO2”
N_H2O = 2   "lbmole"                           “No of mole H2O”

"Molecular weights of the compounds"
MW_CH4=MolarMass(CH4) "lbm/lbmole"             "CH4 molecular weight"
MW_O2=MolarMass(O2) "lbm/lbmole"               "O2 molecular weight"
MW_CO2=MolarMass(CO2) "lbm/lbmole"             "CO2 molecular weight"
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MW_H2O=MolarMass(H2O) "lbm/lbmole"                    "H2O molecular weight"

"Enthalpies of the compounds at reference conditions"
h_CH4 = Enthalpy(CH4,T=T_o) "Btu/lbm"                 "CH4 enthalpy"
h_O2 = Enthalpy(O2,T=T_o) "Btu/lbm"                   "O2 enthalpy"
h_CO2 = Enthalpy(CO2,T=T_o) "Btu/lbm"                 "CO2 enthalpy"
h_H2O = Enthalpy(H2O,T=T_o) "Btu/lbm"                 "H2O enthalpy"

"Enthalpy of liquid water"
h_fg = h_g - h_f "Btu/lbm"                            "Latent heat of vapor."
h_g = Enthalpy(Water, T=T_o, x=1) "Btu/lbm"           "Vapor enthalpy"
h_f = Enthalpy(Water, T=T_o, x=0) "Btu/lbm"           "Liquid enthalpy"
h_H2Oliq = h_H2O - h_fg                               "H2O liquid enthalpy"

"Determine the heating values using equation 9.15. "
LHV = (N_CH4*MW_CH4*h_CH4 + N_O2*MW_O2*h_O2-N_CO2* MW_CO2*h_CO2 –
      N_H2O*MW_H2O*h_H2O)/(N_CH4*MW_CH4)  "Btu/lbm"          "Lower heating value"
HHV = (N_CH4*MW_CH4*h_CH4 + N_O2*MW_O2*h_O2- N_CO2*MW_CO2*h_CO2 –
      N_H2O*MW_H2O*h_H2OLiq)/(N_CH4*MW_CH4) "Btu/lbm"        "Higher heating value"
```

The higher and lower heating values determined from this example equal those given in Table 9.1 and Example 9.1.

SM 9.5 Combustion Reactions with Air

In furnaces and boilers, the fuel combusts with air rather than with just oxygen. Equation 9.14 needs to be modified to include the components of air. Atmospheric air is mainly a mixture of nitrogen and oxygen, with small amounts of carbon dioxide and water vapor and trace amounts of other compounds. It is satisfactory to model **dry air** as only oxygen and nitrogen with a ratio of 1 mole of oxygen to 3.76 moles of nitrogen. The presence of water vapor in the entering air affects the energy release and will be considered later in this section.

Additionally, equation 9.14 represents a **stoichiometric** reaction in which there are no reactants remaining in the products. If, for example, there had been three moles of oxygen introduced as reactants, then there would not have been enough fuel for all of the oxygen to react, and there would have been one mole of oxygen appearing as one of the products. Similarly, if the number of moles of oxygen were less than two there would be some unburned methane in the products. In practice, combustion chambers are designed so that an excess amount of air is always present to ensure that there is a high enough concentration of oxygen in contact with the fuel for complete and self-sustaining combustion. **Incomplete combustion** occurs if there is not enough excess air. There is an energy loss since all of the energy of the fuel is not converted to heat. Additionally, the products of incomplete combustion such as CO and NO are unhealthy and damaging to the environment. To avoid these conditions, for good furnace design the amount of excess air is typically in the range of 5 to 50%, depending on type of fuel and combustion equipment (ASHRAE, 2009).

The chemical equation for complete combustion of methane with some excess air is

$$CH_4 + 2\left(1+E\right)\left(O_2 + 3.76 N_2\right) \rightarrow CO_2 + 2\, H_2O + 2E\left(O_2 + 3.76 N_2\right) + 7.52\, N_2 \qquad (9.18)$$

where the number of moles of excess air is denoted as *E*.

For complete combustion of methane with air, one mole of carbon dioxide and two moles of water are always formed, independent of the amount of excess air in the reactants. The ratios of the moles of carbon dioxide and water formed to the number of moles of the fuel depend only on the chemical composition of the fuel. Therefore, the latent loss term represented in equation 9.13 is independent of the amount of excess air.

However, the stack loss depends on the mass flow rate of the products relative to the fuel flow rate, which is a function of the amount of excess air. The mass flow rates of the different compounds are determined from the number of moles and their respective molecular weights. For the reaction given by equation 9.18, the ratio of the flow rate of the products to that of the fuel is given by:

$$\frac{\dot{m}_p}{\dot{m}_{fl}} = \frac{N_{CO_2}\, M_{CO_2} + N_w\, M_w + N_{O_2}\, M_{O_2} + N_{N_2}\, M_{N_2}}{N_{fl}\, M_{fl}} \qquad (9.19)$$

Typically, the mass flow of the products is 20 to 30 times the mass flow rate of the fuel. The specific heat of the products is close to that of air since the exhaust consists mostly of the compounds that make up air. The stack loss can be determined with the ratio of flow rates from equation 9.19, the specific heat, and the temperature of the products. With both the latent and stack losses evaluated, the furnace efficiency can be determined.

The specific heat of the products can be determined exactly from the chemical composition of the products, the products temperature, and the enthalpy of the products at the products temperature. For the determination of the furnace efficiency, the average specific heat over the temperature range between the reference temperature and the products temperature is needed. This is found using a finite difference approximation for the specific heat as the difference in enthalpy divided by the difference in temperature. The average specific heat is determined from

$$c_{p,p} = \frac{\sum\left(N_i\, M_i\, h_i\right)_{T_p} - \sum\left(N_i\, M_i\, h_i\right)_{T_o}}{\sum\left(N_i\, M_i\right)\left(T_p - T_o\right)} \qquad (9.20)$$

Although it is possible to determine the average specific heat for each furnace calculation, it is probably not necessary to determine the average specific heat too accurately. The energy associated with the products is 10% or less of the fuel energy. The fuel composition is often not precisely known and the amount of excess air is usually approximate. Thus an average specific heat is sufficiently accurate for most calculations. The average specific heat for the products of combustion for methane computed using equation 9.20 for several product temperatures and amounts of excess air are given in Table 9.3. The values are also compared to the values for air over the same temperature range.

Table 9.3 Average specific heat for products of combustion

Products temperature	Excess air	Average $c_{p,p}$ (Btu/lbm-°F)	Average $c_{p,air}$ (Btu/lbm-°F)
200–400 °F	0	0.27	0.24
	1	0.26	0.24
	4	0.25	0.24
3000 °F	0	0.32	0.28
	1	0.30	0.28
	4	0.29	0.28

For the lower product temperature range (200–400 °F or 40–200 °C), the specific heat is slightly above that of air. A value of 0.26 Btu/lbm-°F or 1.09 kJ/kg-°C is a satisfactory approximation. At the higher temperature found directly after combustion (3000 °F or 1600 °C), an average value of 0.30 Btu/lbm-°F or 1.25 kJ/kg-°C is acceptable. If more precise values are desired an exact calculation can be performed. Example 9.2 demonstrates the calculation of furnace efficiency when combustion occurs with excess air. The entering air stream is dry and combustion with humid air is discussed later.

Example 9.2 Determine the efficiency of a furnace in which methane (natural gas) is combusted with 50% excess dry air. The reactants enter at 25 °C and the products leave at 205 °C. The products temperature is higher than the dew point and all of the water leaves as a vapor. The higher heating value of methane calculated in Example 9.1 is 55,492 kJ/kg and the average specific heat of the products is taken as 1.09 kJ/kg-°C.

