Chapter SM 2 Heat Exchangers for Cooling Applications

SM 2.1 Heat and Mass Transfer Processes in a Cooling Coil

The thermodynamic relations for the overall performance of a cooling coil were presented in Chapter 5, Section 5.6, and a simple bypass model was developed. The bypass model is based on the assumption that some of the air stream is in close contact with the coil surfaces and leaves at saturated conditions at the water inlet temperature and that the remainder leaves at the inlet state to the coil. These two flows then mix at the coil exit, and the outlet state is the mixed average of the two flows. The process representation for the bypass model is then a straight line connecting the inlet state and the saturated air state at the water inlet conditions. The actual process line is curved and a straight-line approximation is not accurate for many coil processes.

A more accurate description of the heat and mass transfer processes is based on the mass transfer relations developed in Section 5.7. An "exact" representation for a cooling coil yields three coupled nonlinear differential equations that need to be solved numerically. These equations can be cast in the same form as those for sensible heat exchangers to produce an analogy solution that can be directly solved to yield a simple but accurate representation of coil performance.

A description of the processes that occur inside a cooling coil is important in formulating the heat and mass transfer relations. When moist air enters a cooling coil the first few rows provide mainly sensible cooling. The temperature of the surface is between that of the air and coolant, and when the overall temperature difference is large, as near the entrance, the surface is usually not below the dew point and water in the air does not condense. This section of the coil is called the "dry" section of the coil and the air states are determined as for a sensible exchanger.

Farther along in the coil, the surface temperature drops below the dew point and condensation occurs. This is called the "wet" section and the determination of the states is more complicated than for the dry section. Mass is transferred from the air stream in addition to heat, and the total energy transfer is the sum of the heat transfer and the energy carried by the moisture flow. A cross section of the wet section is shown in Figure 2.1. The temperature, humidity ratio, enthalpy, and relative humidity profiles within the boundary layer on the air side, and the temperature profile through the coil surface and into the water flow are depicted. In the boundary layer on the air side, the driving potential for heat is the temperature difference between the air and the surface, for mass it is the humidity ratio difference, and for the energy it is the enthalpy difference (Section 5.7).

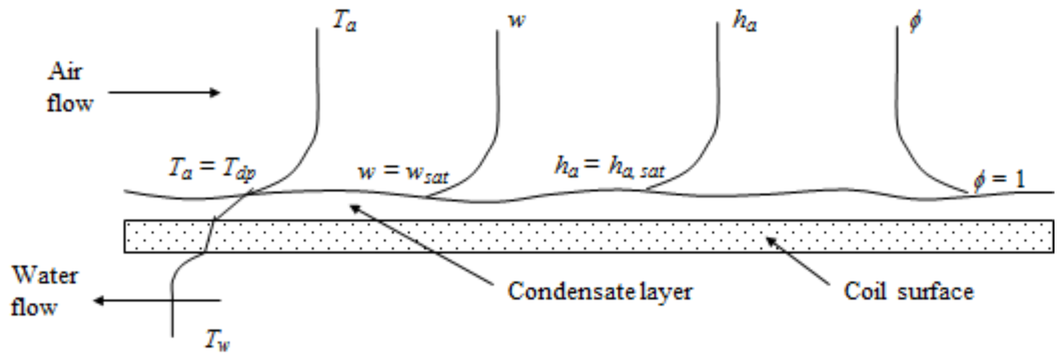

Figure 2.1 Property profiles in the air stream for a cooling coil

The temperature profile is similar to that of a sensible exchanger. The air temperature is relatively high in the fluid far from the surface, decreases rapidly through the air boundary layer, and equals the surface temperature at the top of the condensate layer. The air is saturated at the edge of the condensate layer. The temperature profile is linear through the condensate layer and the coil surface since the energy transfer in these regions is by conduction. There is a thermal boundary layer on the water side, with steep temperature gradients near the surface.

The humidity ratio is highest out in the air stream and decreases through the boundary layer. Moisture diffuses toward the surface where condensation takes place, and to cause this moisture flow the driving potential (humidity ratio) must be greater in the air stream than at the edge of the condensate layer. The humidity ratio at the edge of the condensate layer is the saturation value at the condensate surface temperature.

The enthalpy profile is similar to the humidity ratio profile, being highest in the air stream and decreasing in the boundary layer. The value of enthalpy at the condensate surface is the saturation value at the condensate surface temperature. As with the humidity ratio, energy diffuses toward the surface, and to cause this energy transfer the enthalpy, which is the driving potential, must be higher in the air stream than at the edge of the condensate layer. The relative humidity increases toward the surface and is unity at the edge of the condensate layer. The energy flow in the air stream becomes a heat flow through the water layer and coil surface and into the water stream, and only the temperature profile is relevant in these sections.

The control volume that contains the heat and mass transfer processes in the wet section of the coil is similar to that for the dry section (Section SM 1.1), and is shown in Figure 2.2. The surface is assumed to be completely wet. There is an energy flow carried by the air stream, heat and mass transfer flows to the condensate layer, heat transfer through the condensate layer and the coil surface, and heat transfer to the water flow. There is a flow of condensate out of the coil with an enthalpy equal to that of liquid water at the condensate temperature.

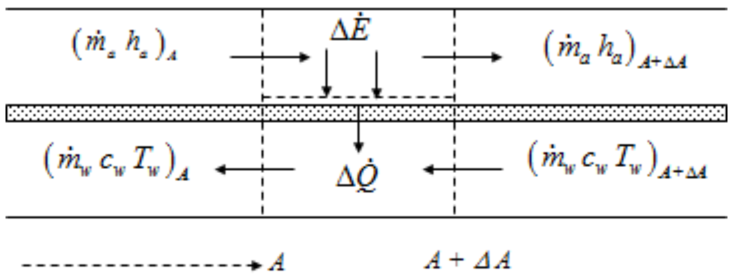

Figure 2.2 Energy flows in the wet section of a cooling coil

The development of the governing relations for the wet section is similar to that for a dry heat exchanger. An overall energy balance is used to relate the enthalpy change of the air stream to that of the water stream and the energy carried by the condensate leaving the coil:

$$\left(\dot{m}_a h_a\right)_A + \left(\dot{m}_w c_{p,w} T_w\right)_{A+\Delta A} - \left(\dot{m}_a h_a\right)_{A+\Delta A} - \left(\dot{m}_w c_{p,w} T_w\right)_A - \Delta \dot{m}_{cond} h_f = 0 \qquad (2.1)$$

As discussed in Section 5.6, the energy flow due to the condensate draining from the coil is small compared to the other energy terms, and can be neglected. Rearranging equation 2.1, dividing by ΔA, and passing to the limit $\Delta A \rightarrow 0$ yields a differential equation for the mean air enthalpy and mean water temperature as functions of area (distance) through the coil:

$$\dot{m}_a \frac{d\,h_a}{d\,A_a} = \dot{m}_w c_{p,w} \frac{d\,T_w}{d\,A_a} \qquad (2.2)$$

An energy balance on the air stream relates the change in mean air enthalpy to the energy transfer from the air stream to the water layer, and brings in the overall conductance:

$$\left(\dot{m}_a h_a\right)_A - \left(\dot{m}_a h_a\right)_{A+\Delta A} - \Delta\dot{E} = 0 \qquad (2.3)$$

The energy transfer from the air to the surface of the condensate layer is the sum of the sensible heat transfer and the energy carried by the flow of moisture that condenses in the layer. The heat flow can be written in terms of the convection heat transfer coefficient h_c and the difference between the air stream temperature and that of the liquid layer. The moisture flow rate can be written in terms of the convection mass transfer coefficient h_m and the difference between the humidity ratio in the air stream w_a and the enthalpy of saturated air at the temperature of the surface of the liquid layer $w_{s,sat}$. The energy of the moisture flow is the enthalpy change between vapor and liquid, $h_{fg,s}$, which is the latent heat of vaporization at the liquid layer temperature. The energy flow is then

$$\Delta\dot{E} = \left[h_c\,(T_a - T_s) + h_m\,(w_a - w_{s,sat})\ h_{fg,s} \right] \Delta A \tag{2.4}$$

The mass transfer coefficient h_m can be related to the heat transfer coefficient through the Lewis number (Section 5.7). For the air–water vapor mixture, the Lewis number can be assumed to be unity with negligible error. The mass transfer coefficient is then simply related to the heat transfer coefficient through the specific heat of the mixture:

$$h_m = \frac{h_c}{c_{p,m}} \tag{2.5}$$

where the specific heat is the weighted sum of the air and water vapor specific heats

$$c_{p,m} = c_{p,a} + w c_{p,v} \tag{2.6}$$

With the mass and heat transfer coefficients related as given by equation 2.6, the energy flow can be expressed in terms of the difference between mean enthalpy of air and the enthalpy of saturated air at the condensate surface temperature (Section 5.7). It is convenient to introduce an overall surface efficiency η_o^* that accounts for the effect of extended surfaces on the air side. The overall energy transfer coefficient reflects the effect of fin efficiency on heat and mass transfer in the same manner as for heat transfer only. The energy transfer fin efficiency, discussed in Section 2.3, is the ratio of the actual energy transfer to that from air to a condensate layer on the fin that is uniform in enthalpy at the same value as on the prime area (at the base of the fin). The overall energy efficiency then accounts for the prime area and the area of the fin. The energy flow then becomes

$$\Delta\dot{E} = \frac{\eta_o^*\, h_c\ \Delta A}{c_{p,m}} (h_a - h_{s,sat}) \tag{2.7}$$

Combining equations 2.3 and 2.7, dividing by the incremental area ΔA, and passing to the limit $\Delta A \to 0$ yields a differential equation for the mean air enthalpy:

$$\dot{m}_a \frac{d\,h_a}{d\,A_a} = -\frac{\eta_o^*\, h_c}{c_{p,m}} \left(h_a - h_{s,sat} \right) \tag{2.8}$$

The energy balance for the water stream is developed in a similar manner. Here only heat transfer is involved, as heat is transferred from the air–condensate layer interface through the surface to the water stream. The heat flow is written in terms of the difference between the surface temperature of the condensate layer and the mean water temperature. The conductance U_w, based on the waterside area A_w, includes the conduction resistance of the condensate layer, the conduction resistance of the coil heat transfer surface, and the heat transfer convection coefficient for the water stream. The resulting equation is:

$$\dot{m}_w \, c_{p,w} \frac{d\,T_w}{d\,A_w} = -U_w \left(T_{s,sat} - T_w \right) \tag{2.9}$$

The performance of a cooling coil is described through equations 2.2, 2.8, and 2.9. These are three coupled equations in terms of the mean enthalpy of the air flow, the mean temperature of the water flow, and the condensate surface temperature (and enthalpy) as variables. The assumptions underlying this formulation are that the Lewis number is unity and the energy flow due to condensate draining from the coil is negligible. The solution to equations 2.2, 2.8 and 2.9 yields the enthalpy of the air stream as a function of coil surface area measured in the air flow direction.

