

## Chapter SM 1 Heat Exchangers for Heating Applications

### SM 1.1 Heat Exchanger Thermal Performance

In designing or analyzing a heat exchanger or selecting one for an application, it is usually necessary to establish the rate of heat transfer for a set of flow rates and entering conditions. The heat flow is typically determined using either the **effectiveness-Ntu** or the **log-mean-temperature-difference** method. The effectiveness-Ntu method is based on thermodynamic limits for exchanger performance and is summarized first. These effectiveness-Ntu performance relations, which are useful for sensible heat exchangers, can be extended to exchangers in which water is condensed or evaporated, such as cooling coils and cooling towers. The development of the relations for heat exchanger performance can be found in standard heat transfer references (e.g., Kays and London, 1964; Incropera et al., 2011) and only the basic relations relevant to HVAC applications are reviewed in this section.

A counterflow exchanger geometry is used as the model for developing the heat transfer relations. A schematic of the cross-section of a counterflow exchanger is shown in Figure 1.1. One stream enters from the left and the other from the right. The coordinate  $A$  represents the heat transfer surface area in the flow direction.

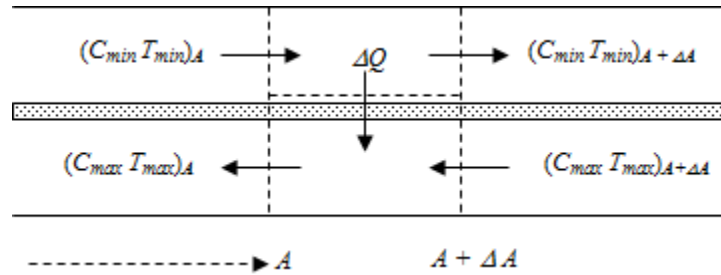


Figure 1.1 Energy flows for a counterflow heat exchanger

The thermal energy carried by either flow can be represented by the product of the mass flow rate and specific heat, termed the thermal capacitance rate. The two streams are identified by whether they have the minimum or the maximum value of capacitance rate. The two thermal flows are then

$$C_{\min} = \dot{m}_{\min} c_{p,\min} \quad \text{and} \quad C_{\max} = \dot{m}_{\max} c_{p,\max} \quad (1.1)$$

The overall energy balance on the control volume in Figure 1.1 that includes both streams is

$$(C_{\min} T_{\min})_A + (C_{\max} T_{\max})_{A+\Delta A} - (C_{\min} T_{\min})_{A+\Delta A} - (C_{\max} T_{\max})_A = 0 \quad (1.2)$$

Rearranging the terms, dividing by  $\Delta A$ , and passing to the limit of  $\Delta A$  approaching 0 yields an overall energy balance equation relating the change in temperature of the two streams as they flow through the exchanger:

$$C_{\min} \frac{dT_{\min}}{dA} = C_{\max} \frac{dT_{\max}}{dA} \quad (1.3)$$

An energy balance is also performed on a control volume that includes the stream with the minimum capacitance rate and the heat flow between the streams. The heat transfer between the two streams is represented using the heat flux, conductance-area product, and temperature difference:

$$\Delta Q = U \Delta A (T_{\min} - T_{\max}) \quad (1.4)$$

The area increment  $\Delta A$  is that on which the conductance is based (i.e., either the area on the minimum or maximum capacitance fluid side). Forming the energy balance, dividing by  $\Delta A$ , and passing to the limit yields a relation between the change of temperature of the minimum capacitance rate stream and the temperature difference between the two streams:

$$C_{\min} \frac{dT_{\min}}{dA} = -U (T_{\min} - T_{\max}) \quad (1.5)$$

Equations 1.3 and 1.5 can be combined to eliminate one of the temperatures and then integrated to yield the temperatures of the streams leaving the heat exchanger. Although the integration is relatively straightforward for counter- and parallel-flow exchangers, it is tedious. Cross-flow and other exchangers are more complex since the temperatures of each fluid vary both in the flow direction and normal to the flow direction and the integration needs to be carried out in two dimensions. The details of the integration are given in Kays and London, 1964.

With the outlet temperatures of each stream determined, the heat transfer can be found from an overall energy balance on either of the streams. The stream that enters with the higher temperature is termed the “hot” stream; that with the colder inlet temperature is the “cold” stream. In terms of the capacitance rates and the temperature change of either stream, the heat transfer rate can be expressed as:

$$\dot{Q} = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i}) \quad (1.6)$$

where the subscripts  $h$  and  $c$  denote the hot and cold streams, respectively, and  $i$  and  $o$  denote the inlet and outlet temperatures, respectively.

The effectiveness is defined as the ratio of the actual heat transfer rate to the maximum possible rate:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} \quad (1.7)$$

The maximum possible heat transfer would occur for a heat exchanger in which the stream with minimum capacitance rate is brought to the inlet temperature of the stream with maximum capacitance rate. The maximum heat transfer is then:

$$\dot{Q}_{\max} = C_{\min} (T_{h,i} - T_{c,i}) \quad (1.8)$$

Using the definition of effectiveness, the overall heat transfer rate can be expressed as:

$$\dot{Q} = \varepsilon C_{\min} (T_{h,i} - T_{c,i}) \quad (1.9)$$

The capacitance rate ratio is the ratio of the minimum capacitance rate to the maximum:

$$C^* = \frac{C_{\min}}{C_{\max}} \quad (1.10)$$

The number of transfer units is defined as the ratio of the overall heat transfer conductance and the minimum capacitance rate:

$$Ntu = \frac{UA}{C_{\min}} \quad (1.11)$$

The heat exchanger effectiveness is a function of  $Ntu$ ,  $C^*$ , and flow arrangement. Graphical presentations and equations for effectiveness in terms of  $Ntu$  and  $C^*$  are given in the text within Chapter 13, Section 13.3.

### ***SM 1.2 Extended Surfaces***

A coil is commonly used to transfer heat between a liquid such as water and a gas such as air. The much higher heat transfer coefficient on the water side leads to a low thermal resistance, while the thermal resistance on the air side is large due to the low heat transfer coefficient. The performance can be significantly improved by reducing the air-side resistance. One method to reduce the air-side resistance is to increase the surface area by adding extended surfaces, commonly called fins. Fins are thin sheets of relatively high conductivity material that conducts heat from the surface and transfers it to the air stream by convection. Figure 1.2 illustrates three

common heat exchanger geometries that are found in HVAC systems. On the left is a tube with circular fins, in the center is the water tube of a heat exchanger with straight metal fins extending from the surface, and on the right is an array of tubes that pass through a series of flat metal sheets.