```
"Problem specifications"
"Combustion reaction for methane and 50% excess air:
CH4 + (2+Ex)(O2 + 3.76N2) = CO2 + 2H2O + 2*Ex(O2+3.76N2)+2*3.76N2"
T_r= 25 "C"                                              "Reactants temp"
T_p = 205 "C"                                            "Products temperature"
```

```
Ex = 0.5                                                              "Excess air"
HHV = 55617 "kJ/kg"                                                   "Higher heating value"
c_p_ave = 1.09  "kJ/kg-C"                                             "Products specific heat"

"Determine the moles of components for the reaction of methane and air."
N_CH4 = 1 "kgmole"                                                    "Moles fuel"
N_CO2 = 1 "kgmole"                                                    "Moles CO2"
N_H2O = 2 "kgmole"                                                    "Moles H2O"
N_O2 = 2*Ex  "kgmole"                                                 "Moles O2"
N_N2 = 2*(1+Ex)*3.76  "kgmole"                                        "Moles N2"

"Determine the molecular weight of the reactants and the products."
M_f = N_CH4*MolarMass(Methane) "kg"                                   "Mass of fuel"
M_w = N_H2O*MolarMass(H2O) "kg/kgmole"                                "Mass of water"
M_p=N_CO2*MolarMass(CO2)+N_H2O*MolarMass(H2O)+N_O2*MolarMass(O2)
 + N_N2*MolarMass(N2)                                                 "Mass products"

"Determine the sensible and latent components of the losses and the efficiency."
Sensible = (M_p/M_f)*c_p_ave*(T_p - T_r)/HHV                          "Sensible loss"
h_fg = Enthalpy(Water,T=T_r,x=1) - Enthalpy(Water,T=T_r,x=0) "kJ/kg"  "Latent heat"
Latent = (M_w/M_f)*h_fg/HHV                                           "Latent loss"
Eff = 1 - Sensible - Latent                                           "Efficiency"
```

Results

The molar mass of the fuel is that of methane, 16.04 kg/kg-mol, and that of the products is 428 kg/kg-mol. The molar mass of the water vapor in the products is 36.03 kg/kg-mol. The latent loss as a fraction of the fuel energy is 0.099. The stack loss due to the heated products being exhausted is 0.094. The resultant furnace efficiency is 0.807.

The latent and stack losses are comparable and are both equal to about 10%. The latent loss can be reduced by condensing the water vapor, and the stack loss can be reduced by lowering the exhaust temperature and decreasing the amount of excess air.

In condensing furnaces, products are cooled below the dew-point temperature, which condenses the water vapor in the exhaust and reduces the latent loss. The dew point depends on the amount of water vapor in the air, which depends on the partial pressure of the water vapor in the products. The partial pressure of a component in a mixture of ideal gases depends on the number of moles of that component relative to the total number of moles. Each of the components in the products can be treated as an ideal gas, and using the equation of state allows the partial pressure of any one component to be written in terms of its mass and the mixture total volume as:

$$p_i V = m_i R T = m_i \frac{R_o}{M_i} T = N_i R T \tag{9.21}$$

The partial pressure of a component can then be related to the total pressure and the number of moles of each of the components. Using Dalton's law, the ratio of the partial pressure to the total pressure is equal to the ratio of the number of moles of that compound to the total number of moles in the mixture:

$$\frac{p_i}{p} = \frac{p_i}{\sum_{j=1}^{n} p_j} = \frac{N_i}{\sum_{j=1}^{n} N_j} = \frac{N_i}{N_T} \tag{9.22}$$

The total pressure p is the sum of the partial pressures p_i of each of the components. With the partial pressure of the water vapor in the products known, the dew-point temperature can be determined from the psychrometric relations. The determination of the dew point of the products in Example 9.2 is illustrated in Example 9.3.

Example 9.3 Determine the dew point of the products of Example 9.2 for two situations. In the first the incoming air is dry and consists only of oxygen and nitrogen. In the second situation the relative humidity of the incoming air is 60%.

```
"Problem specifications"
"Combustion reaction for methane and 50% excess air:
CH4 + (2+Ex)(O2 + 3.76N2) = CO2 + 2H2O + 2*Ex(O2+3.76N2)+2*3.76N2"
p_atm = 101.3 "kPa"                                   "Atmospheric pressure"
T_r= 25 "C"                                           "Reactants temp"
Ex = 0.5                                              "Excess air"

"Determine the moles of components for the reaction of methane and air."
N_CH4 = 1 "kgmole"                                    "Moles fuel"
N_CO2 = 1 "kgmole"                                    "Moles CO2"
N_H2O = 2 "kgmole"                                    "Moles H2O"
N_O2 = 2*Ex  "kgmole"                                 "Moles O2"
N_N2 = 2*(1+Ex)*3.76  "kgmole"                        "Moles N2"

"Determine the dew point of the products when the reactants are only fuel and dry air. With no
water vapor in the reactants, the partial pressure of the water vapor in the products is due only to
the water that is formed by combustion. From the reaction equation, 2 moles of water vapor are
formed, and the total number of moles of the products, including the water vapor, is 1 +2
```

```
+2*Ex*(1 + 3.76) +2*3.76. The partial pressure of the water vapor is given by equation 9.21. The
dew point of the products is the saturation temperature of water at that pressure."
N_total_dry = N_CO2 + N_H2O +N_O2 + N_N2                      "Moles product"
p_w/p_atm = N_H2O/N_total_dry                                 "Partial pressures"
T_dpt_dry = Temperature(Water,x=1,P=p_w) "C"                  "Dew point"

"Determine the number of moles of water vapor in the incoming air. From the relative humidity the
partial pressure of the water vapor is determined. The total number of moles in the incoming air is
then  (1+3.76+N_w_r)."
RH = 0.6                                                      "Relative humidity"
w_z = humrat(airh2o,p=p_atm, T=T_r, R=RH) "kg/kg"             "Humidity ratio"
p_sat = pressure(water, T=T_r ,x=1) "kPa"                     "Saturation pressure"
p_w_r = 0.6*p_sat "kPa"                                       "Partial pressure"
p_w_r/p_atm = N_w_r/(1+3.76+N_w_r)                            "Partial pressures"

"Determine the dew point of the products when the incoming air is humid. The reaction is then
with (2+Ex)(1 + 3.76 + N_w_r), so in the products there are (2+Ex)*N_w_r moles that came in
with the combustion air and 2 moles of water vapor formed by combustion"
p_w_p/p_atm = N_w_p/N_total_wet                               "Partial pressures"
N_w_p = N_H2O+(2+Ex)*N_w_r                                    "Moles water"
N_total_wet = N_CO2 +N_H2O +N_O2 + N_N2+(2+Ex)*N_w_r          "Moles product"
T_dpt_wet = Temperature(Water,x=1,P=p_w_p) "C"                "Dew point"
```

Results

For the combustion of methane with dry air, there are 2 moles of water vapor and 15.28 moles of product. The partial pressure of the water vapor in the products is 13.2 kPa (1.92 psia), and the corresponding dew point is 51.4 °C (124.6 °F). If the products are cooled to below this temperature, some of the water will condense.

For a relative humidity of 60%, the partial pressure of the water vapor in the air is 1.90 kPa. For 1 + 3.96 moles of air, this corresponds to an additional 0.091 moles of water vapor. For the reaction of methane with 50% excess air, there is an additional 0.228 moles of water vapor in the products that was present in the combustion air. The partial pressure of water vapor in the products is now increased to 14.55 kPa (2.11 psia) and the dewpoint is 53.3 °C (128.0 °F). The new dew-point temperature is only 1.9 °C higher than that for dry air. There is a small but usually negligible increase in the dew point of the products when combustion occurs with humid air.

A condensing furnace cannot really condense all of the water vapor in the exhaust flow. If the temperature of the products is reduced to below the dewpoint the air will become saturated but there will still be some water vapor in the air. At a given temperature, the number of moles of water in saturated products is obtained by rearranging equation 9.22:

$$N_w = \frac{p_{sat}}{p_a} \sum_{j=1}^{n} N_j \tag{9.23}$$

The sum includes only the number of moles of water that are vapor since the water that is condensed does not contribute to the partial pressure of the products. Since equation 9.23 has the number of water vapor moles on each side of the equation, it must be algebraically rearranged to solve for the number of water vapor moles. The latent loss is then evaluated using the number of moles that remain as vapor:

$$Latent\ loss = \frac{\dot{m}_w\, h_{fg}}{\dot{m}_{fl}\ HHV} = \frac{N_w\, M_w\ h_{fg}}{N_{fl}\ M_{fl}\ HHV} \tag{9.24}$$

where $\dot{m}_w$ is the flow rate of water vapor in the products. The calculation of the latent loss when the products are cooled below the dew point is illustrated in Example 9.4.

Example 9.4 Determine the latent loss for the combustion of methane with 50% excess air when the products temperature are cooled to 32 °C.