The variables in the energy balances are both enthalpy and temperature. The water temperature T_w can be put in terms of an enthalpy by defining an effective specific heat. The enthalpy representation for the water is chosen to be the enthalpy of saturated air at the water temperature T_{w} and is denoted as $h_{\mathrm{w,sat}}$. This effective specific heat is defined by equating the water temperature change with distance to the change in saturated air enthalpy at the water temperature as

$$c_s \frac{d\,T_w}{d\,A} = \frac{d\,h_{w,sat}}{d\,A} \tag{2.10}$$

The effective specific heat, c_s, is thus defined as the change in enthalpy with respect to temperature along the saturation line at the local water temperature:

$$c_s = \left(\frac{d\,h_{w,sat}}{d\,T_w} \right)_{T_w} \tag{2.11}$$

The appropriate value of the specific heat for a coil process is an average value and can be numerically evaluated using the water inlet and outlet states:

$$c_s = \left(\frac{h_{w,sat,in} - h_{w,sat,out}}{T_{w,in} - T_{w,out}} \right) \tag{2.12}$$

Since the water outlet state may not be known at the start of an analysis, iteration may be needed to evaluate the effective specific heat. Typical values for c_s are in the range of 0.5 to 0.6 Btu/lb-°F (2 to 3 kJ/kg-°C) for coil applications. Using the effective specific heat, the overall energy balance equation 2.2 can be rearranged and written as

$$\frac{d\,h_a}{d\,A_a} = \left(\frac{\dot{m}_w\,c_{p,w}}{\dot{m}_a\;c_s} \right) \frac{d\,h_{w,sat}}{d\,A_a} \tag{2.13}$$

The overall energy balance may be thought of as replacing the water flow with an equivalent air flow that is at saturation conditions. The term in parentheses is the ratio of mass flow rate-specific heat products, and this ratio is equivalent to the capacitance rate ratio of the sensible heat exchanger.

The water energy balance, equation 2.9, can also be put in terms of enthalpy using the effective specific heat, c_s. Multiplying equation 2.9 by c_s allows the water and condensate interface temperatures to be replaced with enthalpies:

$$\dot{m}_w\,c_{p,w}\,\frac{d\,h_{w,sat}}{d\,A_w} = -\,U_w\left(h_{s,sat} - h_{w,sat}\right) \tag{2.14}$$

where again $h_{w,sat}$ is the saturated air enthalpy at the water temperature T_w. Using the average value of c_s as computed by equation 2.12 assumes that the value of c_s at T_s is equal to the value at T_w, which is a reasonable assumption.

The energy balance on the air side, equation 2.8, is in terms of the difference between the enthalpy of the air stream and the saturated value at the condensate layer surface. The enthalpy of the saturated layer can be eliminated from equation 2.8 and replaced by the saturated air enthalpy at the water temperature $h_{w,sat}$ by introducing the thermal resistances between the air stream and the water stream. The thermal resistance for energy flow between the air enthalpy and the saturated air enthalpy at the condensate layer temperature is the reciprocal of the product of the unit thermal conductance and the area on the air side:

$$R_a^* = \frac{c_{p,m}}{\eta_o^* h_c\,A_a} \tag{2.15}$$

The overall thermal resistance for energy flow between the saturated air enthalpy at the condensate layer temperature and the saturated air enthalpy at the water temperature is expressed in terms of the overall thermal conductance, effective specific heat, and water side area as

$$R_w^* = \frac{c_s}{U_w A_w} \tag{2.16}$$

The overall resistance is the sum of the two component resistances:

$$R_o^* = \frac{c_{p,m}}{\eta_o^* h_c A_a} + \frac{c_s}{U_w A_w} \tag{2.17}$$

The overall energy conductance between the air and water streams is the reciprocal of the thermal resistance, or

$$U_o^* A_a = \frac{1}{R_o^*} = \frac{\dfrac{\eta_o^* h_c A_a}{c_{p,m}}}{1 + \dfrac{c_s \eta_o^* h_c A_a}{c_{p,m} U_w A_w}} \tag{2.18}$$

The energy balance for the air side, equation 2.8, can then be written in terms of the overall enthalpy difference between the air stream and the saturated enthalpy at the water temperature. The unit energy conductance U_o^* is the overall value $U_o^* A_a$ from equation 2.18 divided by the air side area A_a:

$$\dot{m}_a \frac{d\,h_a}{d\,A_a} = -U_o^* \left(h_a - h_{w,sat} \right) \tag{2.19}$$

The introduction of the effective specific heat c_s eliminates the temperature (and enthalpy) of the condensate layer as a variable and reduces the number of coupled differential equations from three to two. The overall energy balance, equation 2.13, relates the air enthalpy change with surface area to that for the water and the energy balance equation 2.19 brings in the mass transfer relations. Solving these two coupled differential equations yields the change in air enthalpy and water temperature with distance to be determined.

A direct method for solving these equations is through integration over the area of the coil. In the next section the analogy method will be presented in which the heat and mass transfer equations are solved using the relations developed for sensible heat transfer.

SM 2.2 Cooling Coil Performance Using an Analogy to Heat Transfer

The differential equations describing the overall energy balance and the energy balance on the air side, equations 2.13 and 2.19, are expressed only in terms of enthalpies. They are similar in form to the energy balance relations for a sensible heat exchanger, Section SM 1.1, equations 1.3 and 1.5. The similarity of the energy transfer relations in terms of enthalpies to the heat transfer relations in terms of temperature leads to an analogy between energy transfer and heat transfer. Because the equations are similar in the variables and parameters, the solution for enthalpies will be similar to that for temperature. This analogy then allows all of the sensible heat exchanger relations to be used directly for cooling coils. The analogy method provides generality in terms of flow arrangement and is useful in the design and evaluation of coils.

To employ the analogy, the correspondence of the different variables and parameters needs to be established. Comparing equations 2.13 and 2.19 with 1.3 and 1.5 shows that the air stream enthalpy h_a is analogous to the temperature of the minimum capacitance fluid T_{min} and the saturated air enthalpy at the water temperature $h_{w,sat}$ is analogous to the temperature of the maximum capacitance fluid T_{max}. The solution for air enthalpy will then be the same as that for the temperature of the minimum capacitance fluid.

There is also a correspondence between the parameters of the two sets of equations. The overall energy balance relation equation 2.13 contains the ratio of the product of mass flow rates and specific heats and is analogous to the capacitance rate ratio C_{min}/C_{max} in equation 1.3. The term m^* is used to represent this group, and is defined as

$$m^* = \frac{\dot{m}_a \, c_s}{\dot{m}_w \, c_{p,w}} \tag{2.20}$$

Using m^*, equation 2.13 can be written

$$\frac{d\,h_a}{d\,A_a} = \frac{1}{m^*}\frac{d\,h_{w,sat}}{d\,A_a} \tag{2.21}$$

which is directly analogous to the heat exchanger equation 1.3 with m^* replacing C_{min}/C_{max} (C^*). In equation 2.19, the air mass flow rate is analogous to the minimum capacitance rate C_{min} in equation 1.5 and the overall mass transfer conductance U^* is analogous to the overall conductance U. It is convenient to define a mass transfer number of transfer units Ntu^* that is analogous to the heat transfer Ntu as

$$Ntu^* = \frac{U_o^* \, A_a}{\dot{m}_a} \tag{2.22}$$

With the definition of Ntu^*, equation 2.19 can be written as

$$\frac{d\,h_a}{d\,A_a} = -\,\frac{Ntu^*}{A_a}\left(h_a - h_{w,\,sat}\right) \tag{2.23}$$

Equations 2.21 and 2.23 are identical in form to equations 1.3 and 1.5 for a sensible heat exchanger, with the airflow as the minimum capacitance rate. Since the equations for heat and mass transfer are equivalent to those for heat transfer, the solutions will be the same. The names of the variables are different, but by replacing the sensible heat exchanger variables and parameters of the effectiveness-Ntu solutions with the corresponding variables for the heat and mass transfer cooling coil, the available effectiveness-Ntu relations can be used to determine cooling coil performance. The equivalence of these two sets of equations allows the heat transfer for a cooling coil to be written in a form analogous to that for a sensible heat exchanger using an effectiveness for mass transfer ε*, the inlet enthalpy of the air stream, and the enthalpy of saturated air at the water stream inlet temperature as

$$\dot{Q} = \varepsilon^*\ \dot{m}_a\left(h_{a,in} - h_{w,sat,in}\right) \tag{2.24}$$

which is analogous to equation 1.9. The mass transfer effectiveness is based on the air side enthalpies and is the ratio of the actual enthalpy change of the air to the maximum possible change. The maximum change would be if the air stream exited the coil in equilibrium with the water at the inlet, and the enthalpy would then be the saturation value at the water inlet temperature. The effectiveness of a wet coil is given in terms of the enthalpies as

$$\varepsilon^* = \frac{\left(h_{a,in} - h_{a,out}\right)}{\left(h_{a,in} - h_{w,sat,in}\right)} \tag{2.25}$$

With the heat transfer calculated, the water outlet temperature is determined from an energy balance on the water side:

$$\dot{Q} = \dot{m}_w\, c_{p,w}\left(T_{w,out} - T_{w,in}\right) \tag{2.26}$$

The mass transfer effectiveness is determined using the heat exchanger effectiveness-Ntu relations given in Chapter 13, Table 13.1 in Section 13.3 for the appropriate flow arrangement, which for a cooling coil is typically a cross-flow exchanger with the water flow mixed and the air flow unmixed. The correspondence between the sensible heat exchanger and cooling coil parameters is given in Table 2.1.