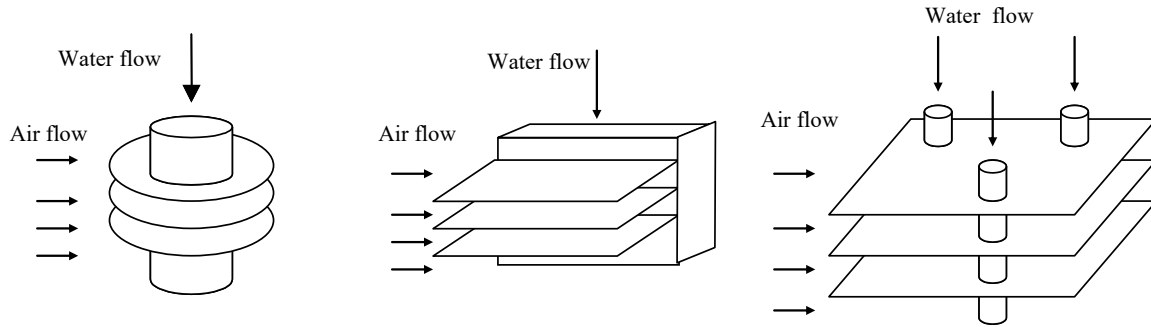


Figure 1.2 Finned tube and straight fin arrangements

The two heat transfer paths from the tube or wall surface to the air stream are convection directly from the bare surface (prime area) to the air and conduction from the base of the fin through the fin material and then convection from the surface of the fin to the air. It is conventional and convenient to represent the combined conduction and convection heat transfer for the fins using a fin efficiency, which is defined as the ratio of the actual heat transfer to that of an ideal fin that has a uniform temperature equal to the base temperature along its length. The heat transfer rate from a fin is then given in terms of the fin efficiency and the temperature at the base of the fin as

$$\dot{Q}_f = \eta_f h_{c,o} A_f (T_b - T_o) \quad (1.12)$$

where  $\eta_f$  is the fin efficiency,  $h_{c,o}$  the convection coefficient for the fin surface,  $A_f$  the total surface area of the fins,  $T_b$  the base temperature, and  $T_o$  the air temperature. Convection from the prime area to the air occurs in parallel with the convection from the fin surface. Assuming that the temperature of the tube surface is the same as that of the base of the fins ( $T_b$ ) and that the heat transfer coefficient is the same for all surfaces gives the total heat flow rate to the air as the sum of that from the fins and the prime area, or

$$\dot{Q} = h_{c,o} A_p (T_b - T_o) + \eta_f h_{c,o} A_f (T_b - T_o) \quad (1.13)$$

It is also convenient to define an overall surface efficiency for the transfer of heat from both the prime and finned areas, which allows the heat transfer to the air to be expressed as

$$\dot{Q} = \eta_o h_o A_o (T_b - T_a) \quad (1.14)$$

where  $A_o$ , the total area on the air side, is the sum of the fin and prime areas. The overall efficiency is given in terms of the fin area, total area, and fin efficiency as

$$\eta_o = 1 - \frac{A_f}{A_o}(1 - \eta_f) \quad (1.15)$$

The overall resistance for heat transfer from the outside surface of the tube to the air can then be expressed in terms of the overall surface efficiency, the outside convection coefficient, and the total outside surface area:

$$R_o = \frac{1}{\eta_o h_{c,o} A_o} \quad (1.16)$$

Equation 1.16 is an especially convenient way to represent the heat transfer from a fin. It allows the combined convection and conduction heat transfer flows from the fin and the bare surface to be represented by a single parameter. The efficiency of an individual fin is a function of the fin geometry, material, thickness, length, and the external heat transfer coefficient. For a rectangular fin attached to a flat surface such as in the center of Figure 1.2, the fin efficiency is given by:

$$\eta_f = \frac{\tanh(m_f L_f)}{m_f L_f} \quad (1.17)$$

where  $L_f$  is the length of the fin, which is the distance from the base to the tip of the fin. The parameter  $m_f$  is a function of the convection heat transfer coefficient  $h_o$ , the thermal conductivity of the fin material  $k_f$ , and the fin thickness  $t_f$ , and is given by

$$m_f = \sqrt{\frac{2h_{c,o}}{k_f t_f}} \quad (1.18)$$

Equation 1.18 applies to fins that do not convect heat from the tip, such as fins that extend between two walls. Heat is not conducted across the centerline and the fin length is one-half the distance between the walls. If the tip of the fin is exposed to air, as for the finned wall of Figure 1.2, the heat transfer from the tip area needs to be accounted for. It is an acceptable approximation to use equation 1.17 with the fin length increased by adding one-half of the fin thickness to the actual length. The effective fin length  $L_c$  when the tip convects heat is then given by

$$L_c = L_f + \frac{t_f}{2} \quad (1.19)$$

The fin area is then computed using the effective fin length  $L_c$ .

The fin efficiency expression for the circular fins shown on the left of Figure 1.2 contains Bessel functions instead of the hyperbolic functions, and the fin efficiency is computed as

$$\eta_f = \left[ \frac{2r_i}{m_f(r_{o,c}^2 - r_i^2)} \right] \left[ \frac{K_1(m_f r_i) I_1(m_f r_{o,c}) - I_1(m_f r_i) K_1(m_f r_{o,c})}{I_0(m_f r_i) K_1(m_f r_{o,c}) + K_0(m_f r_i) I_1(m_f r_{o,c})} \right] \quad (1.20)$$

where  $I_0$ ,  $I_1$ ,  $K_0$ , and  $K_1$  are Bessel functions and  $r_i$  and  $r_o$  are the inner and outer radii of the fin, respectively. There is usually convection from the tips of circular fins, and an effective outer radius  $r_{o,c}$  is used that is defined as the outer radius plus half of the thickness:

$$r_{o,c} = r_o + \frac{t_f}{2} \quad (1.21)$$

The fin area is then computed using the effective outer radius. For a circular fin with both sides exposed to the air, the fin area is:

$$A_f = 2\pi(r_{o,c}^2 - r_i^2) \quad (1.22)$$

The fin efficiency for the fins on the right side of Figure 1.3 is obtained by approximating the fins as circular fins. The total fin area is divided up so that equal areas surround each tube, as shown in Figure 1.3. The rectangular (or other) fin area surrounding each tube is converted into an equivalent circular fin with the same area as the actual area. The parameter  $m_f$  is computed with the equivalent outer radius since the “tip” is insulated, and the fin efficiency is computed with equation 1.20.

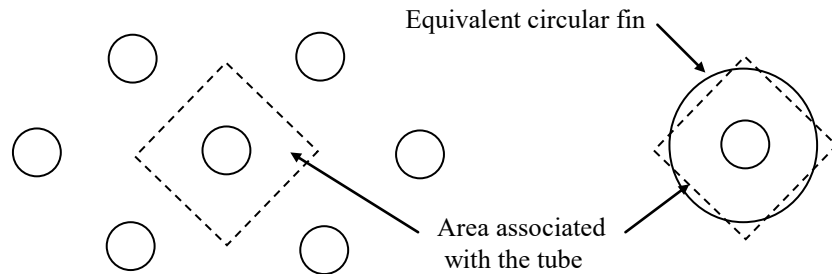


Figure 1.3 Fin area associated with a tube

The fin efficiency is presented in Figure 1.4 as a function of the term  $m_f L_c$  for rectangular fins and  $m_f (r_{o,c} - r_i)$  for circular fins. (Fin efficiencies for additional geometries such as pin and tapered fins are presented in Chapter 3, 2009 *ASHRAE Handbook—Fundamentals*). As shown in Figure 1.4, the fin efficiency decreases as the fin length is increased, as the thickness is decreased, or as the thermal conductivity is reduced. The fin efficiency for a circular fin approaches that of a straight fin as the fin length becomes small relative to the tube radius.

However, fin efficiency is not the only measure for heat transfer, since the area is also important, as shown by equation 1.14. For example, shorter fins will transfer less heat than longer fins even though the efficiency is higher because the surface area is less. There are design tradeoffs between the material costs for added area and the savings due to greater heat transfer. In general, fins are designed to have fin efficiencies of between about 0.6 and 0.9. Example 1.1 illustrates the determination of the fin efficiency and overall resistance for a finned surface.