```
"Problem specifications"
"Combustion reaction for methane and 50% excess air:
CH4 + (2+Ex)(O2 + 3.76N2) = CO2 + 2H2O + 2*Ex(O2+3.76N2)+2*3.76N2"
p_atm = 101.3 "kPa"                                    "Atmospheric pressure"
T_r= 25 "C"                                            "Reactants temp"
T_p = 32 “C”                                           “Products temperature”
Ex = 0.5                                               "Excess air"
HHV = 55492 “kJ/kg”                                    “Higher heating value”
c_p_ave = 1.09 “kJ/kg-C”                               “Products specific heat”
N_CH4 = 1 “kgmole”                                     “Moles fuel”
N_CO2 = 1 “kgmole”                                     “Moles CO2”
N_H2O = 2 “kgmole”                                     “Moles H2O”
N_O2 = 2*Ex  “kgmole”                                  “Moles O2”
N_N2 = 2*(1+Ex)*3.76  “kgmole”                         “Moles N2”

"Determine the mass of the reactants for one mole of fuel."
M_f = N_CH4*MolarMass(Methane) "kg"                    "Molar mass of fuel"
```

```
"Determine the dew point of the products when the reactants are only fuel and dry air. Two moles
of water vapor are formed and the total number of moles of the products is 1+2+2*Ex*(1+3.76)
+2*3.76. The partial pressure of the water vapor is given by equation 9.21. The dew point of the
products is the saturation temperature of water at that pressure."
N_w = 2                                                             "Moles water vapor"
N_total_dry = N_CO2 +N_H2O +N_O2+N_N2                               "Moles product"
p_w/p_atm = N_w/N_total_dry                                         "Partial pressures"
T_dpt_dry = Temperature(Water,x=1,P=p_w) "C"                        "Dew point"

"Determine the partial pressure of the water vapor for saturated products at the products
temperature."
p_w_sat = pressure(Water,x=1,T=T_p) “kPa”                           "Saturation pressure"

"Determine the number of moles relative to the total number of moles in the products for
saturation conditions. The number of moles of water vapor at saturation conditions affects the
total number of moles in the products (equation 9.19)."
p_w_sat/p_atm = N_w_cond/N_total_cond                               "Partial pressure"
N_total_cond = N_CO2 + N_w_cond + N_O2 + N_N2 "kgmole"              "Moles product"

"Determine the mass of the water vapor and the products per kg-mol of fuel. The number of
moles of water vapor is N_w_cond when condensation is present."
M_w = N_w_cond*MolarMass(H2O) "kg/kgmole"                           "Molar mass of water"
M_p = N_CO2*MolarMass(CO2) +N_w_cond*MolarMass(H2O)  +N_O2*MolarMass(O2)+
N_N2*MolarMass(N2)                                                  "Molar mass products"

"Determine the latent loss."
h_fg = Enthalpy(Water,T=25,x=1) - Enthalpy(Water,T=25,x=0) “kJ/kg”  “Latent heat”
Latent = (M_w/M_f)*h_fg/HHV                                         “Latent loss”

"Determine the stack loss and the furnace efficiency."
Sensible = (M_p/M_f)*c_p_ave *(T_p - T_r)/HHV                       “Stack loss”
Eff = 1 - Sensible – Latent                                         “Furnace efficiency”
```

Results

From Example 9.3, the dew point of the products when dry air is the reactant is 51.4 °C. When the products are cooled to 32 °C some of the water vapor will condense. The saturation water vapor pressure for a products temperature of 32 °C is 4.8 kPa (0.698 psia). For 1 atm pressure, the number of moles of water vapor in the products given by equation 9.23 is reduced from 2 kg-mol to 0.65 kg-mol. The mass of water vapor for one kg-mol of fuel is reduced correspondingly, and the latent loss is reduced to 3.3%, compared to the loss without condensation of 9.9% of example 9.2. Even though the exhaust temperature is quite close to the room temperature, there is still some latent loss due to water vapor in the saturated products.

Further, the stack loss is reduced to about 0.4% when the products are cooled to 32 °C (90 °F). The products flow rate is reduced somewhat since some of the water leaves as liquid, but the main reduction in the loss comes from the lower temperature.

In Example 9.2, the furnace efficiency for a non-condensing furnace is 0.807. As a result of condensing the water in the products, the furnace efficiency is increased to 0.964. This is a 20% increase in furnace efficiency and a 20% decrease in fuel consumption.

As shown in Example 9.3, the presence of water vapor in the incoming air increases the vapor pressure of the water vapor in the products. If the combustion air in this example were the humid air of Example 9.3, there would be an additional 0.091 moles of water vapor in the products. This would essentially allow another 0.091 moles of water vapor formed by combustion to condense, and reduce the latent loss to a little less than 3%. There is a positive, but small, increase in the condensing furnace efficiency when the combustion air is humid.

As illustrated in Examples 9.3 and 9.4, the furnace efficiency is a function of the amount of excess air and the temperature of the products. The parametric effect of these variables on furnace efficiency can be determined using the combustion and psychrometric relations described previously. The furnace efficiency is plotted as a function of the temperature of the products for different amounts of excess air in Figure 9.4. The incoming air is dry and at 77 °F (25 °C), and the dew-point temperature of the products is as indicated on the graph

The efficiency is lowest for the highest amount of excess air and the highest products temperature. The efficiency increases as both the amount of excess air and the temperature of the products are decreased. For example, at 100% excess air and a products temperature of 400 °F (205 °C), the stack loss is about 12%, the latent loss is about 10%, and the efficiency is about 78%. For 0% excess air and a products temperature of 400 °F (200 °C), the stack loss is reduced to about 6% because the products flow rate is essentially halved, but the latent loss is the same value because there is the same amount of water vapor in the products. The efficiency is highest with the least amount of excess air and coldest temperature of the products, as expected.

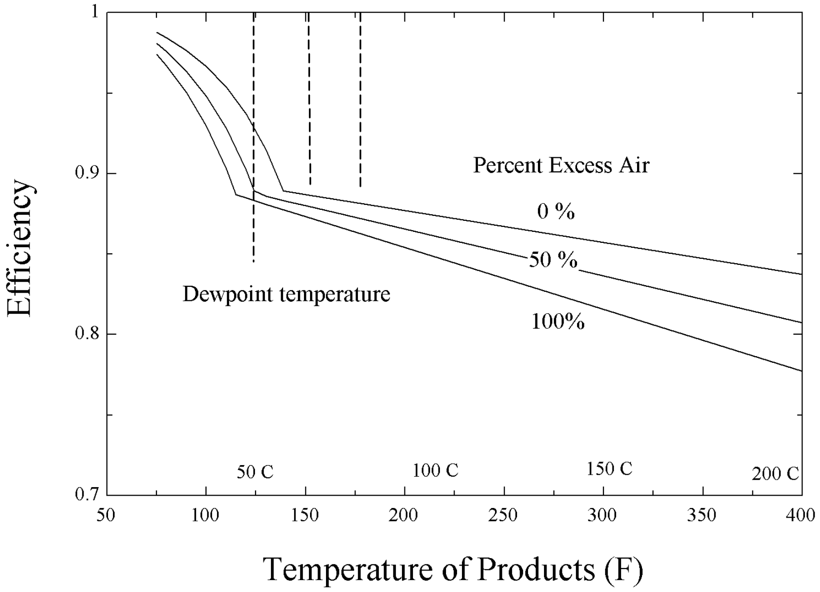


Figure 9.4 Furnace efficiency as function of the products temperature

Reducing the temperature of the products reduces only the stack loss until condensation occurs. Depending on the amount of excess air, the dewpoint is between 120 and 140 °F (50 and 60 °C). At product temperatures slightly above this range, the stack loss is relatively small and about 2%, whereas the latent loss is still 10%. Lowering the products temperature below the dew point then reduces the latent loss significantly as the temperature is reduced. The products temperature needs to be high enough to transfer heat to the return air from the building, which is in the range of 70 to 80 °F (18 to 27 °C). The lowest practical temperature of the products is then in the range of 90 to 100 °F (35 to 40 °C). At least 5% excess air is required for complete combustion and to prevent formation of carbon monoxide. The constraints on operation still allow efficiencies in the range of 90 to 95% to be achieved.

SM 9.6 Simplified Model for Overall Furnace Performance

In Section 9.3, the thermodynamic combustion reaction equations were developed, yielding a set of relations describing the chemical composition of the products, the energy release, and the steady-state efficiency. A simplified model of a furnace can be developed by introducing relations that describe the transfer of heat from the combustion gas to the circulating fluid. Such a simplified model can then be used to understand the effect of the important parameters on furnace performance.

A furnace or boiler can be represented as two thermal components. One is a combustion chamber where the entering fuel and air react to produce high-temperature products. The combustion chamber is insulated, with relatively little heat loss to the environment, allowing the combustion process to be treated as adiabatic. Further, the combustion process can be treated as complete with products that do not contain either unburned fuel or products of incomplete combustion. The second component is a heat exchanger in which heat is transferred from the products to the low temperature circulating fluid that heats the building. The cooled products are then exhausted to the environment at a low temperature. This representation of a furnace is shown in Figure 9.5.

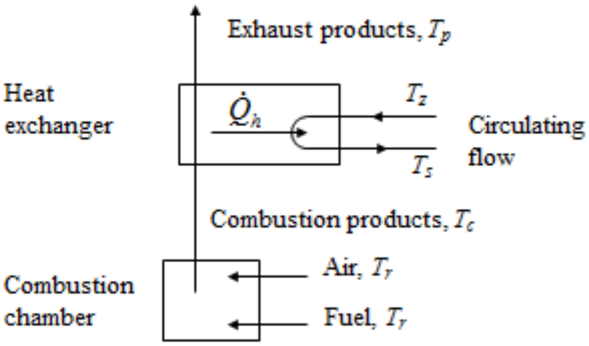


Figure 9.5 Functional representation of a furnace or boiler.

For a complete and adiabatic reaction, the energy release from combustion goes entirely into heating the products. The energy balance for adiabatic combustion is given by equation 9.7 with the heat flow term equal to zero. The reactants are typically either at the reference temperature T_o or close to it. The temperature of the products after this adiabatic reaction is termed the **adiabatic flame temperature**, T_c. The energy balance for adiabatic combustion can be expressed as

$$\left(\dot{m}_a h_a + \dot{m}_{fl} h_{fl} - \dot{m}_p h_p\right)_{T_o} - \dot{m}_p\left(h_{p,T_c} - h_{p,T_r}\right) = 0 \tag{9.25}$$

The term on the left is again the fuel flow rate times the heating value, equation 9.8. The lower heating value is appropriate because the temperature after combustion is high and the water that forms is a vapor. The enthalpy change of the products can be represented by the product of the mass flow rate, specific heat, and the temperature change, as in equation 9.11. The adiabatic combustion energy equation 9.25 can be written as

$$\left(\dot{m}_{fl}\ LHV\right)_{T_o} - \dot{m}_p c_{p,p}\left(T_c - T_o\right) = 0 \tag{9.26}$$

The temperature after combustion, termed the adiabatic flame temperature, can then be determined from equation 9.26 as

$$T_c = T_o + \frac{\left(\dot{m}_{fl}\, LHV\right)_{T_o}}{\dot{m}_p c_{p,p}} \tag{9.27}$$

The adiabatic flame temperature given by equation 9.27 is fictitious in that the temperature of the combustion products in a furnace will actually be considerably lower. During the combustion there is significant heat transfer by radiation from the flames to the heat exchanger surfaces that cool the products. However, the temperature given by equation 9.27 represents the potential for heat transfer to the circulating fluid and is useful in estimating furnace performance.

After combustion, the products enter the heat exchanger and transfer heat to the circulating fluid. For a non-condensing furnace or boiler the heat transfer is sensible and the products cool down without condensation of the water vapor. This heat transfer is given by the flow rate and enthalpy difference of the products, with the enthalpy difference represented by the specific heat and temperature change:

$$\dot{Q}_h = \dot{m}_p \left(h_c - h_{p,T_p}\right) = \dot{m}_p c_{p,p} \left(T_c - T_p\right) \tag{9.28}$$

where the specific heat of the products is the average over the temperature range from T_p to T_c. For a warm air or hot water furnace, the heat transfer can also be represented in terms of the mass flow rate, specific heat, and temperature change of the circulating flow:

$$\dot{Q}_h = \dot{m}_h c_{p,h} \left(T_s - T_z\right) \tag{9.29}$$

For a boiler producing steam, the heat transfer would be represented in terms of the steam flow rate and the latent heat of vaporization.

In addition to these energy balance equations, the heat transfer can be expressed in terms of the relations for a sensible heat exchanger (Chapter 13). The effectiveness relates the actual heat transfer to the maximum possible and is a useful measure of the performance of a heat exchanger. The maximum heat transfer would occur if the fluid with the minimum capacitance rate experienced the maximum possible temperature difference, which is the difference between the temperature of the products after combustion and the temperature of the circulating fluid entering from the zone. The fluid with the minimum capacitance rate is the products flow, since the products are cooled from a very high combustion temperature of 2000 to 3000 °F (1000 to 1700 °C) to a low exhaust temperature of 100 to 400 °F (40 to 200 °C), whereas the circulating flow enters the furnace or boiler at the zone temperature for a warm air furnace or higher for a hot-water boiler and leaves as a supply stream at a moderately warm temperature (120 to 200 °F, or 50 to 90 °C).

Using the adiabatic flame temperature as the maximum possible temperature, the heat transfer then can be expressed in terms of an effectiveness and the capacitance rate of the products as (Section 13.3)

$$\dot{Q}_h = \varepsilon\, \dot{m}_p c_{p,p} \left(T_c - T_z\right) \tag{9.30}$$

The effectiveness is a function of the number of transfer units (*Ntu*) and capacitance rate ratio (*C**). The number of transfer units contains the overall thermal conductance for the heat exchanger, which is the reciprocal of the sum of the convection and conduction thermal resistances between the products flow and circulating fluid flow.

In sizing a furnace for a given situation, the heating output is specified and the mass flow rates of the fuel and products need to be determined. The effectiveness-Ntu method is awkward for this situation in that the unknown mass flow rate appears in both the Ntu and the capacitance rate ratio. The energy balance and rate equations can be combined into the log-mean-temperature-difference form, which is more convenient in this situation. For a counterflow heat exchanger, the heat transfer rate is the product of the overall thermal conductance (*UA*) and log-mean-temperature-difference (*LMTD*).

$$\dot{Q}_h = UA\, LMTD \tag{9.31}$$

It is assumed that the heat exchanger is a counter-flow exchanger, but this is not a critical assumption. The LMTD for counter flow is given in terms of the different fluid temperatures as

$$LMTD = \frac{\left(T_c - T_s\right) - \left(T_p - T_z\right)}{\ln\left(\dfrac{\left(T_c - T_s\right)}{\left(T_p - T_z\right)}\right)} \tag{9.32}$$

Using an *LMTD* based on the adiabatic flame temperature along with a *UA* is an approximation to the heat-exchange process. The use of the adiabatic flame temperature overestimates the *LMTD* and underestimates the *UA* of an actual furnace. However, equation 9.31 is useful for illustrating qualitatively the relation between the variables.

The fuel flow rate is determined from equation 9.9 and 9.10. If the products temperature is higher than the dew point, the water in the products is a vapor and the lower heating value is used. If the furnace is a condensing furnace then the higher heating value is appropriate. The set of equations 9.28 through 9.32 provide a basis for estimating the performance of a furnace.

Example 9.5 illustrates the process of estimating the performance of a furnace with a specified heating capacity and an overall thermal conductance. For this example the entering air is assumed to be totally dry and the products temperature is assumed to be high enough that the water vapor does not condense. The heating value is taken from Table 9.2 and the average specific heat of the products is taken from Table 9.3.

Example 9.5 Determine the required fuel and air flow rates, the steady state efficiency, and the circulation flow rate for a warm air furnace with a design heating capacity of 100,000 Btu/hr. The fuel is natural gas burned with 50% dry excess air. At design conditions, the reactants enter at the zone temperature of 77 °F and the circulating zone air is heated from 77 °F to 130 °F. The heat exchanger has an overall thermal conductance (*UA*) based on the adiabatic flame temperature of 115 Btu/hr-°F.