Table 2.1 Analogous parameters for sensible heat exchangers and cooling coils

Parameter	Sensible heat exchanger	Cooling coil
Capacitance rate ratio	C^*	m^*
Number of transfer units	Ntu	Ntu^*
Effectiveness	$\varepsilon = f\left(C^*, Ntu\right)$	$\varepsilon^* = f\left(m^*, Ntu^*\right)$
Maximum heat flow	$C_{\min}\left(T_{h,i} - T_{c,i}\right)$	$\dot{m}_a\left(h_{a,in} - h_{w,sat,in}\right)$
Heat flow	$\varepsilon\, C_{\min}\left(T_{h,i} - T_{c,i}\right)$	$\varepsilon^* \dot{m}_a\left(h_{a,in} - h_{w,sat,in}\right)$

The analogy between a sensible heat exchanger and a cooling coil yields the air outlet enthalpy. To fix the outlet state an additional set of relations is needed to determine the outlet air temperature or humidity ratio. The outlet temperature is determined assuming that the sensible cooling of the air occurs as a result of convective heat transfer to an appropriately averaged coil surface temperature. It is convenient to define an air side *Ntu* to facilitate this evaluation. The conductance between the air and the condensate film is the product of the heat transfer overall surface efficiency $\hat{\eta}_o$, the heat transfer coefficient h_c, and the surface area. The heat transfer fin efficiency $\hat{\eta}_o$ is different from the heat and mass transfer efficiency η_o^* used in equation 2.7 because it accounts for the effect of the fin temperature distribution on convective heat transfer rather than the effect of the saturated air enthalpy distribution over the fin surface on total energy transfer. Furthermore, $\hat{\eta}_o$ is different from the heat transfer fin efficiency η_f that would occur for a dry fin because the condensation process alters the fin temperature distribution. The relations for these efficiencies are given in Section SM 2.3. The air side *Ntu* for convective heat transfer with a wetted surface is given as

$$Ntu_a = \frac{\hat{\eta}_o\, h_c\, A_a}{\dot{m}_a\, c_{p,m}} \tag{2.27}$$

The air stream outlet temperature is then determined from the solution for air flowing over a surface with a uniform temperature:

$$T_{a,out} = T_{s,eff} + \left(T_{a,in} - T_{s,eff}\right) e^{-Ntu_a} \tag{2.28}$$

The effective surface temperature $T_{s,eff}$ is the saturation temperature at the value of an effective surface enthalpy, $h_{s,eff}$, which is given by the relation similar to that for temperature:

$$h_{s,eff} = h_{a,in} + \frac{h_{a,out} - h_{a,in}}{1 - e^{-Ntu_a^*}} \tag{2.29}$$

where the *Ntu* for the effective surface enthalpy is based on the overall surface energy transfer efficiency:

$$Ntu_a^* = \frac{\eta_o^* \, h_c \, A_a}{\dot{m}_a \, c_{p,m}} \tag{2.30}$$

The effective surface enthalpy is based on the air inlet and outlet enthalpies determined from the analogy method and is thus the value of a constant surface enthalpy that yields the same heat transfer as actually occurs in the coil.

For a given coil and inlet conditions, three operating possibilities exist depending on the amount of dehumidification:

Totally Dry Coil: In this situation the air side surface temperature is above the dew point of the entering air and condensation does not occur anywhere in the coil. The coil surface on the air side is then totally dry and the heat transfer and outlet conditions are determined using the conventional sensible heat exchanger analysis following the ε-Ntu method (Chapter 13, Section 13.3).

Totally Wet Coil: For this situation the coil surface area on the air side is totally wet. The temperature of the surface on the air side is below the dew point throughout the coil and condensation occurs immediately as the air enters the coil. The heat transfer and outlet state are determined using the analogy relations presented in this section.

Partially Dry and Partially Wet Coil: The temperature of the surface on the air side of the coil is initially above the entering air dew point and then drops below the dew point at some position inside the coil. There is then a dry section near the entrance and a wet section for the remainder of the coil. The performance evaluation requires determining the location where condensation first occurs. The calculation procedure is to treat the first fraction of the coil surface area as dry and the remaining fraction of the area as wet.

The fraction of the area where condensation occurs is unknown at the start of the calculation but is the location at which the temperature of the heat transfer surface on the air side reaches the dew point of the entering air. The surface temperature at this location is determined using the energy balance relation so that the heat flow from the air side equals that to the water side. The heat flows are determined using the thermal resistances on the air and water side:

$$\frac{\left(T_{a,x}-T_s\right)}{R_a}=\frac{\left(T_s-T_{w,x}\right)}{R_w} \tag{2.31}$$

where T_s is the surface temperature at the location where condensation occurs and equals the entering air dew point temperature. The unknown temperatures are $T_{a,x}$, the mean air temperature at the location where condensation occurs, and $T_{w,x}$, the water temperature where condensation occurs. The thermal resistances are those on the air side and water side:

$$R_a=\frac{1}{\eta_o\, h_c\, A_a} \tag{2.32}$$

$$R_w=\frac{1}{U_w\, A_w} \tag{2.33}$$

The solution is iterative and involves solving the dry and wet situations simultaneously. Example 2.1 presents the solution method for the partially wet coil.

Approximate Coil Performance Evaluation

When the coil is actually partially wet, the totally dry analysis slightly underpredicts the heat transfer because the latent transfer associated with the condensation is neglected. The totally wet analysis also slightly underpredicts the heat transfer because the coil surface is assumed completely saturated throughout, which would require moisture addition to maintain the condensation layer in the dry section. The result is that the higher of the heat transfer predictions for a totally wet and a totally dry coil provides a good estimate for a partially wet coil. To determine an estimate of the coil performance, both the totally wet and totally dry analyses are conducted and the higher of the two values is used to estimate the heat transfer. The resulting heat transfer is within acceptable engineering accuracy (less than 5%) of the actual answer.

The performance of the same coil for totally dry, totally wet, and partially wet conditions is carried out in Example 2.1. The calculation procedure is illustrated and the results show the small error associated with taking the larger of the totally dry and totally wet heat transfer as representing the actual heat transfer for a partially wet coil.

Example 2.1 Determine the energy transfer, moisture condensate rate, and outlet air and water conditions for a chilled water coil. The air enters the coil at a dry-bulb temperature of 80 °F and a wet-bulb temperature of 64 °F with a flow rate of 21,000 lbm/hr. The water enters at 42 °F with a flow rate of 30,000 lb/hr. For this example the overall surface efficiencies for heat and energy transfer are taken to be the same value and for the air side the heat transfer conductance is 50 Btu/hr-ft^2 and the surface area is 360 ft^2. On the water side the conductance is 1000 Btu/hr-ft^2 and the area is 18 ft^2.

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"Problem Specification"
Air side variables
p_atm = 14.7 "psia"                                                  "Pressure"
T_a_in = 80 "F"                                                      "Temperature"
T_wb_in = 64 "F"                                                     "Temperature"
m_dot_a = 21000 "lbm/hr"                                             "Flow rate"
Water side variables
T_w_in = 42 "F"                                                      "Temperature"
m_dot_w= 30000 "lbm/hr"                                              "Flow rate"

"Coil parameters: The symbol U_a is used for the product of the overall fin effectiveness and heat
transfer coefficient ."
U_a = 50 "Btu/hr-F"                                                  "Unit conductance"
U_w = 1000 "Btu/hr-F"                                                "Unit conductance"
A_a = 360 "ft2"                                                      "Area"
A_w = 18 "ft2"                                                       "Area"

"Air properties."
w_in = HumRat(AirH2O,T=T_a_in , P=p_atm,B=T_wb_in ) "lbm/lbm"        "Humidity ratio"
h_a_in= Enthalpy(AirH2O,T=T_a_in ,P=p_atm,B=T_wb_in ) "Btu/lbm"      "Enthalpy"
cp_a = SpecHeat(AirH2O,T=T_a_in ,P=p_atm,B=T_wb_in ) "Btu/lbm-F"     "Specific heat"

Water properties
h_w_in= Enthalpy(AirH2O,T=T_w_in,P=p_atm,R=1) "Btu/lbm"              "Enthalpy"
cp_w = SpecHeat(water,T=T_w_in,P=p_atm) "psia"                       "Specific heat"

"Totally Dry Coil"
"The performance is evaluated assuming that the surfaces are totally dry. The coil is then a
sensible counterflow heat exchanger. The sensible capacitance rates of the air and water are
determined."
C_w = m_dot_w*cp_w "Btu/hr"                                          "Capacitance rate"
C_a = m_dot_a*cp_a "Btu/hr"                                          "Capacitance rate"
C_min = min(C_w,C_a) "Btu/hr"                                        "Capacitance rate"
C_star = min(C_w,C_a)/max(C_w,C_a)                                   "Capacitance rate ratio"
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"The Ntu is calculated using the overall thermal resistance between the air and water streams. The resistances will also be used in the partially dry and partially wet analysis."

R_dry = R_a + R_w "hr-F/Btu" "Resistance"

R_a = 1/(U_a*A_a) "hr-F/Btu" "Resistance"

R_w = 1/(U_w*A_w) "hr-F/Btu" "Resistance"

U_a_dry*A_a = 1/R_dry "Btu/hr-F" "Overall conductance"

UA_dry = U_a_dry*A_a "Btu/hr-F" "Overall conductance"

Ntu_dry = UA_dry/C_min "Ntu"

"The effectiveness is calculated for a counterflow arrangement using the relations in Table 13.1. The heat transfer is calculated using the minimum capacitance rate and effectiveness."

eff_dry = (1 - exp(-Ntu_dry*(1 - C_star)))/(1 - C_star*exp(-Ntu_dry*(1-C_star)))
"Effectiveness"

Q_dry = eff_dry*C_min*(T_a_in - T_w_in) "Btu/hr" "Heat flow"

Q_dry =C_a*(T_a_in - T_a_out_dry) "Btu/hr" "Heat flow"

Q_dry = C_w*(T_w_out_dry - T_w_in) "Btu/hr" "Heat flow"

Results

The capacitance rates of the air and water are 5134 Btu/hr-°F and 30078 Btu/hr-°F, respectively, resulting in a value of C^* of 0.1707. The *Ntu* of the totally dry coil is 1.753, resulting in an effectiveness of 0.798. The heat transfer rate for the totally dry coil is 155,713 Btu/hr and the outlet air temperature is 48.7 °F. The heat transfer rate will be compared to that for the totally wet and the partially dry and partially wet coil results.

"Totally Wet Coil"

"The effective specific heat c_s needs to be determined. The water outlet temperature and enthalpy are not known and so the solution to determine c_s is iterative. The value of T_w_out obtained from the final energy balance is used to find the saturated air enthalpy and then c_s."

c_s = (h_w_out_wet - h_w_in)/(T_w_out_wet - T_w_in) "Btu/lbm-F" "Specific heat"

"Determine the equivalent capacitance rate m_star from equation 2.20."

m_star = m_dot_a*c_s/(m_dot_w*cp_w) "m_star"

"Determine the energy transfer parameters from equation 2.18 for the conductance and equation 2.22 for the Ntu_star."