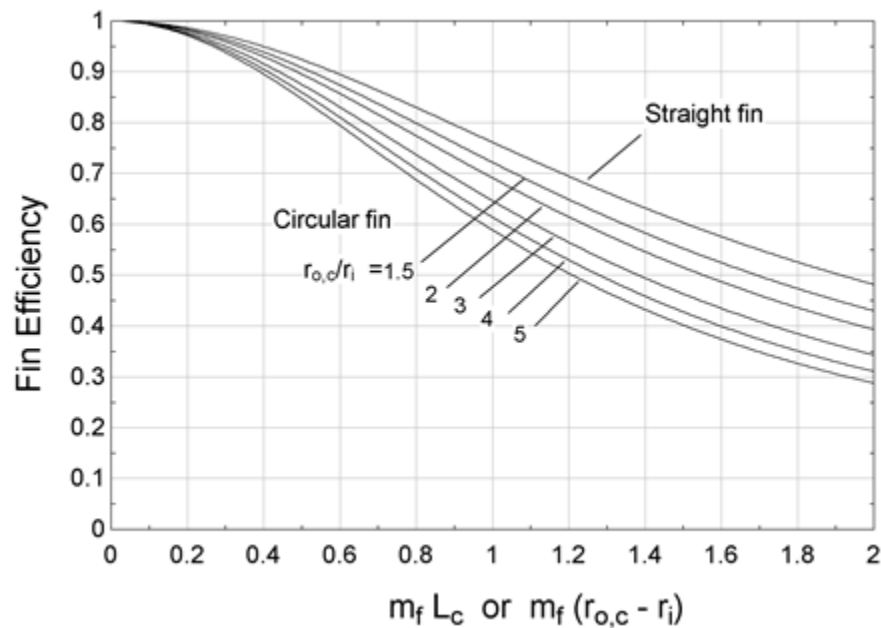


Figure 1.4 Fin efficiency as a function of fin parameters

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Example 1.1 Determine the fin efficiency, overall surface efficiency, and thermal resistance per foot of tube length for a cross-flow heat exchanger using finned tubes. The tube diameter is 0.774 in. The fins are steel with a thickness of 0.012 in., a diameter of 1.463 in., and a pitch of 9.05 fins per inch. The conductivity of steel is 35 Btu/hr-ft<sup>2</sup>-°F. The heat transfer coefficient is 14.4 Btu/hr-ft<sup>2</sup>-°F.

"Problem specifications:"

D<sub>i</sub> = 0.774\*convert(in,ft) "ft"

D<sub>o</sub> = 1.463\*convert(in,ft) "ft"

"Tube diameter"

"Fin outer diameter"

|  |                    |
|--|--------------------|
| $t = 0.012 \cdot \text{convert}(\text{in}, \text{ft}) \text{ "ft"}$      | "Fin thickness"    |
| $p = 9.05 \cdot \text{convert}(1/\text{in}, 1/\text{ft}) \text{ "1/ft"}$ | "Fin pitch"        |
| $k = 35 \text{ "Btu/hr-ft-F"}$   | "Fin conductivity" |
| $h_o = 14.4 \text{ "Btu/hr-ft}^2\text{-F"}$                              | "Heat trans coeff" |

"Determine the fin parameters: effective radius for a fin with convection from the tip (equation 1.21), the equivalent mL of the circular fin (equation 1.18), and the radius ratio for the fin."

|  |                     |
|--|---------------------|
| $r_i = D_i/2 \text{ "ft"}$                               | "Tube radius"       |
| $r_o = D_o/2 \text{ "ft"}$                               | "Fin radius"        |
| $r_c = r_o + t/2 \text{ "ft"}$                           | "Equivalent radius" |
| $m_f = ((2 \cdot h_o)/(k \cdot t))^{0.5} \text{ "1/ft"}$ | "Fin parameter"     |
| $mfr = m_f \cdot (r_c - r_i)$                            | "Fin parameter"     |
| $r_{\text{Ratio}} = r_c/r_i$                             | "Radius ratio"      |

"Determine the fin efficiency using equation 1.20."

|   |                  |
|---|------------------|
| $A = (2 \cdot r_i)/(m_f \cdot (r_c^2 - r_i^2))$   |                  |
| $B = \text{BesselK}(1, m_f \cdot r_i) \cdot \text{BesselI}(1, m_f \cdot r_c) - \text{BesselI}(1, m_f \cdot r_i) \cdot \text{BesselK}(1, m_f \cdot r_c)$ |                  |
| $C = \text{BesselI}(0, m_f \cdot r_i) \cdot \text{BesselK}(1, m_f \cdot r_c) + \text{BesselK}(0, m_f \cdot r_i) \cdot \text{BesselI}(1, m_f \cdot r_c)$ |                  |
| $\eta_{\text{fin}} = A \cdot B/C$   | "Fin efficiency" |

"Determine the fin and total area per foot of length."

|   |                       |
|---|-----------------------|
| $L = 1 \text{ "ft"}$  | "Tube length"         |
| $N_{\text{fin}} = p \cdot L$  | "No. fins per foot"   |
| $A_{\text{fin}} = N_{\text{fin}} \cdot \pi \cdot (r_c^2 - r_i^2) \text{ "ft}^2\text{"}$             | "Fin area per foot"   |
| $A_{\text{prime}} = \pi \cdot 2 \cdot r_i \cdot (L - N_{\text{fin}} \cdot t) \text{ "ft}^2\text{"}$ | "Prime area per foot" |
| $A_{\text{total}} = A_{\text{fin}} + A_{\text{prime}} \text{ "ft}^2\text{"}$                        | "Total area per foot" |

"Determine the overall surface efficiency using equation 1.16 and the thermal resistance per foot of length."

|  |                       |
|--|-----------------------|
| $\eta_o = 1 - (A_{\text{fin}}/A_{\text{total}}) \cdot (1 - \eta_{\text{fin}})$ | "Overall efficiency"  |
| $R = 1/(\eta_o \cdot h_o \cdot A_{\text{total}}) \text{ "hr-ft-F/Btu"}$        | "Resistance per foot" |

## Results

The equivalent radius for the fins is 0.06146 ft and the fin parameter  $m_f$  is 28.7/ft. The non-dimensional fin parameter  $m_f(r_c - r_i)$  is then 0.838. The fin efficiency could be determined from Figure 1.4, but equation 1.20 in terms of Bessel functions is used. The value of the fin efficiency is determined to be 0.763.

For the given fin pitch there are 108.6 fins in a one-foot length. The fin, prime, and total areas per foot are 0.9338 ft<sup>2</sup>, 0.1806 ft<sup>2</sup>, and 1.114 ft<sup>2</sup>, respectively. The overall fin efficiency given using equation 1.20 is 0.801. The thermal resistance per unit length is 0.0778 hr-°F/Btu.

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### ***SM 1.3 Single Phase Heat Transfer Coefficients and Pressure Drop***

The flow situation for heat exchangers is generally forced convection internal flow with the working fluid usually air, water, or refrigerant. Heat transfer coefficients for forced convection internal flow in circular tubes are presented in Chapter 4, Section 4.3, and the friction factor is covered in Chapter 3, Section 3.8. The turbulent flow relations from these sections are included in this chapter to provide more complete coverage of heat exchanger analysis.