```
"This function computes the latent loss for the combustion of CH4 if the temperature of the
products is below the dewpoint of the products."
Function Lat(M_f,M_w,T_p,T_dp,HHV,Ex)
        p_atm = 14.7
        h_fg = Enthalpy(Water,T=77,x=1) - Enthalpy(Water,T=77,x=0)
    If (T_p>T_dp) Then
        Lat: = (M_w/M_f)*h_fg/HHV
    Else
        p_w = Pressure(Water, T=T_p, x=1)
        N_dry = 1 +2*Ex*(1 + 3.76) +2*3.76
        N_wet = (p_w/p_atm)*N_dry/(1 - p_w/p_atm)
        M_wet = N_wet*18
        Lat:= (M_wet/M_f)*h_fg/HHV
    EndIf
end

"Problem specifications."
"Combustion reaction for methane and 50% excess air:
CH4 + (2+Ex)(O2 + 3.76N2) = CO2 + 2H2O + 2*Ex(O2+3.76N2)+2*3.76N2"
p_atm = 14.7 "psia"                         "Atmospheric pressure"
T_z= 77 "F"                                 "Zone temp"
T_o = 77 "F"                                "Reactants temp"
Ex = 0.5                                    "Excess air"
LHV = 21500 "Btu/lbm"                       "Lower heating value"
HHV = 23857 "Btu/lbm"                       "Higher heating value"
c_p_p = 0.30 "Btu/lbm-F"                    "Products specific heat"
N_CH4 = 1 "lbmole"                          "Moles fuel"
N_CO2 = 1 "lbmole"                          "Moles CO2"
N_H2O = 2 "lbmole"                          "Moles H2O"
```

```
N_O2 = 2*Ex  "lbmole"                                  "Moles O2"
N_N2 = 2*(1+Ex)*3.76  "lbmole"                         "Moles N2"
Q_dot = 100000 "Btu/hr"                                "Heating capacity"
T_s = 130 "F"                                          "Supply temperature"

"Determine the adiabatic flame temperature using equation 9.27 for a flow rate of 1 lb-mol of fuel.
The actual fuel and products flow rates will be determined from the heat exchanger relations. The
lower heating value is used because the water formed is a vapor at the high temperature after
combustion."
T_c = T_o +M_f*LHV/(c_p_p*M_p) "F"                     "Adiabatic flame temp"
M_f = N_CH4*MolarMass(Methane) "kg"                    "Mass of fuel"
M_p = N_CO2*MolarMass(CO2) +N_H2O*MolarMass(H2O)  +N_O2*MolarMass(O2)+
N_N2*MolarMass(N2)                                     "Mass products"