U_star = (U_a/cp_a)/(1 + (c_s*U_a*A_a)/(cp_a*U_w*A_w)) "Btu/hr-F" "Unit conductance"

UA_wet = U_star*A_a "Btu/hr-F" "Overall conductance"

Ntu_star = UA_wet/m_dot_a "Ntu"

Determine the coil effectiveness for counterflow using the counterflow expression, with m_star replacing C_star and Ntu_star replacing Ntu

eff_star = (1 - exp(-Ntu_star*(1 - m_star)))/(1 - m_star*exp(-Ntu_star*(1-m_star))) "Effectiveness"

Determine the heat transfer and outlet temperatures. The heat transfer for wet conditions is given by the analogy expression equation 2.24 using the enthalpy of the inlet air stream and the enthalpy of saturated air at the water inlet temperature. Energy balances on the air and water flows are used to determine the outlet temperatures and enthalpy.

```
Q_wet = eff_star*m_dot_a*(h_a_in - h_w_in) "Btu/hr"                "Heat flow"
Q_wet = m_dot_a*(h_a_in - h_a_out_wet) "Btu/hr"                    "Heat flow"
Q_wet = m_dot_w*cp_w*(T_w_out_wet - T_w_in) "Btu/hr"               "Heat flow"
```

Determine the saturated air enthalpy at the water outlet temperature for use in determining the effective specific heat c_s.

```
h_w_out_wet= Enthalpy(AirH2O,T=T_w_out_wet,P=p_atm,R=1) "Btu/lbm"  "Enthalpy"
"
```

"Determine the outlet air temperature and humidity ratio using the approximation that the surface is at a uniform temperature and enthalpy. Ntu_a_star is from equation 2.30 and the effective surface enthalpy is from equation 2.29. The value of Ntu_a_hat, which for this example is the same as Ntu_a_star, is from equation 2.27 and the outlet temperature is from 2.28"

```
Ntu_a_star = U_a*A_a/(m_dot_a*cp_a)                                 "Ntu"
h_s_eff = h_a_in +(h_a_out_wet - h_a_in )/(1 - exp(-Ntu_a_star)) "Btu/lbm"  "Enthalpy"
Ntu_a_hat = U_a*A_a/(m_dot_a*cp_a)                                  "Ntu"
T_a_out_wet = T_s_eff +(T_a_in - T_s_eff)*exp(-Ntu_a_hat) "F"       "Temperature"
T_s_eff = temperature(AirH2O,P=p_atm,h=h_s_eff,R=1) "F"             "Temperature"
```

Determine the rate of condensation from the air flow rate and humidity ratio change.

```
w_out_wet = humrat(AirH2O, P=p_atm, T=T_a_out_wet, R=1) "lbm/lbm"  "Humidity ratio"
m_dot_cond_wet = m_dot_a*(w_in - w_out_wet) "lbm/hr"               "Mass flow rate"
```

Results

The value of the effective specific heat is found to be 0.504 Btu/lbm-°F. With this value the value of m^* is 0.352, the overall mass transfer conductance is 24,039 lbm/hr, the Ntu* is 1.145, and the effectiveness eff* is 0.629. The outlet water temperature is 47.7 °F. The heat transfer, based on the inlet air enthalpy of 29.2 Btu/lbm and saturated air at the inlet water temperature of 42 °F of 16.2 Btu/lbm, is 171,708 Btu/hr.

The value of the heat transfer for the totally wet coil is higher than that for the totally dry coil, even though the effectiveness is lower. For the wet coil the effectiveness is based on enthalpies and accounts for the latent energy transfer. According to the criteria, the totally wet coil assumption produces a heat transfer that is closer to the actual value for a partially dry and partially wet coil. This will be shown at the end of this example.

The outlet air conditions are a temperature of 51.7 °F and a humidity ratio of 0.008122 lbm_w/lbm_a. The outlet air temperature is higher than that for the totally dry coil but the heat transfer rate is more because of the energy transfer due to condensation. The condensate flow rate is 19.5 lbm/hr.

"Partially dry and partially wet"

"The partially dry and partially wet analysis is carried out to verify that the totally wet analysis is close to the actual heat transfer. The air is at a temperature T_ax at the location that condensation starts and T_s is the surface temperature at that point. T_s equals the dew point of the incoming air at the start of condensation.

The relation among the air, water, and surface temperatures at the location where condensation starts is determined using equation 2.31. Neither the air nor water temperature is known at this point and the solution is iterative. The resistances are those computed for the dry coil."

```
T_s = DewPoint(AirH2O,T=T_a_in ,B=T_wb_in ,P=p_atm) "F"          "Temperature"
(T_a_x - T_s)/R_a=(T_s - T_w_x)/R_w                               "Temperatures"
```

"Dry section"

"X is the fraction of the air side area that is dry For this section, the air enters at T_a_in and leaves at T_a_x. The water enters at T_w_x and leaves at T_w_out_p. The Ntu for the dry section is the same as that for the totally dry coil times the fraction X. The capacitance rate ratio is the same as that for the totally dry coil and the effectiveness is given by the expression for counterflow."

```
Ntu_p_dry = Ntu_dry*X                                              "Ntu"
eff_p_dry = (1 - exp(-Ntu_p_dry*(1 - C_star)))/(1 - C_star*exp(-Ntu_p_dry*(1-C_star))) "Eff."
```

"The heat transfer for the dry section is given in terms of the effectiveness and energy balances that yield the air and water temperatures at the location that condensation starts."

```
Q_p_dry = eff_p_dry*C_min*(T_a_in - T_w_x) "Btu/hr"               "Heat flow"
Q_p_dry =C_a*(T_a_in - T_a_x) "Btu/hr"                             "Heat flow"
Q_p_dry = C_w*(T_w_out_p-T_w_x) "Btu/hr"                           "Heat flow"
```

"Wet section"

"The Ntu of this section is the remaining fraction of the coil. For this section, the air enters at T_a_x and leaves at T_a_out_p. The water enters at T_w_in and leaves at T_w_x. The air side area is used in the definitions of Ntu and the wet Ntu for the partially wet section is Ntu_star times the remaining fraction of the area. The effectiveness is given by the expression for counterflow."

```
Ntu_p_wet = Ntu_star*(1-X)                                         "Ntu"
eff_p_wet = (1 - exp(-Ntu_p_wet*(1 - m_star)))/(1 - m_star*exp(-Ntu_p_wet*(1-m_star))) "Eff."
```

"The energy transfer for the wet section is given in terms of the effectiveness and energy balances that yield the air and water temperatures at the location that condensation starts."

```
Q_p_wet = eff_p_wet*m_dot_a*(h_a_x - h_w_in) "Btu/hr"              "Heat flow"
```

"The air enthalpy at the location where condensation starts is the enthalpy at the temperature T_a_x and the inlet humidity ratio."

h_a_x = enthalpy(airh2o,p=p_atm,T=T_a_x, w=w_in) "Btu/lbm" "Enthalpy"

"The temperatures of the air and water at the location of condensation are determined from the energy transfer in the wet section and energy balances on the air and water streams."

Q_p_wet =m_dot_a*(h_a_x - h_a_out_p) "Btu/hr" "Heat flow"

Q_p_wet = C_w*(T_w_x-T_w_in) "Btu/hr" "Heat flow"

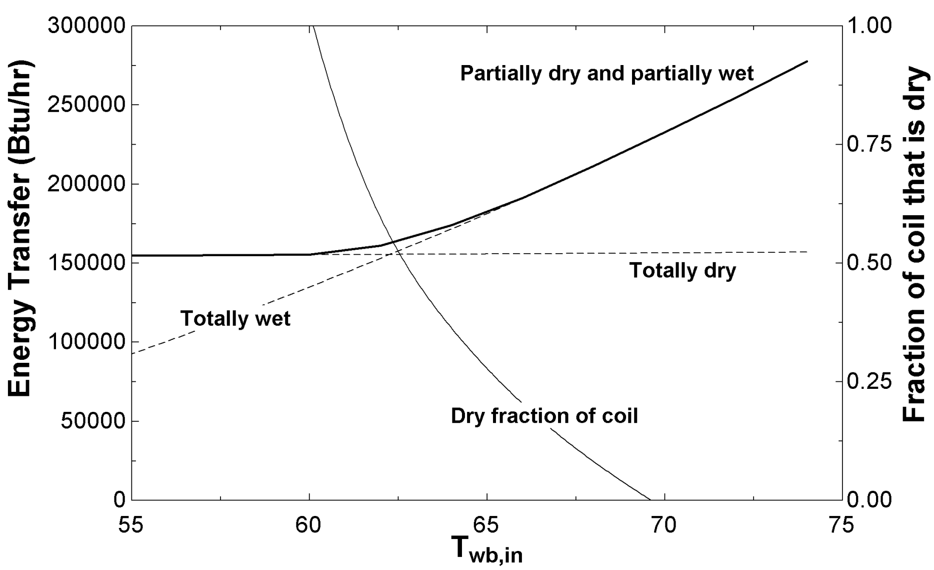
Q_p_total = Q_p_wet +Q_p_dry "Btu/hr" "Heat flow"

Results

The calculation procedure yields a final value of the fraction of the area that is dry of 0.363. The air temperature at this location is 64.1 °F and the water temperature is 45.1 °F. The heat flow for the dry section is 81,748 Btu/hr and that for the wet section is 92,234 Btu/hr. The total heat flow for the partially dry and partially wet coil is the sum of the two values, or 173,983 Btu/hr.

The totally wet coil analysis gave a heat transfer rate of 171,708 Btu/hr, which is 1.3% below the more exact value. This shows that the heat flow from the totally wet analysis, which is higher than that for the totally dry analysis, closely represents the actual heat flow.

For this example, the effect of the inlet wet bulb temperature on the heat transfer and the fraction of the coil surface that was dry was also carried out, with the results shown in the figure below.



For an inlet air temperature of 80 °F, the coil is dry (x = 1) for wet-bulb temperatures less than 60 °F. The partially dry and partially wet analysis yields the same heat flow as the totally dry analysis. The totally wet analysis significantly underpredicts the heat transfer. For wet-bulb temperatures greater than 70 °F, the coil is totally wet and the totally and partially wet analyses give the same energy transfer. The energy transfer for the totally dry analysis is independent of wet-bulb temperature.

It is only in the range of wet-bulb temperatures between 60 and 70 °F that the totally dry and totally wet analyses give somewhat lower than the correct value. In this range, the higher of the two values is closer to the actual value and the maximum error is less than 2%.

Between a wet-bulb temperature of 60 and 70 °F the dry section of the coil decreases from 100% to zero. For many operating conditions a coil can be expected to have both dry and wet sections, but the procedure presented in this section accurately predicts the heat transfer.

The analogy formulation can be extended to an analogous Log-Mean-Temperature-Difference representation. For constant values of the parameters U^*, c_w, and c_s, the following expression for the heat transfer rate in terms of an enthalpy difference can be obtained:

$$\dot{Q} = U^* A_a \ LMED \tag{2.34}$$

where $LMED$ is a log-mean enthalpy difference. This is the average enthalpy difference between the air stream and a saturated air stream whose temperature everywhere along the cooling coil surface is equal to temperature of the water stream. This relation will then give the heat transfer that actually occurs in the coil. The log-mean enthalpy difference is given as

$$LMED = \frac{(h_{a,in} - h_{w,out}) - (h_{a,out} - h_{w,in})}{Ln\left[\dfrac{h_{a,in} - h_{w,out}}{h_{a,out} - h_{w,in}}\right]} \tag{2.35}$$

where h_w is the enthalpy of saturated air at the water temperature.