The Reynolds number is defined in terms of the fluid properties and hydraulic diameter as

$$Re_{D_H} = \frac{\rho_f V D_H}{\mu_f} \quad (1.23)$$

where  $\rho_f$  is the fluid density,  $\mu_f$  the fluid viscosity,  $V$  the fluid velocity, and  $D_H$  the hydraulic diameter. The hydraulic diameter is defined in terms of the cross-sectional area of the tube  $A_c$  and the wetted perimeter  $WP$ :

$$D_H = \frac{4A_c}{WP} \quad (1.24)$$

The hydraulic diameter is the actual diameter for a circular tube. For other tube geometries such as the oval tubes or rectangular passages often found in heat exchangers, the hydraulic diameter allows the circular tube relations to be used to estimate the heat transfer coefficient. It is often convenient to express the Reynolds number in terms of the mass flow rate and the wetted perimeter as

$$Re_{D_H} = \frac{4\dot{m}}{WP\mu_f} \quad (1.25)$$

The heat transfer coefficient is expressed in terms of the Nusselt number, defined as

$$Nu_{D_H} = \frac{h_c D_H}{k_f} \quad (1.26)$$

where  $h_c$  is the convection coefficient and  $k_f$  is the thermal conductivity of the fluid. The friction factor  $f$  is a nondimensional representation of the pressure drop for the flow. With the friction factor, the pressure drop over the flow length can be determined as

$$\Delta p = f \frac{L}{D_H} \frac{\rho_f V^2}{2} \quad (1.27)$$

where  $L$  is the length in the flow direction and  $V$  is the fluid velocity. It is sometimes more convenient to use the mass flux, which is the mass flow rate per unit area, defined as

$$G = \frac{\dot{m}}{A_c} \quad (1.28)$$

The pressure drop relation can then be expressed as

$$\Delta p = f \frac{L}{D_H} \frac{G^2}{2\rho} \quad (1.29)$$

This set of relations allows the heat transfer coefficient and pressure drop for the single-phase flow of a fluid in the tubes of a heat exchanger to be determined.

Table 1.1 Heat transfer and friction factor relations for internal turbulent flow  
(Reynolds number greater than 2,500)

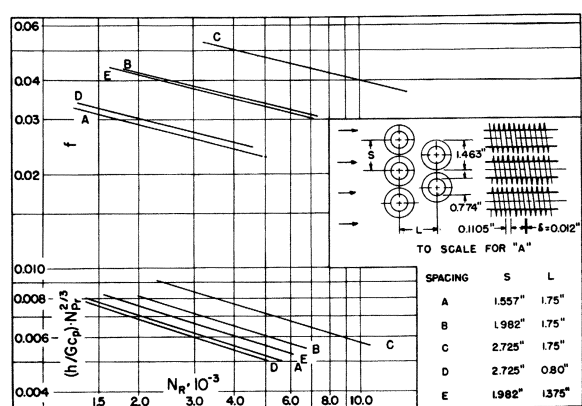
|                      |   |  |
|----------------------|---|--|
| Smooth tubes         | $Nu_{D_H} = 0.023 Re_{D_H}^{0.8} Pr^n$<br>$n = 0.4$ heating or $n = 0.3$ cooling              | $f = \frac{0.3164}{Re_{D_H}^{0.25}}$   |
| Or (larger Re range) | $Nu_{D_H} = \frac{(f/2)(Re_{D_H} - 1000)Pr}{1 + 12.7(f/2)^{1/2}(Pr^{2/3} - 1)}$               | $f = 0.0032 + \frac{0.221}{Re_{D_H}^{0.237}}$  |
| Rough tubes          | $Nu_{D_H} = \frac{(Re_{D_H} Pr (f/2))}{1 + (f/2)^{1/2} (4.5 Re_{D_H}^{0.2} Pr^{0.5} - 8.48)}$ | $\overline{Re_{D_H} < 10^6}$<br>$f^{-0.5} = 1.14 + 2 \log \left( \frac{D_H}{\varepsilon} \right) - 2 \log \left( 1 + \frac{9.3}{Re_{D_H} \left( \frac{\varepsilon}{D_H} \right)} f^0 \right)$<br>$\overline{Re_{D_H} > 10^6}$<br>$f^{-0.5} = 1.14 + 2 \log \left( \frac{D_H}{\varepsilon} \right)$ |

### SM 1.4 Extended Surface Heat Transfer Coefficients and Pressure Drop

Heat exchangers are often composed of finned tubes with a fluid, usually air, flowing over the outside of the tubes. The convection relations for the simple geometries of plates and circular tubes presented in Chapter 4, Section 4.3, are for a single surface in an undisturbed flow and do not accurately represent the heat transfer coefficients for a heat exchanger made of a number of finned tubes because there are flow interactions between the flow over a fin and the other fins and the tubes. The second row of tubes, for example, is in the wake region behind the first row, and the heat transfer coefficient is increased due to turbulence. The relations for the heat transfer

coefficients for the air side of the type of heat exchanger surface found in coils are presented in this section.

The heat transfer coefficients for the surfaces found in coils are usually determined experimentally and presented either graphically or as equations in terms of governing dimensionless parameters. Figure 1.5, from *Compact Heat Exchangers* (Kays and London, 1964), gives heat transfer and friction factor data as a function of Reynolds number for a number of finned tube configurations representative of heating and cooling coils. Each figure presents data for tubes of the same diameter, fin length, and fin pitch, but with different longitudinal ( $L$ ) and transverse ( $S$ ) pitches of the tubes. The tubes in Figure 1.5a have smaller diameters than those in Figure 1.5b. A number of the important geometric parameters that describe the surface are given below the graphs.



Tube outside diameter = 0.774 in.

Fin pitch = 9.05 per in.

Fin thickness = 0.012 in.

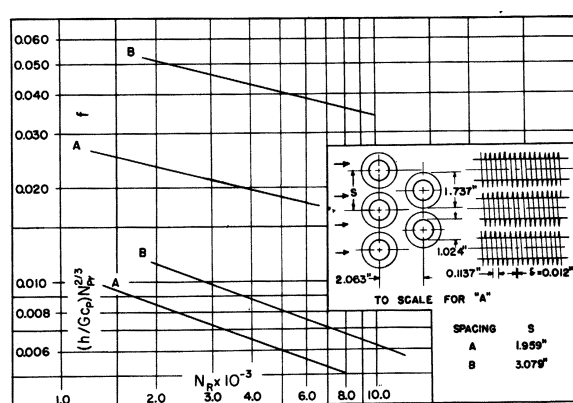
Fin area/total area = 0.835

|   | A       | B       | C      | D       | E          |
|---|---------|---------|--------|---------|------------|
| Flow passage hydraulic diameter, $4r_h$ = | 0.01681 | 0.02685 | 0.0445 | 0.01587 | 0.02108 ft |

|   | A     | B     | C     | D     | E     |
|---|-------|-------|-------|-------|-------|
| Free-flow area/frontal area, $\sigma$ = | 0.455 | 0.572 | 0.688 | 0.537 | 0.572 |

|   | A   | B    | C    | D   | E                                    |
|---|-----|------|------|-----|--------------------------------------|
| Heat transfer area/total volume, $\alpha$ = | 108 | 85.1 | 61.9 | 135 | 108 ft <sup>2</sup> /ft <sup>3</sup> |

Note: Minimum free-flow area in all cases occurs in the spaces transverse to the flow, except for D, in which the minimum area is in the diagonals.



Tube outside diameter = 1.024 in.

Fin pitch = 8.8 per in.

Fin thickness = 0.012 in.

Fin area/total area = 0.825

|   | A       | B        |
|---|---------|----------|
| Flow passage hydraulic diameter, $4r_h$ = | 0.01927 | 0.443 ft |

|                                    | A     | B     |
|------------------------------------|-------|-------|
| Free-flow/frontal area, $\sigma$ = | 0.439 | 0.643 |

|   | A    | B                                     |
|---|------|---------------------------------------|
| Heat transfer area/total volume, $\alpha$ = | 91.2 | 58.1 ft <sup>2</sup> /ft <sup>3</sup> |

Note: Minimum free-flow area is in spaces transverse to flow.

Figure 1.5 Friction factor and Stanton number for finned tube geometries: (a) tube diameter of 0.774 in., (b) tube diameter of 1.024 in.