"Determine the products flow rate required to transfer the desired heat transfer to the supply air.
The circulation air flow rate is determined from the heat transfer to the supply air and the
temperature rise of the zone air to the supply temperature."
Q_dot = m_dot_s*c_p_air*(T_s - T_z) "Btu/hr"           "EB on supply air"
c_p_air = SpecHeat(Air, T=T_s) "Btu/lbm-F"             "Air specific heat"

"Determine the exhaust temperature of the products using the LMTD expression, equation 9.31.
The LMTD is determined using equation 9.30."
Q_dot = UA*LMTD "Btu/hr"                               "HT by LMTD method"
LMTD = ((T_c - T_s) - (T_p-  T_z))/(ln((abs(T_c - T_s))/(abs(T_p - T_z)))) "F"          "LMTD"

"Determine the heat exchanger effectiveness."
eff_HX = (T_c - T_p)/(T_c - T_z)                       "Heat exchanger effectiveness"

"Determine the products flow rate from the heat transfer rate, the adiabatic flame temperature,
and the products temperature. "
Q_dot = m_dot_p*c_p_p*(T_c - T_p) "Btu/hr"             "EB on products flow"

"Determine the sensible loss based on the products temperature."
Sensible = (M_p/M_f)*c_p_p*(T_p - T_o)/HHV             "Sensible loss"

"Determine the dewpoint of the products."
N_p = N_CO2 +N_H2O +N_O2+ N_N2                         "Moles product"
p_w/p_atm = N_H2O/N_p                                  "Partial pressures"
T_dp = Temperature(Water,x=1,P=p_w)                    "Products dewpoint"
```

```
"Determine the latent loss. This includes the effect of condensation if the temperature is below
the dew point temperature."
M_w = N_H2O*MolarMass(H2O)                          "Moles water"
Latent = Lat(M_f,M_w,T_p,T_dp,HHV,Ex)               "Latent loss"

"Determine the furnace efficiency and the fuel flow rate from the heat transfer rate and the
heating value."
eff = 1 - Sensible - Latent                         "Furnace efficiency"
eff = Q_dot/(m_dot_f*HHV)                           "Furnace efficiency"