Equation 2.35 is the exact enthalpy difference for counterflow coils. For cross-flow or other geometry coils, the enthalpy difference needs to be modified using the correction factor F developed for sensible heat exchangers:

$$\dot{Q} = F \cdot U^* A_a \ LMED \tag{2.36}$$

where F is the correction factor that accounts for the flow arrangement (see, e.g., Incropera et al). Since most cooling coils are cross-flow geometries with the air unmixed and the water flow mixed, the performance is close to that of a counterflow coil for which the factor F is essentially unity.

SM 2.3 Heat and Mass Transfer Fin and Surface Efficiencies

The sensible fin efficiency accounts for the effect of the varying temperature along the fin surface and is the ratio of the convective heat transfer to the fin with a varying temperature to that for a fin of a uniform temperature that is equal to the temperature at the base of the fin. The development of the fin efficiency relations is given in Section SM 1.2. For heat transfer only, the fin efficiency is a function of the non-dimensional parameter:

$$m_f\, L_c = \sqrt{\frac{2\,h_c}{k_f\, t_f}}\; L_c \tag{2.37}$$

where h_c is the convection coefficient, k_f is the thermal conductivity of the fin, t_f is the fin thickness, and L_c is the characteristic fin length. The characteristic length depends on the fin geometry (straight, circular, plate, etc.) and on whether the fin tip is exposed or insulated.

The overall surface efficiency is used to represent the effect of the surface area of the fin and the unfinned area (prime area) on the heat transfer. The overall surface efficiency is the ratio of the actual heat transfer to that when the fin and the prime area are at a uniform temperature. The overall heat transfer surface efficiency is given by

$$\eta_0 = 1 - \frac{A_f}{A_0}\left(1 - \eta_f\right) \tag{2.38}$$

where η_o is the overall efficiency, η_f is the fin efficiency, A_f is the fin surface area, and A_o is the sum of the fin and prime surface area. The fin efficiency can be combined with the surface area and convective heat transfer coefficient to give the overall heat transfer conductance U_a on the air side of a coil:

$$U_a = \eta_o\, h_c \tag{2.39}$$

The energy transfer conductance for a cooling coil in which condensation occurs also depends on fin and overall surface efficiencies, but these efficiencies are different from those for heat transfer only. The fin energy transfer efficiency is defined similar to that for heat transfer only. The fin efficiency is the ratio of the actual energy transfer to the fin to that if the air at the surface of the condensate layer were at a uniform saturation enthalpy equal to that at the base of the fin. The distribution of saturated air enthalpy along the fin is different from the distribution of temperature, leading to a different heat and mass transfer fin efficiency.

The fin efficiency for energy transfer can be determined using the heat transfer only fin efficiency equations and applying the analogy approach of Section SM 2.2 (Zhou et al., 2007). The fin efficiency for wetted surface energy exchange is determined using the heat transfer fin efficiency equations and replacing the heat transfer convection coefficient with mass transfer convection coefficient $\left(h_c / c_{p,m}\right)$ and introducing the effective specific heat c_s:

$$m_f^* L_c = \sqrt{\frac{2h_c}{k_f t_f} \frac{c_s}{c_{p,m}}} L_c \tag{2.40}$$

The heat and mass transfer fin efficiency can be found using the relations for heat transfer only fins given in Section SM 1.2 using the modified fin parameter in Equation 2.40.

Analogous to the overall heat transfer surface efficiency, the overall heat and mass transfer surface efficiency is related to the fin efficiency, fin area, and total area as

$$\eta_o^* = 1 - \frac{A_f}{A_o}(1 - \eta_f^*) \tag{2.41}$$

A fin efficiency is also needed to determine the exit air temperature using the relations of Section SM 2.2. The conventional heat transfer fin efficiency is not strictly applicable because the temperature distribution for a wet fin is different from that for a dry fin. Zhou et al. (2007) developed a correction factor that allows estimation of the convective heat transfer fin efficiency from the heat and mass transfer fin efficiency. The correction factor is defined as

$$C_f = \frac{1}{c_s} \frac{h_{s,b} - h_a}{T_b - T_a} \tag{2.42}$$

where $h_{s,b}$ is saturation air enthalpy at the base temperature, h_a is the free stream air enthalpy, T_b is the base temperature, and T_a is the air temperature. The values of $h_{s,b}$ and T_b are averages over the surface area. The overall heat transfer surface efficiency $\hat{\eta}_o$ is evaluated using the heat and mass transfer efficiency and correction factor as

$$\hat{\eta}_o = 1 - C_f\left(1 - \eta_f^*\right) \tag{2.43}$$

Example 2.2 presents the calculation procedures for the different fin efficiencies and the magnitude of the effects of mass transfer on the efficiency. The fin geometry and heat transfer coefficient are the same as for Example 1.2, Section SM 1.2.

Example 2.2 Determine the fin and overall surface efficiencies for a cooling coil. The tube diameter is 0.774 inch and the fins are steel with a thickness of 0.012 inch, a diameter of 1.463 inch, and a pitch of 9.05 fins per inch. The conductivity of steel is 35 Btu/hr-ft-°F and the heat transfer coefficient is 2. 4 Btu/hr-ft^2-°F. The air temperature is 70 °F and saturated and the surface temperature is 50 °F.

```
"Problem specifications"
T_a = 70 "F"                                    "Temperature"
T_s = 50 "F"                                    "Temperature"
h_c = 14.4 "Btu/hr-ft2-F"                       "Heat trans coefficient"
k = 35 "Btu/hr-ft-F"                            "Thermal conductivity"

"Fin specifications"
r_i = 0.774/2*convert(in,ft) "ft"               "Inner radius"
r_o = 1.463/2*convert(in,ft) "ft"               "Outer radius"
t = 0.012*convert(in,ft) "ft"                   "Thickness"
r_c = r_o + t/2 "ft"                            "Effective radius"
r_Ratio = r_c/r_i                               "Radius ratio"

"Determine the fin, prime, and overall surface areas on a per-foot-of-length basis."
L = 1 "ft"                                      "Length"
p = 9.05*convert(1/in,1/ft) "1/ft"              "Fin pitch"
N_f = p*L                                       "Number fins per foot"
A_f = N_f*Pi*(r_c^2 - r_i^2) "ft2"              "Fin area"
A_p =Pi*2*r_i*( L-N_f*t) "ft2"                  "Prime area"
A_o = A_f + A_p "ft2"                           "Total area"

"Heat transfer only efficiencies"
"Determine the fin parameter for heat transfer only using equation 2.37."
m_f = ((2*h_c)/(k*t))^0.5 "1/ft"                "Fin parameter"

"Determine the fin efficiency from the expression for circular fins, equation 1.25, Section SM 1.2."
A = (2*r_i)/(m_f*(r_c^2-r_i^2))
B = BesselK(1,m_f*r_i)*BesselI(1,m_f*r_c) - BesselI(1,m_f*r_i)*BesselK(1,m_f*r_c)
C = BesselI(0,m_f*r_i)*BesselK(1,m_f*r_c) + BesselK(0,m_f*r_i)*BesselI(1,m_f*r_c)
eta_f_ht = A*B/C                                "Fin efficiency"
```

"Determine the overall surface efficiency using equation 2.38."
eta_o_ht = 1 - (A_f/A_o)*(1 - eta_f_ht) "Overall efficiency"

Results

The fin parameter for heat transfer only is 28.7/ft and the corresponding fin efficiency is 0.763. The overall heat transfer efficiency is 0.801. This result will be compared to the results for the wet fin.

```
"Energy transfer efficiencies"
"Determine the saturation enthalpies at the air and surface temperatures and the air specific
heat."
p_atm = 14.7 "psia"                                              "Pressure"
h_a = enthalpy(AirH2O,p = p_atm, T=T_a, R=1) "Btu/lbm"           "Enthalpy"
h_s = enthalpy(AirH2O,p = p_atm, T=T_s, R=1) "Btu/lbm"           "Enthalpy"
cp_m = specheat(AirH2O,p = p_atm, T=T_a, R=1) "Btu/lbm-F"        "Specific heat"

"Determine the effective specific heat using the surface enthalpy and temperature. A numerical
approximation is used for the derivative at the surface temperature using a 1 °F increment."
T_ss = T_s + 1 "F"                                               "Temperature"
h_ss = enthalpy(AirH2O,p = p_atm, T=T_ss, R=1) "Btu/lbm"         "Enthalpy"
c_s = (h_ss - h_s)/(T_ss - T_s) "Btu/lbm-F"                      "Eff specific heat"

"Determine the energy transfer parameter m_f_star using equation 2.40."
m_f_star = ((2*h_c*c_s/cp_m)/(k*t))^0.5 "1/ft"                   "Fin parameter"

"Determine the fin efficiency from the expression for circular fins using the mass transfer
parameters."
A_star = (2*r_i)/(m_f_star*(r_c^2-r_i^2))
B_star = BesselK(1,m_f_star*r_i)*BesselI(1,m_f_star*r_c) -
    BesselI(1,m_f_star*r_i)*BesselK(1,m_f_star*r_c)
C_star= BesselI(0,m_f_star*r_i)*BesselK(1,m_f_star*r_c) +
    BesselK(0,m_f_star*r_i)*BesselI(1,m_f_star*r_c)
eta_f_star = A_star*B_star/C_star                                "Fin efficiency"

"Determine the overall surface efficiency using equation 2.41."
eta_o_star= 1 - (A_f/A_o)*(1 - eta_f_star)                       "Overall efficiency"
```

Results

The fin parameter for energy transfer is 43.2/ft. This is significantly larger than that for heat transfer and reflects the increased energy transfer potential due to moisture condensation. The corresponding fin efficiency is 0.599 and the overall heat transfer efficiency is 0.664. The overall surface efficiency is almost 20% lower than that for heat transfer only.

```
"Heat transfer fin efficiency for a wet fin"
"Determine the correction factor using equation 2.42."
C_f = (1/c_s)*(h_a - h_s)/(T_a - T_s)                          "Correction factor"

"Determine the overall surface efficiency for heat transfer to a wet fin using equation 2.43."
eta_o_hat = 1 - C_f*(1-eta_f_star)                             "Overall efficiency"
```

<u>Results</u>

The correction factor for the heat transfer to a wet fin is 1.277, which reduces the heat transfer efficiency to 0.508. This is a significant reduction from the value for a dry fin of 0.801. The results show the strong effect that combined heat and mass transfer have on the fin and overall efficiencies. It can be important to make these corrections to avoid errors in evaluating cooling coils.