Many heating and cooling coils use heat exchanger surfaces that are similar to the circular finned tubes of Figure 1.5. The tubes may be flattened or oval or with fins in the form of a continuous sheet extending between the tubes. The fins may be wavy, slotted, or perforated in an attempt to increase the heat transfer. The heat transfer relations are usually presented in a format similar to Figure 1.5 and with the same type of information as provided at the bottom of the figure.

A number of terms refer to the geometric properties of these surfaces. On the air side of a coil, the **face** or **frontal area** is the area normal to the flow and the **face velocity** is the velocity of the air perpendicular to this area. The **number of tube rows** is in the direction of the airflow,

with each row containing a number of parallel tubes that are the same distance from the inlet. **Fin pitch** is the number of fins per unit length along the tube.

These surfaces have a flow area and a flow velocity that characterize the heat transfer. Usually, the velocity is based on the average cross-sectional area in the coil, often called the **free-flow area**, rather than the frontal area. This area is referenced to the frontal area by the ratio of the free-flow to frontal area,  $\sigma$ , defined as

$$\sigma = \frac{A_c}{A_{fr}} \quad (1.30)$$

where  $A_c$  is the free-flow area and  $A_{fr}$  the frontal area. The value of  $\sigma$  is given in Figure 1.5 for each of the coil configurations. The characteristic velocity is a **mass flux** defined as the mass flow rate divided by the free-flow area (equation 1.28). The heat transfer and friction data are correlated in terms of a Reynolds number based on mass flux, hydraulic diameter, and fluid viscosity. The hydraulic diameter, which equals four times the hydraulic radius, is tabulated on Figure 1.5. The Reynolds number is defined as

$$N_R = Re_{D_H} = \frac{D_H G}{\mu} \quad (1.31)$$

where  $N_R$  is the notation used in Figure 1.5 for Reynolds number and  $G$  is the mass velocity.

The heat transfer coefficient is often represented by the Stanton number rather than the Nusselt number. As seen in Figure 1.5, both Stanton number and friction factor have the same dependency on Reynolds number. The Stanton number,  $St$ , is defined in terms of the heat transfer coefficient, mass flux, and fluid specific heat as

$$St = \frac{h_c}{G c_p} \quad (1.32)$$

where  $h_c$  is the convection coefficient and  $c_p$  is the specific heat. The Prandtl number raised to the two-thirds power is incorporated to correlate the properties of different fluids.

The pressure drop through the heat exchanger is represented by the friction factor, which is defined in terms of an equivalent shear stress that represents the fluid force on the heat exchanger surfaces. The equivalent shear stress is a combination of viscous shear forces (skin friction) and pressure forces (form drag). The friction factor  $f$  is defined as

$$f = \frac{\rho \tau_0}{G^2 / 2} \quad (1.33)$$

where  $\tau_0$  is the equivalent shear stress. The friction factor is related to the pressure drop

$$\Delta p = f \frac{A}{A_c} \frac{G^2}{2 \rho} \quad (1.34)$$

where  $A$  is the total heat transfer area of the exchanger on the air side and  $\rho$  is the density. The ratio  $A/A_c$  is the equivalent to  $L/D_H$  in equation 1.29. There are also entrance and exit pressure drops associated with the flow through an exchanger, and these are not included in either the equivalent shear stress or the friction factor.

A representation such as Figure 1.5 gives the measured overall convective heat transfer coefficient for the entire surface. In using these values for design or performance evaluation, the effect of the fin efficiency must be taken into account.

The information presented in Figure 1.5 is for a type of heat transfer surface that has been used for many years. It is relatively simple to construct, but it is not the most effective in terms of the amount of material, size, and pressure drop. Heat exchanger design has evolved to include many heat transfer enhancements through improvements in surface geometry. Modern fin designs produce high heat transfer rates with low pressure drop. These designs include louvered, interrupted, or staggered fin geometries that promote thin boundary layers and mixing of the flow, and oval or flattened tubes that reduce pressure drop. However, the basic methods for determining the heat transfer coefficient and pressure drop are the same. Example 1.2 demonstrates the use of the relations in Figure 1.5 to determine the performance of the surface.

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**Example 1.2** Determine the air-side convective heat transfer coefficient, thermal resistance, and pressure drop for a coil made of finned tubes with configuration B of Figure 1.5a. The coil frontal area is 4 ft<sup>2</sup>, there are four rows of coils, and the fins are made of aluminum. The airflow is 4000 cfm at a temperature of 75 °F and 50% relative humidity.

"Problem specifications"

p\_atm = 14.7 "psia"

T\_a = 75 "F"

Rh = 0.5

V\_dot = 4000 "cfm"

A\_fr = 4 "ft2"

N\_row = 4

"Atm pressure"

"Air temperature"

"Air relative humidity"

"Volume flow rate"

"Coil frontal area"

"No. of tube rows"

"From Figure 1.5a, determine the surface parameters sigma, alpha, hydraulic diameter, and spacing for each row."

sigma = 0.572

alpha = 85.1 "ft"

D\_H = 0.02685 "ft"

Spacing = 1.75\*convert(in,ft) "ft"

"Free-flow/frontal area"

"HT area/volume"

"Hydraulic diameter"

"Row spacing"

"Determine the volume from the product of the frontal area and flow length of a four-row coil. Determine the free flow area from the parameter sigma and the frontal area, and the heat transfer area from alpha and the frontal area."

|  |                  |
|--|------------------|
| $L = N_{\text{row}} * \text{Spacing}$ "ft"                 | "Flow length"    |
| $\text{Vol} = A_{\text{fr}} * L$ "ft <sup>3</sup> "        | "Volume"         |
| $A_{\text{c}} = \sigma * A_{\text{fr}}$ "ft <sup>2</sup> " | "Free flow area" |
| $A_{\text{a}} = \alpha * \text{Vol}$ "ft <sup>2</sup> "    | "HT area"        |

"Determine the air properties and the mass flow rate of the air."

|  |                  |
|--|------------------|
| $cp = \text{specheat}(\text{airh2o}, p=p_{\text{atm}}, T=T_{\text{a}}, R=\text{RH})$ "Btu/lbm-F"             | "Specific heat"  |
| $\rho = \text{density}(\text{airh2o}, p=p_{\text{atm}}, T=T_{\text{a}}, R=\text{RH})$ "lbm/ft <sup>3</sup> " | "Density"        |
| $\mu = \text{viscosity}(\text{airh2o}, p=p_{\text{atm}}, T=T_{\text{a}}, R=\text{RH})$ "lbm/hr-ft"           | "Viscosity"      |
| $k = \text{conductivity}(\text{airh2o}, p=p_{\text{atm}}, T=T_{\text{a}}, R=\text{RH})$ "Btu/hr-ft-F"        | "Cond."          |
| $Pr = cp * \mu / k$  | "Prandtl number" |
| $m_{\text{dot}} = V_{\text{dot}} * \rho * \text{convert}(1/\text{min}, 1/\text{hr})$ "lbm/hr"                | "Mass flow rate" |

"Determine the mass velocity G (equation 1.28) for the flow and the Reynolds number (equation 1.31). The product of the Stanton number and Prandtl number raised to the 2/3 power is found from Figure 1.5a to be 0.0063. Using this value with the definition of the Stanton number, determine the overall heat transfer coefficient."