"Determine the airflow rate from a mass balance on the furnace."
m_dot_f +m_dot_a = m_dot_p                           "Mass flow rate"
```

<u>Results</u>

The flame temperature is determined using equation 9.27. For the reaction of 1 lb-mol (16 lb) of methane with 50% excess air, the mass of the products produced is 428 lbm. The adiabatic flame temperature is found to be 2769 °F. The value for the average specific heat of the products is taken to be 0.30 Btu/lb-°F, and is higher than that used in Example 9.2 because the average temperature is higher.

Since the heat transfer rate and the overall thermal conductance (UA) are specified for this furnace, the LMTD necessary to transfer the desired amount of heat can be determined directly. Using 9.32, the LMTD is found to be 870 °F. With the LMTD specified, the products temperature is found using equation 9.31 to be 229 °F. This is higher than the dewpoint of 124.7 determined in example 9.3, and so water vapor does not condense in the heat exchanger.

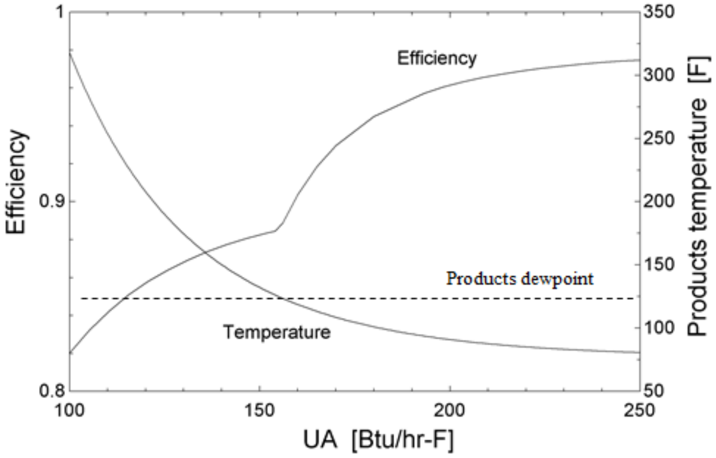
The mass flow rate of the products is now determined from equation 9.30, which relates the heat transfer rate to the change in temperature of the products. The products mass flow rate is found to be 131.5 lbm/hr. The flow rate of fuel required to produce the products flow rate is 4.93 lbm/hr and the air mass flow rate is 126.6 lbm/hr. The steady state furnace efficiency is 0.85 based on the higher heating value (equation 9.12).

The mass flow rate of circulating air required to transfer the 100,000 Btu/hr to the zone is determined using the energy balance, equation 9.29 The flow rate is 7850 lbm/hr, corresponding to about 1870 cfm.

The heat exchanger effectiveness is then determined from equation 9.30 as 0.94 and the number of transfer units (*Ntu*) is 2.9. The minimum capacitance rate is that of the combustion products, as can be seen in that the temperature drop of the products is much greater than the temperature rise of the circulating air.

The heat exchanger has a high value of the effectiveness, which means that the heat transfer rate is close to the maximum possible for the given capacitance rates. However, the furnace efficiency is considerably lower (87%). The reason for this difference is that the effectiveness used in this model is based on sensible heat exchange and does not include latent energy. A more complete model would use an effectiveness based on the combination of latent and sensible energy, as for cooling coils (Chapter 13).

The effect of increasing the heat exchanger's overall thermal conductance on the performance of the furnace is shown in the figure below. As the overall thermal conductance is increased the temperature of the products drops and there is a corresponding increase in efficiency. For this furnace, at an overall thermal conductance of about 160 Btu/hr-°F, the products temperature is below 124.7 °F and condensation occurs. The furnace efficiency increases significantly. There is a diminishing increase in efficiency as the overall thermal conductance is increased beyond 200 Btu/hr-°F, and this is probably close to the economic maximum value of overall thermal conductance.



SM 9.7 Seasonal Performance

The seasonal performance of a furnace or boiler reflects the operating cost over the year, and it may be less than the steady-state efficiency due to a number of factors. The measure of the seasonal performance of heating equipment is the annual fuel utilization efficiency (AFUE), which is the ratio of the total heat delivered to the building divided by the total fuel energy supplied. One factor that adversely affects the seasonal efficiency is oversizing the furnace capacity relative to the load. Some modulation of heating equipment is an inherent result of variation of heating loads over the course of a season. The installed capacity is selected on the basis of the design load, which is the maximum expected load, while in operation the system needs only to meet the actual load, which is considerably less than the maximum most of the time. The unit needs to be regulated to produce only the heat that is needed. In residential units and small commercial buildings the unit is cycled on and off so that heat is supplied intermittently. In larger heating systems the heating output is modulated continuously through control of the fuel flow rate.

An estimate of the time that a furnace is on during a heating season helps explain the decrease in seasonal efficiency from the steady-state value. The installed capacity can be compared to the average heating load. The installed capacity is based on the design heating load, which can be estimated as the product of the overall thermal conductance of the building and the temperature difference between the zone and the ambient at design conditions:

$$\dot{Q}_{des} = UA\left(T_{z,des} - T_{A,\,des}\right) \tag{9.33}$$

A typical design temperature for a zone is 75 °F (24 °C). The ambient design temperature is a low temperature that is usually selected to be at the 99% level for that location, which means that the ambient temperature will be no less than this value for more than 1% of the time during a typical year.

The actual load on the heating system at any time can also be estimated using the overall thermal conductance for the building, but the temperature difference is that between the zone balance temperature and the ambient temperature. The balance temperature reflects that heat is supplied by internal and solar gains, and it is typically 10 to 20 °F (5 to 10 °C) lower than the thermostat setting (text, Section 18.3). For example, a zone thermostat setting of 75 °F (24 °C) and a balance temperature of 60 °F (15 °C) means that heating is not required until the ambient temperature drops below 60 °F (15 °C). Furthermore, the actual ambient temperature is higher than the design ambient temperature most of the time, and thus the average temperature during the heating season is considerably higher than the design ambient temperature. An estimate of the average load on a building is

$$\overline{\dot{Q}}_h = UA\left(T_{z,bal} - \overline{T}_A\right) \tag{9.34}$$

Equation 9.34 is accurate only for months in which the ambient temperature is always below the balance temperature, such as winter. It does not account for the times when the ambient temperature is greater than the balance temperature on some days, such as during spring and fall months. Equation 9.34 is an approximation that is useful in estimating furnace on-time.

For a system with on-off control, the unit runs at the full-load capacity when it is on and produces no heat when it is off. It meets the total heating requirement over the season, but the time that it is on is less than that during the heating season. The relation between the total heating requirement and the amount of time that the system is on is approximately the product of the furnace capacity and the on-time, or in terms of the design heat flow

$$Q_T = Capacity\ t_{on} = \dot{Q}_{des}\, t_{on} \tag{9.33}$$

The total heating requirement can also be approximated as the product of the average heating load and the total time of the heating season. In most locations the heating season is from October through April.

$$Q_T = \overline{\dot{Q}}_h \; t_{sea} \tag{9.34}$$

The fraction of the time that a heating system is on during the season can be estimated in terms of the relevant temperature differences using equations 9.31 to 9.34 as

$$f = \frac{t_{on}}{t_{sea}} = \frac{\left(T_{z,\,bal} - \overline{T}_A\right)}{\left(T_{z,\,des} - \overline{T}_A\right)} \tag{9.35}$$

The average and design temperatures for two locations, Chicago and Washington, DC, are given in Table 9.3. An estimate of the fraction of the time that a heating system with on-off control would operate for a building with a thermostat setting of 75 °F (24 °C) and a balance temperature of 60 °F (15 °C) is also given. An indication of the fraction of the time a furnace is on is given in Table 9.4.

Table 9.4 Operating time for heating equipment during heating season

Location	Design ambient temperature	Average ambient temperature	Fraction of time *f* furnace is on
Chicago	-8 °F (-22 °C)	32 F (0 °C)	35%
Washington, DC	10 °F (-12 °C)	40 °F (4 °C)	30%

The estimates in Table 9.4 show that heating units with on-off controls operate at full capacity only about one-third of the time during the heating season. The performance of a heating system when the furnace is off has a significant effect on the seasonal performance. In older residential units and in some current lower-performance models, a pilot light is used to initiate ignition. The pilot light burns continuously and the heat generated during the time that the furnace is off does not contribute to heating the building. Most modern furnaces and boilers employ electronic ignition devices to ignite the flame and eliminate this loss.

Older furnaces are often natural draft units with a chimney to induce air into the burners, and many chimneys do not have dampers to restrict warm air from flowing out of the building when the furnace is off. Newer units use exhaust dampers and controlled burner fans that bring in air from the outdoors for combustion only when the furnace is on. These improvements reduce the exfiltration loss associated with the furnace when it is off.

In older units the circulating fan does not turn on until the heat exchanger has reached a certain temperature, and the fan turns off as soon as the burner is turned off. Some of the heat transfer during the warm-up period heats air that exhausts through the chimney, and the energy in the hot exchanger heats air that is exhausted. This cyclic loss degrades performance significantly. Modern furnace controllers turn the circulating fans on as soon as the heat exchanger is warm but before the furnace has reached a high operating temperature, and they turn the fans off after the furnace metal has cooled to a relatively low value. This fan control helps minimize the thermal energy storage losses during startup and cool down. The design of modern high-efficiency furnaces allows the AFUE to closely approach the steady-state efficiency values even though the furnace operates in an on-off mode.

Table 9.4 also implies that large furnaces that modulate the heating output by controlling the fuel flow rate run at about one-third capacity on average. The AFUE of these units have also increased over time and are close to the steady-state values. The design of combustion chambers and fuel nozzles has improved so that the performance at part load is near that at full load. Modulation of the combustion air flow along with the fuel flow improves the heat exchanger performance and actually increases the efficiency. Simultaneous modulation of both air and fuel maintains the proper excess air level and reduces stack losses. The AFUE of these units is also close to the steady-state efficiency.

SM 9.8 Furnace Emissions

The use of hydrocarbon fuels to heat buildings generates exhaust products that have detrimental environmental impacts. As shown by the chemical reaction relations, carbon dioxide is produced and exhausted into the atmosphere, where it adds to the global atmospheric level. Chemicals are also produced through dissociation and by interactions with other compounds as a result of the high temperatures experienced during combustion As a result, the products can include oxides of nitrogen (NO_x), sulfur dioxide (SO_2), carbon monoxide (CO), volatile organic compounds (VOCs), and particulate matter, all of which are exhausted to the atmosphere. Although the levels of some of these compounds in the exhaust are very low, they can have adverse environmental and health effects.

In the atmosphere, carbon dioxide does not absorb the short-wave radiation emanating from the sun, allowing it to penetrate the atmosphere and reach Earth. However, carbon dioxide is opaque to long-wave thermal radiation and is an added thermal resistance for heat transfer from Earth's surface through the atmosphere. This means that the temperature of the Earth's surface must be higher to transfer heat to space than without carbon dioxide in the atmosphere, a mechanism termed the “greenhouse effect.” The consensus of the scientific field is that increases in the temperature of Earth and the atmosphere in the last few decades are due, in part, to the increased levels of carbon dioxide resulting from combustion of hydrocarbon fuels.