SM 2.4 Extension of Catalog Information

The use of manufacturer's data to select a cooling coil for a given situation is illustrated in Chapter 13, Section 13.7. In Example 13.4, the desired operating conditions were those available from the catalog. In general, it would be unusual if the catalog contained exactly the design inlet conditions for a particular application and it would be desirable to extend the available catalog information to the appropriate inlet conditions. The analogy relations given in section SM 2.2 provide a method for extrapolation to different conditions. Typically, insufficient information is given to evaluate all of the parameters required for the analogy approach, such as the detailed data on the coil areas and heat and mass transfer coefficients. However, the effectiveness values can be determined from the catalog information at the design conditions and then used to estimate the performance for other operating conditions. The coil enthalpy effectiveness at the conditions given in a catalog is determined from the analogy representation, equation 2.25, Section SM 2.2:

$$\varepsilon^* = \frac{\left(h_{a,in} - h_{a,out}\right)}{\left(h_{a,in} - h_{a,win,sat}\right)} \tag{2.25}$$

Using the effectiveness value calculated at one operating condition to determine the performance at another condition is based on the assumption that the number of transfer units (Ntu^*, Ntu_a, and Ntu_a^*) and the equivalent capacitance rate ratio (m^*) are the same for both sets of air and water states. Although the transfer coefficients and equivalent specific heat are somewhat dependent on temperature and humidity, the assumption of constant effectiveness turns out to be a satisfactory approximation.

Equation 2.25 is useful for computing the outlet enthalpy and the coil capacity, but it does not yield the outlet humidity level or the wet and dry bulb temperatures at the outlet. A humidity effectiveness can also be determined from the catalog data and used to estimate the outlet humidity at the new conditions. The humidity effectiveness is defined as:

$$\varepsilon_w^* = \frac{\left(w_{a,in} - w_{a,out}\right)}{\left(w_{a,in} - w_{w,in,sat}\right)} \tag{2.44}$$

where $w_{w,in,sat}$ is the humidity ratio of saturated air at the water inlet temperature. This effectiveness is the ratio of the actual humidity of the air relative to the maximum change in humidity. With the outlet enthalpy and humidity ratio determined using the respective effectivenesses, the leaving dry- and wet-bulb temperatures can be obtained.

The extrapolation to other operating conditions will be illustrated using the performance given in Chapter 13, Table 13.4 as a base, and extended to conditions for which catalog information is available to provide a comparison of the accuracy of the extrapolation. The performance for entering conditions at an air velocity of 500 fpm is given in Table 2.2.

Table 2.2. Coil performance information

	Entering condition 95/70 °F			Entering condition 70/60 °F		
No. of rows	MBH	LDB	LWB	MBH	LDB	LWB
2	13.4	74.3	68.1	5.7	59.6	56.0
4	23.1	64.3	62.4	9.8	54.0	53.0
6	30.1	58.4	57.8	12.7	51.1	50.8
8	35.3	54.3	54.2	14.9	49.1	49.0

The inlet conditions of 70 °F dry-bulb and 60 °F wet-bulb are at the lower extreme of the coil design data. The design performance for the inlet conditions of 70/60 °F will be estimated from the entering conditions of 95/70 °F using the analogy relations and then compared to the values in Table 2.2. The calculations are illustrated in Example 2.3.

Example 2.3 Extrapolate the performance data given in Table 2.2 for entering conditions of 95/70 °F to 70/60 °F. The extrapolation will be based on the enthalpy and humidity effectivenesses. The two effectivenesses are determined for the design performance of coils with 2, 4, 6, and 8 rows at inlet conditions of 95 °F/75 °F and 500 ft/min velocity. These effectivenesses are assumed to be constant for all inlet conditions and are used to predict the outlet dry- and wet-bulb temperatures and heat flow at 70 °F/60 °F. Two lookup tables are created with the design performance from the catalog. Lookup 1 is for 95 °F/75 °F and Lookup 2 is for 70 °F/60 °F.

```
"Determine the air properties for both sets of inlet conditions."
p_atm = 14.7 "psia"                                                     "Pressure"
"For 95 F/75 F"
T_a_in_1 = 95 "F"                                                       "Dry-bulb temperature"
Twb_in_1 = 75 "F"                                                       "Wet-bulb temperature"
w_a_in_1 = HumRat(AirH2O,T=T_a_in_1, P=p_atm,B=Twb_in_1) "lbm/lbm" "Humidity ratio"
h_a_in_1= Enthalpy(AirH2O,T=T_a_in_1,P=p_atm,B=Twb_in_1) "Btu/lbm"   "Enthalpy"
For 70 °F/60 °F:
T_a_in_2 = 70  "F"                                                      "Dry-bulb temperature"
Twb_in_2 = 60 "F"                                                       "Wet-bulb temperature"
w_a_in_2 = HumRat(AirH2O,T=T_a_in_2, P=p_atm,B=Twb_in_2) "lbm/lbm" "Humidity ratio"
h_a_in_2= Enthalpy(AirH2O,T=T_a_in_2,P=p_atm,B=Twb_in_2) "Btu/lbm"   "Enthalpy"

"Determine the saturated air enthalpy and humidity ratio at the water inlet temperature. The water
inlet temperature is the same for both sets of inlet conditions."
T_w_in = 42
h_w_in= Enthalpy(AirH2O,T=T_w_in,P=p_atm,R=1) "Btu/lbm"                 "Enthalpy"
w_w_in = HumRat(AirH2O,T=T_w_in, P=p_atm,R=1) "lbm/lbm"                 "Humidity ratio"

"Input the catalog performance at the reference condition of 95 °F/75 °F from Lookup Table 1."
Rows = Lookup('Lookup 1',Tablerun#,'rows')                              "Number of rows"
T_a_out_1 = Lookup('Lookup 1',Tablerun#,'LDB') "F"                      "Dry-bulb temperature"
Twb_out_1 = Lookup('Lookup 1',Tablerun#,'LWB') "F"                      "Wet-bulb temperature"
MBH_1 = Lookup('Lookup 1',Tablerun#,'MBH') "Btu/hr-ft2"                 "Heat flow/area"

"Determine the outlet enthalpy and humidity ratio for the 95 °F/75 °F condition."
h_a_out_1= Enthalpy(AirH2O,T=T_a_out_1,P=p_atm,B=Twb_out_1) "Btu/lbm"   "Enthalpy"
w_a_out_1 = HumRat(AirH2O,T=T_a_out_1, P=p_atm,B=Twb_out_1) “lbm/lbm” "Humidity ratio"

"Determine the enthalpy and humidity effectivenesses."
eff_h_1 = (h_a_in_1 - h_a_out_1)/(h_a_in_1 - h_w_in)                   "Enthalpy effectiveness"
eff_w_1 = (w_a_in_1 - w_a_out_1)/(w_a_in_1 - w_w_in)                   "Humidity effectiveness"
```

```
"Assume that the enthalpy and humidity effectiveness at 95 °F/75 °F are the same as at 70 °F/60
°F and use the definitions of effectiveness to determine the outlet enthalpy and humidity ratio."
eff_h_2 = eff_h_1                                                   "Enthalpy effectiveness"
eff_h_2 = (h_a_in_2 - h_a_out_2)/(h_a_in_2- h_w_in)                 "Enthalpy effectiveness"
eff_w_2 = eff_w_1                                                   "Humidity effectiveness"
eff_w_2 = (w_a_in_2 - w_a_out_2)/(w_a_in_2 - w_w_in)                "Humidity effectiveness"

"Determine the outlet air dry- and wet-bulb temperatures from the extrapolated outlet enthalpy
and humidity ratio."
T_a_out_2= Temperature(AirH2O,h=h_a_out_2,P=p_atm,w=w_a_out_2) "F"  "Dry-bulb temp"
Twb_out_2 = WetBulb(AirH2O,T=T_a_out_2, P=p_atm,w=w_a_out_2) "F"    "Wet-bulb temp"

"Determine the mass flow rate and determine the heat flow per unit area from the extrapolated
outlet enthalpy."
Vel = 500 "ft/min"                                                  "Velocity"
rho = 0.075                                                         "standard air density"
m_dot_a= rho*Vel*convert(1/min,1/hr) "lbm/hr-ft2"                   "Mass flow rate/are"
Q_2 = eff_h_2*m_dot_a*(h_a_in_2 - h_w_in)/1000 "Btu/hr-ft2"         "Heat flow/area"

"Input the values of dry- and wet-bulb temperatures and heat flow per unit area from the catalog
data for comparison."
LDB_2 = Lookup('Lookup 2',Tablerun#,'LDB') "F"                      "Dry-bulb temperature"
LWB_2 = Lookup('Lookup 2',Tablerun#,'LWB') "F"                      "Wet-bulb temperature"
MBH_2 = Lookup('Lookup 2',Tablerun#,'MBH') "Btu/hr-ft2"             "Heat flow/area"
Ratio_2 = Q_2/MBH_2                                                 "Heat flow ratio"
```

<u>Results</u>

The outlet states and the heat transfer rate are compared in Table 2.3. The values under the heading "Analogy" are the calculated values based on the assumption that the effectivenesses for enthalpy and humidity ratio are not dependent on the inlet conditions. The values under the heading "Catalog Data" are those from the catalog.

The heat transfer is within 9% of that given in the catalog. The effectiveness, which is based on hot and humid entering conditions of 95 °F dry bulb and 75 °F wet bulb, tends to overpredict the heat transfer for the lower temperature conditions. The dry- and wet-bulb temperatures are all within 1 °F of the catalog. Within about 10% accuracy, the analogy relations can be used to estimate the coil performance at other operating conditions.

Table 2.3 Comparison of analogy results to the catalog data of Table 2.2

	Analogy				Catalog data		
No. of rows	ε_h	MBH	LDB	LWB	MBH	LDB	LWB
2	0.268	6.2	60.1	55.7	5.7	59.6	56.0
4	0.462	10.6	54.7	52.4	9.8	54.0	53.0
6	0.603	13.8	51.4	49.9	12.7	51.1	50.8
8	0.705	16.2	49.0	48.0	14.9	49.1	49.0

The air velocity affects the coil effectiveness. As the velocity of the air increases, the convection heat and mass transfer coefficient increase. For forced convection, the heat transfer coefficient is proportional to the velocity to an exponent that is between 0.5 and 0.6. The overall coil conductance then increases, but, because the air and water side resistances are in series, the increase is to a lesser power than that for the air side alone. The mass flow rate of the air stream is proportional to the velocity to the first power, and therefore the number of transfer units (Ntu), which is the ratio of the overall conductance and the mass flow rate of the air stream, decreases. The enthalpy and humidity effectiveness thus decrease. However, the total energy transfer and the condensation rate, which are the products of the effectiveness and the airflow rate, increase.

The enthalpy and humidity effectiveness as a function of air velocity are shown in Figure 2.5 for all velocities at an inlet condition of 95/70 °F as given in Chapter 13, Table 13.4.