|   |                       |
|---|-----------------------|
| $G = m_{\text{dot}} / A_{\text{c}}$ "lbm/hr-ft <sup>2</sup> " | "Mass velocity"       |
| $Re = G * D_{\text{H}} / \mu$                                 | "Reynolds number"     |
| $StPr = 0.0063$   | "Stanton-Prandtl No." |
| $StPr = (h_{\text{o}} / (G * cp)) * Pr^{(2/3)}$               | "Stanton-Prandtl No." |

"The surface is the same as used in Example 1.1 and the overall heat transfer coefficient is the same. The fin efficiency determined in Example 1.1 is 0.801 and is used here. The overall thermal resistance is based on the total heat transfer area for the coil."

|   |                      |
|---|----------------------|
| $\eta_{\text{a}} = 0.801$   | "Overall efficiency" |
| $R_{\text{a}} = 1 / (\eta_{\text{a}} * h_{\text{o}} * A_{\text{a}})$ "hr-F/Btu" | "Overall resistance" |

"Determine the pressure drop from equation 1.34. The friction factor is determined from the Figure 1.5a to be 0.0034. The pressure drop is converted to in water to allow comparison to other HVAC component pressure drops."

|   |                   |
|---|-------------------|
| $f = 0.0034$  | "Friction factor" |
| $\Delta p = f * (A_{\text{a}} / A_{\text{c}}) * G^2 / (2 * \rho) * \text{convert}(\text{lbm/hr}^2\text{-ft}, \text{psi})$ "psi" | "Pressure drop"   |
| $\Delta p_{\text{in}} = \Delta p * \text{convert}(\text{psi}, \text{inh2o})$ "in. water"  | "Pressure drop"   |

## Results

Using the flow length for the coil and the frontal area, the volume is found to be 2.33 ft<sup>3</sup>. Using the values of  $\sigma$  and  $\alpha$  from Figure 1.5a, the free-flow area is found to be 2.29 ft<sup>2</sup> and the heat transfer area is 198.6 ft<sup>2</sup>.

The mass flow rate is 17,550 lbm/hr and the mass velocity is determined to be 7,670 lbm/hr-ft<sup>2</sup>. From the hydraulic diameter, the Reynolds number is 4620. From surface B on Figure 1.5a, the value of  $St Pr^{2/3}$  is 0.0063. The heat transfer coefficient is determined using equation 1.32 is 14.4 Btu/hr-ft<sup>2</sup>-°F.

The fin and overall efficiency were calculated in Example 1.1, which uses the fin and tube geometry and heat transfer coefficient of this example. The fin efficiency is 0.763 and the overall efficiency is 0.801. The total thermal resistance on the air side determined from the total area and the overall surface efficiency is 0.000435 hr-F/Btu. This value could be combined with the convection thermal resistance at the inside surface of the tube and the thermal resistance of the tube wall to yield the overall conductance-area product for the coil.

At a Reynolds number of 4620, the friction factor for configuration B in Figure 1.9a is 0.034. The pressure drop determined using equation 1.34 is 0.00198 psi or 0.055 in. water. This pressure drop is only for the coil core. There would be entrance and exit losses in addition. In an HVAC system, the total pressure drop through the ductwork would probably be in the range of one to three inches of water. At this airflow rate the pressure drop through the coil would be a relatively small contributor to the overall pressure drop.

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### **SM 1.5 Summary**

Heat exchangers are a common component of HVAC and refrigeration systems. Cross flow exchangers are commonly employed when one or both of the fluids is a gas, as in water heating coils, chilled water cooling coils, and direct expansion coils. When both of the fluids are liquids, counter flow exchangers are typically used. For all heat exchanger types, the overall heat transfer conductance is an important parameter in determining the performance. The overall heat transfer conductance combines the thermal resistances due to convection from each fluid to the surface and conduction through the heat transfer surfaces. The convection heat transfer coefficients for heat exchanger surfaces are generally determined experimentally and presented graphically. Extended surfaces such as fins are often used to add area on the air or gas-side of the exchanger.

The fin efficiency is an important parameter that characterizes the effectiveness of extended surfaces in transferring heat to a fluid. The fin efficiency depends on the geometry of the fin, the fin material, and the convection heat transfer coefficient.

The effectiveness-Ntu approach provides a unified method for modeling sensible heat exchangers and evaluating the thermal performance. The effectiveness is a thermodynamic measure that relates the actual heat transfer to the maximum possible. The number of transfer units (Ntu) is a measure of the thermal size of the exchanger relative to the ability of the flow stream to transfer or receive energy. Another important parameter is the ratio of the thermal capacitance of the two fluid streams. These parameters facilitate the design and analysis of sensible heat exchangers.

### SM 1.6 Nomenclature

|             |   |                   |  |
|-------------|---|-------------------|--|
| $A$         | area  | $St$              | Stanton number                               |
| $A_c$       | cross-sectional area, free-flow area        | $t_f$             | fin thickness                                |
| $A_{fr}$    | frontal area                                | $T$               | temperature                                  |
| $c_p$       | specific heat                               | $U$               | overall unit thermal conductance             |
| $C$         | capacitance rate                            | $UA$              | overall thermal conductance                  |
| $C^*$       | capacitance rate ratio                      | $V$               | velocity                                     |
| $D_H$       | hydraulic diameter                          | $WP$              | wetted perimeter                             |
| $f$         | friction factor                             |                   |  |
| $F$         | heat exchanger correction factor            | $\Delta p$        | pressure drop                                |
| $G$         | mass flux                                   | $\varepsilon$     | effectiveness                                |
| $h_c$       | convection heat transfer coefficient        | $\eta_f$          | fin efficiency                               |
| $I_0, I_1$  | Bessel functions                            | $\eta_o$          | overall surface efficiency for heat transfer |
| $k$         | thermal conductivity                        |                   |  |
| $K_0, K_1$  | Bessel functions                            | $\mu_f$           | fluid viscosity                              |
| $L$         | length                                      | $\rho_f$          | fluid density                                |
| $L_c$       | equivalent fin length                       | $\sigma$          | free flow to frontal area ratio              |
| $L_f$       | fin length                                  | $\tau_o$          | wall shear stress                            |
| $m_f$       | fin parameter for heat transfer             |                   |  |
| $\dot{m}$   | mass flow rate                              | <b>Subscripts</b> |  |
| $N_R$       | Reynolds number                             | $b$               | base   |
| $Nu_{D_H}$  | Nusselt number based on hydraulic diameter  | $c$               | cold, equivalent                             |
|             |   | $f$               | fin, fluid                                   |
| $Ntu$       | number of transfer units for heat transfer  | $h$               | hot  |
| $\dot{Q}$   | heat flow rate                              | $i$               | inside, inner                                |
| $\dot{Q}''$ | heat flux (heat flow rate per unit area)    | $f$               | fin  |
| $r$         | radius                                      | $max$             | maximum                                      |
| $R$         | thermal resistance                          | $min$             | minimum                                      |
| $Re_{D_H}$  | Reynolds number based on hydraulic diameter | $o$               | outside, outlet, outer, overall              |
|             |   | $p$               | prime  |
|             |   | $t$               | tube   |
|             |   | $T$               | total  |

### SM 1.7 References

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### **Problems**

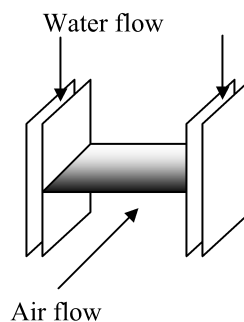
These problems are in addition to those in Chapter 13 and require the material in this chapter.