The high temperatures produced when hydrocarbon fuels react with air causes oxygen to react with nitrogen, producing a number of highly reactive compounds grouped as NO_x. The nitrogen dioxide in the group has a direct effect on human health in that it can react with moisture in the respiratory tract to form nitric acid and damage lung tissue. An indirect effect on human health is that NO_x contributes to the formation of ground-level ozone that, in the presence of sunlight, can form smog and cause respiratory problems. One environmental impact is that NO_x goes into solution in rain, snow, and fog, which damages forests and causes water bodies to be too acidic for many fish. Nitrous oxide, which is one of the compounds in NO_x, is a “greenhouse gas” that also contributes to global warming. Even though the levels of NO_x are so low that they are termed “trace levels,” it is important to minimize the amount that is produced.

Small amounts of the additional compounds present in liquid and solid hydrocarbon fuels are also sources of pollution. Sulfur reacts with oxygen to form sulfur dioxide, and, when dissolved in water, an acid solution is formed that is harmful to humans and that corrodes equipment. The oxides of trace amounts of metals in the fuel are harmful to human health. Unburned products can form small ash particles, and these can penetrate the lungs to cause respiratory diseases.

The potentially harmful effects resulting from fuel combustion requires careful design of the equipment and attention to safe operation. It is inevitable that carbon dioxide is produced. The amount is related to the amount of heat required, the efficiency of the unit, and the chemical composition of the fuel. The combustion relation, equation 9.18, establishes that the amount of carbon dioxide produced relates directly to the fuel flow rate. The ratio of the flow rate of carbon dioxide in the products to the flow rate of fuel entering the combustion chamber is

$$\frac{\dot{m}_{CO_2}}{\dot{m}_{fl}} = \frac{N_{CO_2}\, M_{CO_2}}{N_{fl}\, M_{fl}} \tag{9.36}$$

The fuel flow rate is related directly to the heating rate and furnace efficiency through equation 9.12:

$$\dot{m}_{fl} = \frac{\dot{Q}_h}{\eta_{fr}\, HHV} \tag{9.37}$$

Combining equations 9.36 and 9.37 allows the carbon dioxide produced by combustion to be expressed as

$$\dot{m}_{CO_2} = \left(\frac{\dot{Q}_h}{\eta_{fr}\, HHV}\right)\left(\frac{N_{CO_2}\, M_{CO_2}}{N_{fl}\, M_{fl}}\right) \tag{9.38}$$

where the first set of terms in parentheses is a function of the furnace efficiency, heating required by the building, and fuel heating value, and the second set of terms is a function of the chemical composition of the fuel. From a design or operating point of view, the only parameter that can be altered is the furnace efficiency. The amount of carbon dioxide produced by heating a building is then essentially directly proportional to the heating supplied. Example 9.6 illustrates the magnitude of the amount of carbon dioxide produced by a furnace through combustion.

Example 9.6. Estimate the carbon dioxide emissions from a natural gas–fired furnace used to supply heat to a building. The furnace capacity is 150 kW (500,000 Btu/hr) and the total heating requirement over the course of the year is 300,000 kWh (1×10^9 Btu). The furnace efficiency is 90%.

The higher heating value for natural gas, obtained from Table 9.1, is about 55,000 kJ/kg. Natural gas is mostly methane, and the combustion reaction relation for methane, equation 9.18, shows that the combustion of one mole of fuel produces one mole of carbon dioxide. The molecular weights of carbon dioxide and fuel are 44 kg/kg-mol and 16 kg/kg-mol, respectively. The rate at which carbon dioxide is emitted is given by equation 9.38 as

$$\dot{m}_{CO_2} = \left(\frac{150(kW)}{0.9 * 55,000(kWh)} \right)\left(\frac{1 * 44\, kg/kgmole}{1 * 16\, kg/kgmole} \right) = 30\ kg/hr$$

The annual mass of carbon dioxide produced is obtained with equation 9.38, using the seasonal heating requirement of 300,000 kWh. The total amount of carbon dioxide produced is 60,000 kg/year, or 65 tons per year. This is a large number, but it needs to be put in perspective.

The mass of carbon dioxide emitted by a typical automobile is about five tons per year. The amount emitted by the furnace of 60,000 kg or 65 tons is as much as 10 to 15 automobiles emit in a year. A building with this size of heating unit would have a floor area in the range of 2500 to 5000 m^2 (25,000 to 50,000 ft^2), which is only a medium-sized office building. It is apparent that the emissions of carbon dioxide from fuel-fired heating devices has an environmental impact comparable to that of automobiles.

A typical residential furnace would have a capacity of 30 kW (100,000 Btu/hr). The annual heating load would be between 10,000 and 30,000 kWh (50 and 150 $\times10^6$ Btu) for houses in the northern tier of states. The amount of carbon dioxide emitted by a residential furnace would then be between 2000 and 6000 kg (2 and 6 tons) of carbon dioxide per year, or about the same as that from an automobile.

Fuel oil is often used in furnaces and boilers in many areas of the country that do not have natural gas service. Coal is also used, especially in buildings with older equipment.

As shown in Table 9.1, the heating values of these fuels are lower than that of methane, with more of the energy coming from the combustion of carbon than from hydrogen. Thus the amount of carbon dioxide produced per unit of heating is greater than that from natural gas. In addition, the efficiency of a fuel oil or coal-fired furnace is generally lower than that of a natural gas–fired furnace since the temperature of the products needs to be higher than the dew point due to the presence of sulfur in the fuel. Sulfur dioxide would combine with condensate to form sulfuric acid, which is corrosive. As a consequence, furnaces used with these fuels are non-condensing furnaces with lower efficiencies than comparable capacity natural gas units.

Fuel oil is a mixture of different compounds that contain carbon and hydrogen. The general chemical formula for a fuel oil is CH_xO_y, where x is the ratio of the moles of hydrogen in the fuel to those of carbon and y is the corresponding ratio for the moles of oxygen bound up in the fuel. Light weight fuel oil has about 84 to 86% carbon, and heavier oils have up to 8% carbon. Coal is essentially all carbon and x and y are essentially zero. For these two fuels, as with methane, one mole of carbon dioxide is produced for each mole of carbon in the fuel. Equation 9.38 can be used to estimate the amount of carbon dioxide produced during combustion. Table 9.5 gives the total amount of carbon dioxide produced for three common fuels for the situation given in Example 9.6.

Table 9.5 Annual carbon dioxide production for an annual building heating requirement of 10^9 Btu (300,000 kWh)

Fuel	Furnace efficiency	HHV (Btu/lbm)	Carbon dioxide (tons)	HHV (kJ/kg)	Carbon dioxide (kg)
Natural gas	0.9	24,000	65	55,000	60,000
Fuel oil	0.85	19,000	100	45,000	90,000
Coal	0.85	12,000	170	30,000	160,000

In addition to the carbon dioxide emissions, very small amounts of oxides of nitrogen (NO_x) are formed when fuels are burned in furnaces and boilers. Although the concentrations do not affect the thermal performance, the levels are significant in terms of their impact on health. The principal mechanism for formation of NO_x in natural gas combustion is termed the thermal NO_x, or Zeldovich, mechanism (Hsieh et al., 1998). Dissociation of nitrogen and oxygen molecules due to the high combustion temperatures and their subsequent reaction with each other produces oxides of nitrogen in the region near the burners. The amount produced increases as the oxygen concentration, peak temperature, and time of exposure to the high temperature increase. These combustion chamber design parameters allow furnaces to be built to minimize the generation of NO_x.

The second mechanism of formation is termed the "prompt mechanism," and it occurs through the reactions of nitrogen with hydrocarbon radicals in the fuel in the early stages of combustion. Generally, the levels produced are much smaller than those due to the thermal NO_x mechanism. The third mechanism is the reaction of nitrogen compounds in the fuel with oxygen. Since natural gas has very low levels of nitrogen compounds, the amount of NO_x produced by this mechanism is insignificant. However, it can contribute significantly when fuel oil or coal is burned.

The analytical prediction of NO_x levels in combustion products is complicated and depends on modeling both the flow and chemical reactions accurately. Even the experimental determination of the levels of NO_x is difficult, and measurements on the same equipment by different investigators are not always in agreement. The concentration levels of NO_x are very low, which contributes to the difficulty of measurement. Nevertheless, the range of the levels and the effect of the important operating parameters on the levels have been established.

One useful measure of the amount of the oxides of nitrogen formed is termed the NO_x emission factor. This is the mass of NO_x formed per unit of heat supplied by the fuel. The Environmental Protection Agency (EPA) has compiled levels of the emission factor and recommended allowable levels. Another useful measure is the emission-indexed NO_x concentration (EINOx), which is defined as the ratio of the mass of NO_x in the flue gas to the mass of fuel entering the combustion chamber. This measure has been correlated with furnace design and operating parameters and is useful for scaling the results from one unit to another.

A comprehensive experimental test program on continuously modulated natural gas fired furnaces and boilers determined the NO_x emissions from modern furnaces over a range of heating capacities (Hsieh et al., 1998). The amount of NO_x produced was found to be a function of the full load capacity and to be more for units with a larger heating capacity. This is in contrast to older units in which the emissions were found to be higher in smaller units. For a given capacity unit, emissions were found to increase as the fuel firing rate was decreased, which means that there are proportionally more emissions at part load. The emissions were found to decrease as combustion temperature was decreased through increased amounts of excess air.

Measured values of emission factors levels for gas and fuel oil–fired furnaces have been compiled by EPA and are shown in Table 9.6 along with the corresponding values of the EINOx. Older furnaces can be expected to have greater than these values, while modern units with advanced combustion controls have lower levels. Example 9.7 illustrates the magnitude of NOx production for the same furnace as in Example 9.6.

Table 9.6 EPA emission factor limits for combustion furnaces and boilers

Fuel	Application	NO_x emission factor	EINOx ($\text{lbm } NO_x/\text{lbm fuel}$)

		($lbm/10^6$ Btu)	(kg NO_x/kg fuel)
Natural gas	Residential	0.09	0.0022
Natural gas	Commercial	0.10	0.0024
Fuel oil	Residential/Commercial	0.14	0.0027

(Emission factors from EPA, 1995)

Example 9.7. Estimate the NO_x emissions from a natural gas fired furnace used to supply heat to the commercial office building of example 9.6. The capacity is 500,000 Btu/hr (150 kW) and the total heating requirement over the course of the year is 1×10^9 Btu (300,000 kWh). The furnace efficiency is 90%.

The total amount of NO_x produced over a year is given by the product of the emission factor and the total heating requirement:

$$m_{NO_x} = 1x10^9\left(Btu\right) * 0.1\, lbm / 10^6\left(Btu\right) = 100\, lbm = 45\, kg$$

This value can be put in perspective by comparing this value to the NO_x emitted by a passenger automobile. The EPA emission standard for passenger cars built after 1999 is 0.3 grams/mile. For an automobile that is driven 10,000 miles in a year, the amount of NO_x produced is