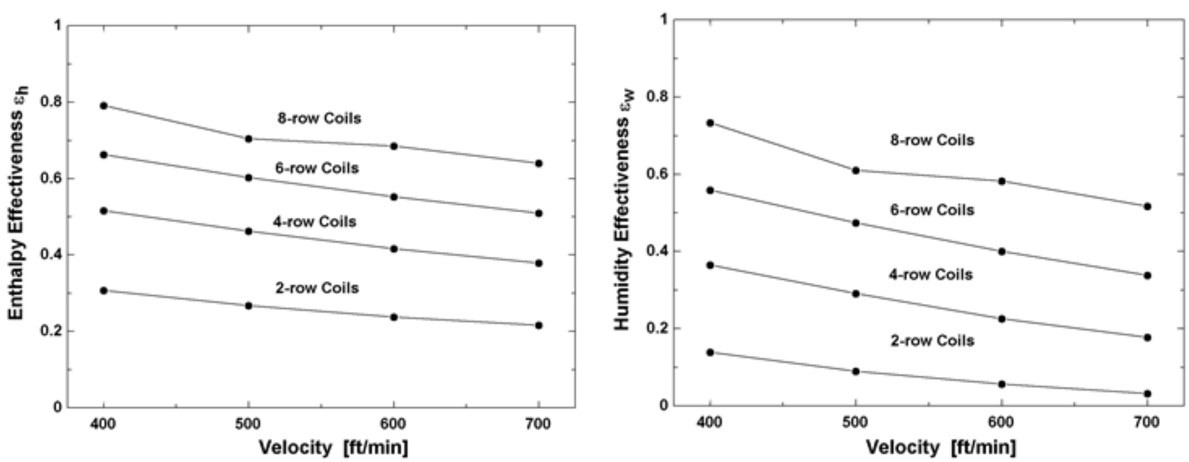


Figure 2.5 Enthalpy and humidity effectiveness for the coils of Table 2.2

The enthalpy and humidity effectiveness both decrease with velocity. The enthalpy effectiveness is a little higher than the humidity effectiveness. This is probably due to the sensible heat transfer in the first section of a coil. Over the range of the velocities for which catalog information is available, the decrease is essentially linear. For a given series of coil surfaces, it would be possible to correlate the effectiveness determined from catalog information as a function of air velocity and then estimate the performance at other velocities and conditions.

SM 2.5 Summary

In a cooling coil, the air is cooled by heat transfer to the cold coil surfaces and moisture is removed by mass transfer to the condensate layer on the coil surfaces. The heat and mass transfer flows are coupled, and together produce an energy flow that becomes the heat transfer to the cooling fluid. The fundamental relationships describing these energy flows for a coil are differential equations describing the change of temperature, enthalpy, and humidity ratio with position through the coil. These governing differential equations can be integrated over the entire coil surface to yield the performance.

The analogy approach to coil performance is based on the effectiveness-Ntu method developed for sensible heat exchangers. The performance is determined using algebraic relations rather than integration of the governing equations, and is useful for simulation studies or extension of coil performance to other operating conditions. The overall transfer coefficient for the coil is combined with an effective saturated air enthalpy that represents the coolant flow. The analogy approach yields the heat transfer for a completely wet coil, and a sensible heat exchanger approach yields the heat transfer for a completely dry coil. These greater of the two heat flows approximates the actual coil heat transfer.

The fin efficiency for a coil with a wetted surface is different from that for a dry surface. The efficiency applies to the energy transfer and depends is on the fin geometry, fin material and convection coefficient.

The analogy approach can be used to extend catalog performance to other operating conditions. The effectiveness determined for one set of operating conditions can be used to estimate the outlet state and heat transfer at another set of conditions.

Manufacturers produce a wide variety of coil geometries, and present performance for a wide range of air and water velocities. In selecting a coil for a given application, a small subset of coils will be satisfactory in terms of providing the desired heat transfer. However, the air and water flow rates, pressure drops, and physical size will be different for each coil. The final selection of the coil will involve consideration of these factors, which often introduces the economic trade-off between first and operating costs.

SM 2.6 Nomenclature

A	area
c_p	specific heat
$c_{p,m}$	mixture specific heat
c_s	effective specific heat
C_f	correction factor
C^*	thermal capacitance rate ratio
EDB	entering air dry bulb temperature
EWB	entering air wet-bulb temperature

EWT	entering water temperature
$\dot{E}$	energy flow
F	heat exchanger correction factor
h	specific enthalpy
h_c	convection heat transfer coefficient
h_{fg}	latent heat of vaporization
h_m	convection mass transfer coefficient
k_f	thermal conductivity of fin material
L_c	characteristic fin length
LDB	leaving dry-bulb temperature
LMED	log-mean enthalpy difference
LWB	leaving wet-bulb temperature
m_f	fin parameter for heat transfer
m_f^*	fin parameter for mass transfer
m^*	mass transfer capacitance rate ratio
$\dot{m}$	mass flow rate
MBH	coil heat transfer per ft^2 of coil face area
Ntu	number of transfer units for heat transfer
Ntu_a	air-side number of transfer units
Ntu^*	number of transfer units for mass transfer
Ntu_a^*	air-side number of transfer units for energy transfer
$\dot{Q}$	heat flow rate
R	thermal resistance
R^*	mass transfer resistance
t_f	fin thickness
T	temperature
U	overall unit heat transfer conductance
U^*	overall unit mass transfer conductance
w	humidity ratio
WT*R*	temperature rise of the water flow

Δ	difference
ε	heat transfer effectiveness
ε^*	energy transfer effectiveness
ε_w^*	effectiveness for moisture transfer
	fin efficiency for heat transfer
η_f^*	fin efficiency for energy transfer
η_o	overall surface efficiency for heat transfer
η_o^*	overall surface efficiency for energy transfer
$\hat{\eta}_o$	overall surface efficiency for heat transfer for a wet surface
ϕ	relative humidity

<u>Subscripts</u>

a	air
b	base
con*d*	condensate
dp	dew point
eff	effective
f	liquid phase, fin
in	in, inlet
min	minimum
o	overall
out	out, outlet
s	surface
sat	saturated conditions
v	vapor phase
w	water
x	location between dry and wet

SM 2.7 References

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Problems

Problems in English Units

Problems 2.1, 2.3, 2.5, and 2.10 are the same as in Chapter 13 and can be solved using the material in that chapter. The remaining problems require the material presented in this supplementary chapter.

2.1 A cooling coil is designed for an airflow of 20,000 cfm at 90 °F and 50% RH entering the coil. The chilled water design flow rate is 80,000 lb/hr at a temperature of 45 °F. The design conductances are a U_a of 100 Btu/hr-ft^2-°F and a U_w of 300 Btu/hr-ft^2-°F. The heat transfer area is 3000 ft^2 on the air side and 400 ft^2 on the water side. Using the analogy method, determine the heat transfer rate, the coil effectiveness, the outlet temperatures of the air and water flows, and the condensate flow rate under the assumptions that (a) the coil is completely wet and (b) the coil is completely dry. Draw some conclusions from your results.

2.2 A cooling coil is designed for an airflow of 20,000 cfm at 90 °F and 50% RH entering the coil. The chilled water design flow rate is 80,000 lb/hr at a temperature of 45 °F. The design conductances are a U_a of 100 Btu/hr-ft^2-°F and a U_w of 300 Btu/hr-ft^2-°F. The heat transfer area is 3000 ft^2 on the air side and 400 ft^2 on the water side. Using the analogy method, determine the heat transfer rate, the coil effectiveness, the outlet temperatures of the air and water flows, and the condensate flow rate under the assumptions that (a) the coil is completely wet, (b) the coil is completely dry, and (c) the coil is partially wet and partially dry. Draw some conclusions from your results.

2.3 A cooling coil operates with an airflow of 20,000 cfm and a chilled water flow rate of 80,000 lb/hr at a temperature of 45 °F. The conductances are a U_a of 100 Btu/hr-ft^2-°F and a U_w of 300 Btu/hr-ft^2-°F. The heat transfer area is 3000 ft^2 on the air side and 400 ft^2 on the water side. During operation, the inlet conditions are in the range of 70 to 85 °F and 30 to 60% relative humidity. Determine the heat transfer rate, the outlet temperature and humidity ratio of the air, and the condensation rate over the range of operation. Draw some conclusions from your results.

2.4 A cooling coil is designed for an air flow of 20,000 cfm with entering conditions of 85 °F and 60% RH. The chilled water design flow rate is 80,000 lb/hr at a temperature of 42 °F. The design conductances are a U_a of 65 Btu/hr-ft^2-°F and a U_w of 175 Btu/hr-ft^2-°F and the heat transfer areas are 4500 ft^2 on the air side and 2000 ft^2 on the water side. Determine the air outlet temperature and humidity ratio, the water outlet temperature, the heat transfer rate, and the condensate flow rate for

a. Design conditions
b. An entering air state of 85 °F and 20% RH
c. Air and water flow rates of one-half design values with the same conductances as at design conditions.
d. Air and water flow rates of one-half design values with the conductances proportional to flow rate to the 0.8 power:

$$\frac{U}{U_{design}} = \left(\frac{\dot{m}}{\dot{m}_{design}}\right)^{0.8}$$

e. Draw some conclusions from your results.

2.5 Select a coil using the performance information given in Tables 2.2 and 2.3 for a commercial application with a building design load of 20 tons with a sensible heat ratio of 0.75. The design conditions are entering dry- and wet-bulb temperatures of 95/75 °F and an entering water temperature of 42 °F. The zone is to be maintained at 70 °F and within the ASHRAE comfort region. Specify the face area, airflow rate (cfm), water flow rate (lb/hr), coil load (tons), pressure drop, and fan power (hp). Draw some conclusions from your results.

2.6 The aluminum circular fins on the tubes of a cooling coil are 1.5 in. diameter and 0.006 in. thick with a pitch of 10 fins/inch. The tube diameter is 0.5 in. The heat transfer coefficient is 15 Btu/hr-ft^2-°F. For an air temperature of 70 °F and saturated and a surface temperature of 50 °F, determine the fin and overall efficiencies for heat transfer, energy transfer, and heat transfer for a wet fin. Draw some conclusions from your results.

2.7 The aluminum circular fins on the tubes of a cooling coil are 1.5 in. diameter and 0.010 in. thick with a pitch of 10 fins/inch. The tube diameter is 0.5 in. The heat transfer coefficient varies from 10 to 40 Btu/hr-ft^2-°F. For an air temperature of 70 °F and saturated and a surface temperature of 50 °F, determine the fin and overall efficiencies for heat transfer, energy transfer, and heat transfer for a wet fin. Draw some conclusions from your results.