#### Problems in English units

- 1.1 A hot water heating coil is constructed of finned steel tubes that are 0.652 in. ID and 0.750 in. OD. There are copper fins with OD 1.5 inch, thickness 0.003 inches, and pitch 8 fins/inch. The air-side heat transfer coefficient is 17 Btu/hr-ft<sup>2</sup>-°F and the water heat transfer coefficient is 650 Btu/hr-ft<sup>2</sup>-°F.
  - a. Determine the unit conductances  $U_i$  and  $U_o$  based on the inside and outside areas, respectively.
  - b. Determine the total length of tubing necessary to have an overall conductance-area product of 20,000 Btu/hr-°F.
  - c. Draw some conclusions from your analysis.
  
- 1.2 A hot water heating coil is constructed of copper tubes that are 0.652 ID and 0.750 OD. The fins have OD 1.5 inch and thickness 0.003 inches. The air-side heat transfer coefficient is 17 Btu/hr-ft<sup>2</sup>-°F and the water transfer coefficient is 125 Btu/hr-ft<sup>2</sup>-°F. For a design that uses the surface in a cost-effective manner, the thermal resistances on the inside and outside of the tube should be about the same.
  - a. Determine the fin pitch and fin diameter that will produce about equal inside and outside resistances. The pitch should be less than 12/in. and the fin diameter less than 2 inches.
  - b. Determine the total length of tubing necessary to have an overall conductance-area product of 20,000 Btu/hr-°F.
  - c. Draw some conclusions from your analysis.
  
- 1.3 A hot water coil with two passes and an overall conductance of 100 Btu/hr-ft<sup>2</sup>-°F is designed to heat a flow of 2000 cfm of air from 75 °F to 90 °F. The design water flow rate is 20 gpm and the water enters at 140 °F.
  - a. Determine the heat transfer rate and the required surface area for design conditions.
  - b. Determine the variation in air outlet temperature as the air flow rate is varied between 500 and 2000 cfm under the assumption that the value of  $U$  is the design value.

- c. Determine the variation in air outlet temperature as the air flow rate is varied between 500 and 2000 cfm under the assumption that the  $Ntu$  is constant at the design value. The value of  $U$  will not be the design value.
  - d. Determine the variation in air outlet temperature as the air flow rate is varied between 500 and 2000 cfm under the assumption that the effectiveness is constant at the design value. The value of  $U$  will not be the design value.
  - e. Determine the variation in air outlet temperature as the air flow rate is varied between 500 and 2000 cfm with the assumption that the  $UA$  varies as the air flow rate to the 0.4 power. This models the effect on the heat transfer coefficient on the air side. The value of  $U$  will not be the design value.
  - f. Draw some conclusions from your analysis.
- 1.4 A coil is made of two rows of the finned tubes of surface A of Figure 1.5a. The air flow is 5000 cfm at a temperature of 75 °F and 50% relative humidity. The coil frontal area is 5 ft<sup>2</sup> and the fins are made of aluminum. The detailed dimensions of the fin are given in the table below the figure. Determine the convective heat transfer coefficient, the overall thermal resistance, the air-side conductance, and the pressure drop. Draw some conclusions from your analysis.
- 1.5 A coil is made using finned tubes of surface A of Figure 1.5a. The air flow of 5000 cfm enters at a temperature of 75 °F and 50% relative humidity. The coil frontal area is 5 ft<sup>2</sup>, there are two rows, and the fins are made of aluminum. The water flow is 20 gpm and enters at 140 °F.
- a. Determine the convective heat transfer coefficient and the overall thermal resistance for the air side of the exchanger.
  - b. Determine the convective heat transfer coefficient and the overall thermal resistance for the water side of the exchanger.
  - c. Determine the overall conductance area product, the exchanger effectiveness, and the air and water outlet temperatures.
  - d. Draw some conclusions from your analysis.
- 1.6 A coil is made with steel tubes and aluminum fins 0.012 in thick. The tubes have an inner diameter of 1.5 in. and an outer diameter of 1.6 in. and are oriented in a triangular grid (Figure 1.3). The pitch is 10 fins per inch of tube length. The heat transfer coefficient for the airflow over the fins is 20 Btu/hr-ft<sup>2</sup>-°F and that for the water inside the coil is 450 Btu/hr-ft<sup>2</sup>-°F. Determine the fin efficiency, overall surface efficiency, the overall transfer coefficient, and the transfer coefficient-area product for a coil with four rows in the flow direction, 20 rows high, and 4 feet in width over a range of spacing between the tubes of 2 to 4 inches. Draw some conclusions from your analysis.

- 1.7 The fins on a baseboard convector are made of 2 in. diameter aluminum sheets bonded to a 1 in. diameter copper pipe. The temperature of the hot water is 120 °F, the zone air temperature is 75 °F, and the natural convection heat transfer coefficient is 4 Btu/hr-ft<sup>2</sup>-°F.
- Determine the thickness of the aluminum plates necessary to have a fin efficiency of 85%.
  - Determine the overall fin efficiency for a spacing of 10 fins per inch of length.
  - Determine the length needed to provide 2000 Btu/hr to a room, which is a typical value.
  - If steel plates of the same thickness were substituted for aluminum, determine the heat transfer rate.
  - Draw some conclusions from your analysis.
- 1.8 Plate fin heat exchangers are made as series of parallel water channels connected by aluminum fins with an air flow is over the fin surface as shown in Problem 1.10 Figure.



Problem 1.10 Figure

- Determine the fin and overall efficiencies for a design with a channel spacing of 0.3 in., a fin length in the flow direction of 0.2 in., and a fin thickness of 0.004 in. for (a) aluminum fins and (b) steel fins. The fins are spaced 8 fins per inch. The air velocity varies over the range of 500 to 3000 fpm. Draw some conclusions from your results.
- 1.9 A hot water heating coil is constructed of copper tubes that are 0.652 ID and 0.750 OD. The fins have an OD of 1.5 in. and a thickness of 0.003 in. The air-side heat transfer coefficient is 17 Btu/hr-ft<sup>2</sup>-°F and the water transfer coefficient is 650 Btu/hr-ft<sup>2</sup>-°F. For a design that uses the surface in a cost-effective manner, the thermal resistances on the inside and outside of the tube should be about the same.
- Determine the fin pitch that will produce about equal inside and outside resistances.
  - Determine the overall conductance  $U$  based on the inside surface areas.
  - For an air flow of 5000 cfm entering at 60 °F and a water flow of 100 gpm entering at 120 °F, determine the total length of tubing necessary to heat the air to 80 °F in a two-row coil configuration.
  - Draw some conclusions from your analysis.

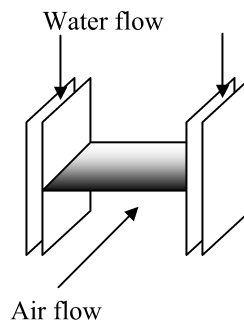
- 1.10 Design a coil made of copper tubes with steel fins in configuration C of Figure 1.5a to heat an air flow of 10,000 cfm from an entering temperature of 55 °F to a leaving temperature of 75 °F. The hot water is delivered from a boiler at a temperature of 160 °F and drops 15 °F through the coil.
- Determine the heating capacity of the coil
  - Determine the air-side area, heat transfer coefficient, velocity, overall thermal resistance, and pressure drop. It will help to curve fit the heat transfer relation from Figure 1.5a. This involves varying the number of rows and the face velocity so that the  $Ntu$  based on the air- and water-side resistances equals the  $Ntu$  required for the capacity. The air velocity should be in the range of 600 to 1200 ft/min.
  - Determine the water flow rate, heat transfer coefficient, and overall thermal resistance.
  - Determine the coil dimensions.
  - Draw some conclusions from your analysis.
- 1.11 Design a coil made of copper tubes with steel fins in configuration A of Figure 1.5a to heat an air flow of 10,000 cfm from an entering temperature of 55 °F to a leaving temperature of 80 °F. The hot water is delivered from a boiler at a temperature of 160 °F and drops 15 °F through the coil.
- Determine the heating capacity of the coil
  - Determine the air-side area, heat transfer coefficient, velocity, overall thermal resistance, and pressure drop. It will help to curve fit the heat transfer relation from Figure 1.5a. This involves varying the number of rows and the face velocity so that the  $Ntu$  based on the air and water-side resistances equals the  $Ntu$  required for the capacity. The air velocity should be in the range of 600 to 1200 ft/min.
  - Determine the water flow rate, heat transfer coefficient, and overall thermal resistance.
  - Determine the coil dimensions.
  - Determine the capacity of the coil, the outlet air temperature, and the air-side pressure drop as the airflow is varied from 5000 to 15000 cfm.
  - Draw some conclusions from your analysis.