$$m_{NO_x} = 10{,}000(miles) * 0.3(gm / mile) * \frac{2.2\left(lbm\right)}{1\left(kg\right)} = 6.6\, lbm = 3\, kg$$

As with the carbon dioxide emissions, the amount emitted by a furnace in a commercial office building is more than that from a single car, and about equal to that from 15 cars. However, the amount emitted from a residential furnace would be about 5 to 10 lbm per year (2 to 5 kg), which is about that emitted from an automobile. A residential furnace and an automobile have comparable environmental impacts in terms of emissions.

Examples 9.6 and 9.7 show that the emissions from combustion heating equipment have significant health and environmental impacts. The carbon dioxide produced can accelerate global climate change. The efficiency of heating equipment is close to unity, and so significant reductions in the amount of carbon dioxide emitted can occur only through reduced heating demands. The oxides of nitrogen produced are detrimental to human and plant health, and for these compounds combustion chamber design can influence the levels.

SM 9.9 Summary

Combustion boilers and furnaces are widely used to heat air and water in the residential, commercial, and industrial sectors. Their use represents a major consumption of fossil fuels such as oil, natural gas, and, to a lesser extent, coal. The efficiency of modern natural gas units is in the range of 90 to 95%, which is close to the practical upper limit on conversion efficiency. For heating equipment that uses fuel oil or coal, and for high-temperature units, efficiency is limited by the presence of water vapor in the products. The temperature of the exhaust needs to be greater than the dew point of the products to avoid corrosion of the metal parts of the furnace, which results in operating efficiencies in the range of 75 to 80%. Modern designs achieve annual fuel utilization efficiencies (AFUE) comparable to the steady-state efficiency.

Emissions from combustion heating equipment have significant environmental and health consequences. The carbon dioxide produced through combustion of hydrocarbon fuels is a greenhouse gas that can contribute to global climate change. Oxides of nitrogen and of sulfur are damaging to the human respiratory system. These compounds combine with water in rain or snow to produce acid rain that can damage forests and lakes. The emissions from a residential furnace are comparable to those emitted from an automobile. The control of the combustion processes in furnaces and boilers can reduce the emissions.

SM 9.10 Nomenclature

$c_{p,p}$	specific heat of products
f	fraction of time furnace is on
h	specific enthalpy
h_{fg}	latent heat of vaporization of water
HV	heating value
HHV	higher heating value
LHV	lower heating value
$LMTD$	log-mean-temperature-difference
m	mass
$\dot{m}$	mass flow rate
M	molecular weight
N	number of moles
p	pressure
p_i	partial pressure
p_{sat}	saturation pressure
Q_T	total or seasonal heat flow
$\dot{Q}_h$	house heat flow rate
$\bar{\dot{Q}}$	average house heat flow rate
R	specific gas constant
R_o	universal gas constant

t	time
T	temperature
$\bar{T}$	average temperature
T_o	reference temperature
UA	overall thermal conductance
V	volume
ε	effectiveness
η_{fr}	furnace efficiency

Subscripts

A	ambient
a	air
bal	balance
c	combustion
des	design
fl	combustion fuel
p	products
r	reactants
s	supply
T	total

w	water	N_2	nitrogen
CH_4	methane	NO_x	oxides of nitrogen
CO_2	carbon dioxide	O_2	oxygen
H_2O	water		

SM 9.11 References

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Problems

<u>Problems in English Units</u>

9.1 A furnace burns natural gas (methane) with 75% excess air. The reactants enter at 75 °F and the products leave at 350 °F.

   a. Determine the furnace efficiency.
   b. Determine the fuel, air, and products flow rate for a heating output of 100,000 Btu/hr.

9.2 A furnace burns natural gas (methane) with 75% excess air. The reactants enter at 75 °F.

a. Determine and plot the furnace efficiency and fuel flow rate over the range of product temperatures of 75 to 500 °F.
b. Draw some conclusions from your results.

9.3 Propane (C_3H_8) is combusted in a furnace. The propane enters the furnace at 75 °F, the products leave at 300 °F, and combustion occurs with 100% excess air.
a. Determine the steady-state efficiency and the dew point of the products.
b. Determine the heating capacity for a fuel flow rate of 10 lbm/hr.

9.4 Reactants enter a furnace at 75 °F. Combustion occurs with 50% excess air and the products leave at 250 °F. Determine the products dew-point temperature for combustion with methane, propane, fuel oil, and coal for dry air and air with 50%RH. Assume that the fuel oil is $CH_{1.8}$ and the coal is only carbon. Draw some conclusions from your results and discuss why the products temperature for fuel oil and coal is typically much higher than the dew point of the products.

9.5 Natural gas is burned with 75% excess air in a warm-air furnace. The reactants enter at the room temperature of 75 °F. The heating output is 100,000 Btu/hr and the seasonal energy use is 2×10^8 Btu. The heat exchanger in the furnace has an overall thermal conductance of 120 Btu/hr-°F and the circulating air is heated from 75 °F to 140 °F. The average specific heat of the products is 0.28 Btu/lb-°F.
a. Determine the furnace efficiency and the products temperature.
b. Determine the fuel, air, products, and circulating air flow rate.
c. Determine the annual fuel cost for a natural gas cost of $1.1 per therm.
d. Determine the total CO_2 and NO_x emissions produced.
e. Draw some conclusions from your results.

9.6 Fuel oil is burned in a boiler with 100% excess air to produce steam at 300 °F. The combustion air and fuel is at 75 °F. The products temperature is 400 °F. Assume that the fuel oil is $CH_{1.8}$. The heating output is 500,000 Btu/hr, the seasonal energy use is $2x10^9$ Btu, and fuel oil costs $ 2.50/gallon.
a. Determine the required overall heat exchanger conductance
b. Determine the furnace efficiency.
c. Determine the fuel, air, products, and steam flow rates.
d. Determine the total CO_2 and NO_x emissions produced.
e. Determine the annual fuel cost.
f. Draw some conclusions from your results.

9.7 In a warm-air furnace natural gas is burned with 50% excess air. The reactants enter at room temperature of 75 °F. The heating output is 100,000 Btu/hr. The heat exchanger in the furnace is designed so that condensation occurs and the products leave at 85 °F. The circulating air is heated from 75 °F to 140 °F

a. Determine the furnace efficiency.
b. Determine the fuel, air, products and circulating air flow rates.
c. Determine the *UA* product of the heat exchanger.
d. Draw some conclusions from your results.

9.8 Natural gas is burned with 75% excess air in a warm air furnace. The reactants enter at the room temperature of 75 °F. The heating output is 100,000 Btu/hr, the seasonal energy use is 4×10^8 Btu, and the fuel cost is \$1.00/therm. The circulating air is heated from 75 F to 140 F. As a function of the heat exchanger (*UA*) in the furnace, determine:

a. The furnace efficiency.
b. The fuel flow rate and temperature of the products.
c. The seasonal cost of operation.
d. The allowable added cost of heat exchanger overall conductance, based on a 10-year life span.
e. Draw some conclusions from your results.

9.9 A natural gas warm-air furnace is designed to have a heating output of 100,000 Btu/hr with a heat exchanger *UA* of 150 Btu/hr-°F and an excess air fraction of 0.5. The temperature for the reactants is taken to be 75 °F. The design values for the circulating air are heating from 75 °F to 140 °F.

a. Determine the sensitivity of the performance of the furnace to reasonable (+/-10% or so) variations in the excess air, supply air temperature, and supply flow rate.
b. Draw some conclusions from your results.

<u>Problems in SI Units</u>

9.10 A furnace burns natural gas (methane) with 75% excess air. The reactants enter at 25 °C and the products leave at 200 °C.

a. Determine the furnace efficiency.
b. Determine the fuel, air, and products flow rate for a heating output of 25 kW.

9.11 A furnace burns natural gas (methane) with 75% excess air. The reactants enter at 25 °C.

a. Determine and plot the furnace efficiency and fuel flow rate over the range of product temperatures of 25 to 250 °C.
b. Draw some conclusions from your results.

9.12 Propane (C_3H_8) is combusted in a furnace. The propane enters the furnace at 25 °C, the products leave at 175 °C, and combustion occurs with 100% excess air.

   a. Determine the steady-state efficiency and the dew-point of the products.
   b. Determine the heating capacity for a fuel flow rate of 5 kg/hr.

9.13 Reactants enter a furnace at 25 °C. Combustion occurs with 50% excess air and the products leave at 150 °C. Determine the products dew-point temperature for combustion with ethane, propane, fuel oil, and coal for dry air and air with 50% RH. Assume that the fuel oil is $CH_{1.8}$ and the coal is only carbon. Draw some conclusions from your results and discuss why the products temperature for fuel oil and coal is typically much higher than the dew point of the products.

9.14 Natural gas is burned with 75% excess air in a warm air furnace. The reactants enter at the room temperature of 25 °C. The heating output is 30 kW and the seasonal energy use is 60,000 kWh. The heat exchanger in the furnace has an overall thermal conductance of 150 Btu/hr-°F and the circulating air is heated from 25 °C to 55 °C. The average specific heat of the products is 1.2 kJ/kg-°C. Determine:

   a. The furnace efficiency and the products temperature.
   b. The fuel, air, products, and circulating air flow rate.
   c. The annual fuel cost for a natural gas cost of $1.1 per therm.
   d. The total CO_2 and NO_x emissions produced.
   e. Draw some conclusions from your results.

9.15 Fuel oil is burned in a boiler with 100% excess air to produce steam at 150 °C. The combustion air is at room temperature (25 °C) and the products temperature is 220 °C. The heating output is 150 kW, the seasonal energy use is 100,000 kWh, and fuel oil costs $0.80/L.

   a. Determine the overall heat exchanger conductance
   b. Determine the furnace efficiency.
   c. Determine the fuel, air, products, and steam flow rates.
   d. Determine the total emissions produced.
   e. Determine the annual cost for fuel.
   f. Draw some conclusions from your results.

9.16 In a warm air furnace natural gas is burned with 50% excess air. The reactants enter at room temperature of 25 °C. The heating output is 30 kW. The heat exchanger in the furnace is designed so that condensation occurs and the products leave at 30 °C. The circulating air is heated from 25 °C to 60 °C.

   a. Determine the furnace efficiency.
   b. Determine the fuel, products and circulating air flow rates.

c. Determine the *UA* product of the heat exchanger.
d. Draw some conclusions from your results.

9.17 Natural gas is burned with 50% excess air in a warm air furnace. The reactants enter at the room temperature of 25 °C. The heating output is 30 kW, the seasonal energy use is 120,000 kWh, and the fuel cost is \$0.35/m^3. The circulating air is heated from 25 °C to 60 °C. As a function of the heat exchanger (*UA*) in the furnace, determine:
a. The furnace efficiency.
b. The fuel flow rate and products temperature.
c. The seasonal cost of operation.
d. The allowable added cost of heat exchanger overall conductance based on a 10 year life span.
e. Draw some conclusions from your results.

9.18 A natural gas warm air furnace is designed to have a heating output of 100,000 Btu/hr with a heat exchanger *UA* of 80 W/C and an excess air fraction of 0.5 The temperature for the reactants is taken to be 25 °C. The design values for the circulating air are heating from 25 °C to 60 °C.
a. Determine the sensitivity of the performance of the furnace to reasonable (+/-10% or so) variations in the excess air, supply air temperature, and supply flow rate.
b. Draw some conclusions from your results.

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