2.8 A cooling coil designed to cool and dehumidify air has the following parameters at design conditions: inside tube surface area 60 ft^2, total outside surface area (fins and tube) of 1000 ft^2, inside heat transfer coefficient of 400 Btu/hr-°F-ft^2, outside heat transfer coefficient of 6.5 Btu/hr-°F-ft^2, overall surface efficiency of 0.875 (assume the same value for both heat transfer only and for heat and mass transfer). The design conditions are water entering the cooling coil at 45 °F, air entering at 80 °F and 50% relative humidity, a water flow rate of 8000 lbm/hr, and a dry air flow rate of 10,000 lbm/hr.

a. Assume that no dehumidification occurs and that the heat transfer coefficients are constant. Determine and plot the total coil cooling capacity (tons) and the coil heat transfer effectiveness as a function of the mass flow rate of air over the range 7500 lbm/hr and 15,000 lbm/hr. Explain the dependence of these quantities on the airflow rate.
b. Assume that dehumidification occurs and that the heat transfer coefficients are constant. Determine and plot the total coil cooling capacity (tons), the coil heat transfer effectiveness, and the coil sensible load fraction as functions of the mass flow rate over the range of 5000 lbm/hr and 15,000 lbm/hr. Compare and contrast the results with those for the case of no dehumidification. Explain the dependence of the sensible load fraction on the airflow rate.
c. For the given design and inlet air state, explore changes in operating conditions that would improve dehumidification by lowering the sensible heat fraction without lowering the cooling capacity. Suggest design changes that could improve dehumidification performance.

2.9 A cooling coil is designed for an air flow of 20,000 cfm with entering conditions of 85 °F and 50% RH. The chilled water design flow rate is 80,000 lb/hr at a temperature of 42 °F. The design conductances are a U_a of 65 Btu/hr-ft^2-F and a U_w of 175 Btu/hr-ft^2-°F and the heat transfer areas are 4500 ft^2 on the air side and 2000 ft^2 on the water side. Generate a performance map of the coil heat transfer rate, outlet air temperature and humidity as functions of

a. The air flow rate over the range of 5000 to 30,000 cfm. The air side conductance varies with flow rate as:

$$\frac{U}{U_{design}} = \left(\frac{\dot{m}}{\dot{m}_{design}} \right)^{0.8}$$

b. Air inlet conditions varying from 70 °F and 30% RH to 90 °F and 60% RH.
c. Draw some conclusions from your results.

2.10 For the cooling coils of Table 2.2,

a. Extrapolate the performance for a velocity of 400 fpm to conditions of 75 °F/65 °F.
b. For the 6-row coil, determine the heat transfer rate per unit face area and the outlet conditions for 400 fpm over a range of inlet dry-bulb temperatures of 70 to 90 °F and a wet-bulb temperatures of 60 °F.
c. Draw some conclusions from your analysis.

2.11 Design a cooling coil that transfers 30 tons of cooling for design conditions that are inlet dry- and wet-bulb temperatures of 85 °F and 65 °F, respectively. The air flow rate is 8,000 cfm and the supply temperature should be in the range of 50 to 55 °F. Chilled water is available at 42 °F and the chilled water return should be in the range of 55 to 60 °F. Choose one of the surfaces given in Figure 13.9 for your design. Account for the different fin efficiencies for heat and energy. Summarize your design and specify the air face velocity, coil frontal area and thickness, the air outlet temperature and humidity ratio, the water flow rate, and the air side pressure drop. Draw some conclusions from your results.

<u>Problems in SI Units</u>

Problems 2.12, 2.14, 2.16, and 2.21 are the same as in Chapter 13 and can be solved using the material in that chapter. The remaining problems require the material presented in this supplementary chapter.

2.12 A cooling coil is designed for an airflow of 10,000 L/s at 32 °C and 50% RH entering the coil. The chilled water design flow rate is 10 kg/s at a temperature of 10 °C. The design conductances are a U_a of 550 W/m^2-°C and a U_w of 1700 W/m^2-°C. The heat transfer area is 300 m^2 on the air side and 40 m^2 on the water side. Using the analogy method, determine the heat transfer rate, the coil effectiveness, the outlet temperatures of the air and water flows, and the condensate flow rate under the assumptions that (a) the coil is completely wet and (b) the coil is completely dry. Draw some conclusions from your results.

2.13 A cooling coil is designed for an airflow of 10,000 L/s at 32 °C and 50% RH entering the coil. The chilled water design flow rate is 10 kg/s at a temperature of

10 °C. The design conductances are a U_a of 550 W/m^2-°C and a U_w of 1700 W/m^2-°C. The heat transfer area is 300 m^2 on the air side and 40 m^2 on the water side. Using the analogy method, determine the heat transfer rate, the coil effectiveness, the outlet temperatures of the air and water flows, and the condensate flow rate under the assumptions that (a) the coil is completely wet, (b) the coil is completely dry, and (c) the coil is partially wet and partially dry. Draw some conclusions from your results.

2.14 A cooling coil operates with an airflow of 10,000 L/s and a chilled water flow rate of 10 kg/s at a temperature of 8 °C. The conductances are a U_a of 550 W/m^2-°C and a U_w of 180 W/m^2-°C. The heat transfer area is 300 m^2 on the air side and 40 m^2 on the water side. During operation, the inlet conditions are in the range of 20 to 30 °C and 30 to 60% relative humidity. Determine the heat transfer rate, the outlet temperature and humidity ratio of the air, and the condensation rate over the range of operation. Draw some conclusions from your results.

2.15 A cooling coil is designed for an air flow of 10,000 L/s with entering conditions of 30 °C and 60% RH. The chilled water design flow rate is 10 kg/s at a temperature of 5 °C. The design conductances are a U_a of 350 W/m^2-C and a U_w of 1000 W/m^2-°C and the heat transfer areas are 200 m^2 on the air side and 100 m^2 on the water side. Determine the air outlet temperature and humidity ratio, the water outlet temperature, the heat transfer rate, and the condensate flow rate for:

a. Design conditions
b. An entering air state of 30 °C and 20% RH
c. Air and water flow rates of one-half design values with the same conductances as at design conditions.
d. Air and water flow rates of one-half design values with the conductances proportional to flow rate to the 0.8 power:

$$\frac{U}{U_{design}} = \left(\frac{\dot{m}}{\dot{m}_{design}} \right)^{0.8}$$

e. Draw some conclusions from your results.

2.16 Select a coil using the performance information given in Tables 2.2 and 2.3 for a commercial application with a building design load of 80 kW with a sensible heat ratio of 0.75. The design conditions are entering dry- and wet-bulb temperatures of 35/21 °C, respectively, and an entering water temperature of 5 °C. The zone is to be maintained at 22 °C and within the ASHRAE comfort region. Specify the face area, airflow rate, water flow rate, coil load, pressure drop, and fan power. Draw some conclusions from your results.

2.17 The aluminum circular fins on the tubes of a cooling coil are 40 mm diameter and 0.15 mm thick with a pitch of 350/m. The tube diameter is 15 mm. The heat transfer coefficient is 100 W/m^2-°C. For an air temperature of 20 °C and saturated and a surface temperature of 10 °C, determine the fin and overall efficiencies for heat transfer, energy transfer, and heat transfer for a wet fin. Draw some conclusions from your results.

2.18 The aluminum circular fins on the tubes of a cooling coil are 40 mm diameter and 0.15 mm thick with a pitch of 350/m. The tube diameter is 15 mm The heat transfer coefficient varies from 60 to 250 W/m^2-°C. For an air temperature of 20 °C and saturated and a surface temperature of 10 °C, determine the fin and overall efficiencies for heat transfer, energy transfer, and heat transfer for a wet fin. Draw some conclusions from your results.

2.19 A cooling coil designed to cool and dehumidify air has the following parameters at design conditions: inside tube surface area 6 m^2, total outside surface area (fins and tube) 100 m^2, inside heat transfer coefficient of 2000 W/m^2-°C, outside heat transfer coefficient of 40 W/m^2-°C, overall fin efficiency of 0.875 (assume the same value for both heat transfer only and for heat and mass transfer). The design conditions are water entering the cooling coil at 5 °C, air entering at 30 °C and 50% relative humidity, a water flow rate of 1 kg/s, and a dry air flow rate of 1.5 kg/s. Assuming a counterflow arrangement and that dehumidification occurs throughout the coil, determine the following:

a. Assume that no dehumidification occurs and that the heat transfer coefficients are constant. Determine and plot the total coil cooling capacity and the coil heat transfer effectiveness as a function of the mass flow rate of air over the range 0.5 to 2 kg/s. Explain the dependence of these quantities on the airflow rate.
b. Assume that dehumidification occurs and that the heat transfer coefficients are constant. Determine and plot the total coil cooling capacity, the coil heat transfer effectiveness, and the coil sensible load fraction as functions of the mass flow rate over the range of 0.5 to 2 kg/s. Compare and contrast the results with those for the case of no dehumidification. Explain the dependence of the sensible load fraction on the airflow rate.
c. For the given design and inlet air state, explore changes in operating conditions that would improve dehumidification by lowering the sensible heat fraction without lowering the cooling capacity. Suggest design changes that could improve dehumidification performance.

2.20 A cooling coil is designed for an air flow of 10,000 L/s with entering conditions of 30 °C and 50% RH. The chilled water design flow rate is 10 kg/s at a temperature of 5 °C. The design conductances are a U_a of 300 W/m^2-°C and a U_w of 1800 W/m^2-°C and the heat transfer areas are 200 m^2 on the air side and 40 m^2 on the water side. Generate a performance map of the coil heat transfer rate, outlet air temperature, and humidity as functions of

a. The air flow rate over the range of 3000 to 15,000 L/s. The air side conductance varies with flow rate as:

$$\frac{U}{U_{design}} = \left(\frac{\dot{m}}{\dot{m}_{design}}\right)^{0.8}$$

b. Air inlet conditions varying from 25 °C and 30% RH to 40 °C and 60% RH.
c. Draw some conclusions from your results.

2.21 For the cooling coils of Table 2.2,

a. Extrapolate the performance for 2 m/s to conditions of 25 °C/15 °C.
b. For the 6-row coil, determine the heat transfer rate per unit face area and the outlet conditions for 400 fpm over a range of inlet dry-bulb temperatures of 20 to 35 °C and a wet-bulb temperature of 15 °C.
c. Draw some conclusions from your analysis.

2.22 Design a cooling coil that transfers 100 kW of cooling for design conditions that are inlet dry- and wet-bulb temperatures of 30 °C and 20 °C, respectively. The air flow rate is 4000 L/s and the supply temperature should be in the range of 10 to 25 °C. Chilled water is available at 5 °C and the chilled water return should be in the range of 10 to 15 °C. Choose one of the surfaces given in Figure 13.9 for your design. Account for the different fin efficiencies for heat and energy. Summarize your design and specify the air face velocity, coil frontal area and thickness, the air outlet temperature and humidity ratio, the water flow rate, and the air side pressure drop. Draw some conclusions from your results.

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