### Problems in SI Units

- 1.12 A hot water heating coil is constructed of finned copper tubes that are 16 mm ID and 19 mm OD. The copper fins have an OD of 40 mm, a thickness of 0.07 mm, and a pitch of 2 fins/cm. The air-side heat transfer coefficient is  $95 \text{ W/m}^2\text{-}^\circ\text{C}$  and the water heat transfer coefficient is  $3500 \text{ W/m}^2\text{-}^\circ\text{C}$ .
- Determine the unit conductances  $U_i$  and  $U_o$  based on the inside and outside areas, respectively.
  - Determine the total length of tubing necessary to have an overall conductance-area product of  $10,000 \text{ W/}^\circ\text{C}$ .
  - Draw some conclusions from your analysis.

- 1.13 A hot water heating coil is constructed of copper tubes that are 1.6 mm ID and 1.9 mm OD. The fins have a thickness of 0.07 mm. The air-side heat transfer coefficient is  $95 \text{ W/m}^2\text{-}^\circ\text{C}$  and the water heat transfer coefficient is  $700 \text{ W/m}^2\text{-}^\circ\text{C}$ . For a design that uses the surface in a cost-effective manner, the thermal resistances on the inside and outside of the tube should be about the same.
- Determine the fin pitch and diameter that will produce about equal inside and outside resistances. The pitch should be less than 6/cm and the fin diameter less than 10 cm.
  - Determine the total length of tubing necessary to have an overall conductance-area product of  $10,000 \text{ W/}^\circ\text{C}$ .
  - Draw some conclusions from your analysis.
- 1.14 A hot water coil with an overall conductance of  $500 \text{ W/m}^2\text{-}^\circ\text{C}$  is designed to heat a flow of  $1000 \text{ L/s}$  of air from  $25^\circ\text{F}$  to  $35^\circ\text{C}$ . The design water flow rate is  $1.5 \text{ L/s}$  and the water enters at  $60^\circ\text{C}$ .
- Determine the heat transfer and the required surface area for design conditions.
  - Determine the required water flow rate for air flow rate values between 200 and  $1000 \text{ L/s}$  under the assumption that the value of  $U$  is the design value
  - Determine the variation in air outlet temperature as the air flow rate is varied between 200 and  $1000 \text{ L/s}$  under the assumption that the  $Ntu$  is constant at the design value. The value of  $U$  will not be the design value
  - Determine the variation in air outlet temperature as the air flow rate is varied between 200 and  $1000 \text{ L/s}$  under the assumption that the effectiveness is constant at the design value. The value of  $U$  will not be the design value.
  - Determine the variation in air outlet temperature as the air flow rate is varied between 200 and  $1000 \text{ L/s}$  with the assumption that the  $UA$  varies as the air flow rate to the 0.4 power. This models the effect on the heat transfer coefficient on the air side. The value of  $U$  will not be the design value.
  - Draw some conclusions from your analysis.
- 1.15 A coil is made of two rows of the finned tubes of surface A of Figure 1.5a. The air flow is  $2500 \text{ L/s}$  at a temperature of  $24^\circ\text{C}$  and 50% relative humidity. The coil frontal area is  $0.5 \text{ m}^2$  and the fins are made of aluminum. The detailed dimensions of the fin are given in the table below the figure. Determine the convective heat transfer coefficient, the overall thermal resistance, the air-side conductance, and the pressure drop. Draw some conclusions from your analysis.
- 1.16 A coil is made using finned tubes of surface A of Figure 1.7a. The air flow of  $2500 \text{ L/s}$  enters at a temperature of  $24^\circ\text{C}$  and 50% relative humidity. The coil frontal area is  $0.5 \text{ m}^2$ , there are two rows, and the fins are made of aluminum. The water flow is  $1.3 \text{ L/s}$  and enters at  $60^\circ\text{C}$ .

- a. Determine the convective heat transfer coefficient, the overall thermal resistance, and the pressure drop for the air side of the exchanger.
  - b. Determine the convective heat transfer coefficient, the overall thermal resistance, and the pressure drop for the water side of the exchanger.
  - c. Determine the overall conductance area product, the exchanger effectiveness, and the air and water outlet temperatures.
  - d. Draw some conclusions from your analysis.
- 1.17 A coil is made with steel tubes and aluminum fins 0.6 mm thick. The tubes have inner diameter 30 mm and outer diameter 32 mm and are oriented in a triangular grid (Figure 1.3). The pitch is 4 fins per centimeter of tube length. The heat transfer coefficient for the airflow over the fins is  $120 \text{ W/m}^2\text{-}^\circ\text{C}$  and that for the water inside the coil is  $2000 \text{ W/m}^2\text{-}^\circ\text{C}$ . Determine the fin efficiency, overall surface efficiency, the overall transfer coefficient, and the transfer coefficient-area product for a coil with four rows in the flow direction, 20 rows high, and 1 m in width over a range of spacing between the tubes of 5 to 10 cm. Draw some conclusions from your analysis.
- 1.18 The fins on a baseboard convector are made of 50 mm diameter aluminum sheets bonded to a 25 mm diameter copper pipe. The temperature of the hot water is  $60^\circ\text{C}$ , the zone air temperature is  $24^\circ\text{C}$ , and the natural convection heat transfer coefficient is  $25 \text{ W/m}^2\text{-}^\circ\text{C}$ .
- a. Determine the thickness of the aluminum plates necessary to have a fin efficiency of 85%.
  - b. Determine the overall fin efficiency for a spacing of 4 fins/cm of length.
  - c. Determine the length needed to provide 1000 W to a room, which is a typical value.
  - d. If steel plates of the same thickness were substituted for aluminum, determine the heat transfer rate.
  - e. Draw some conclusions from your analysis.
- 1.19 Plate fin heat exchangers are made as series of parallel water channels connected by aluminum fins with an air flow is over the fin surface as shown in Problem 1.23 Figure.



Problem 1.23 Figure

Determine the fin and overall efficiencies for a design with a channel spacing of 8 mm, a fin length in the flow direction of 5 mm, and a fin thickness of 0.1 mm for (a) aluminum fins and (b) steel fins. The fins are spaced 3 fins/cm. The air velocity varies over the range of 2 to 10 m/s. Draw some conclusions from your results.

- 1.20 A hot water heating coil is constructed of copper tubes that are 16 mm ID and 19 mm OD. The fins have an OD of 40 mm and a thickness of 0.07 mm. The air-side heat transfer coefficient is  $110 \text{ W/m}^2\text{-}^\circ\text{C}$  and the water heat transfer coefficient is  $900 \text{ W/m}^2\text{-}^\circ\text{C}$ . For a design that uses the surface in a cost-effective manner, the thermal resistances on the inside and outside of the tube should be about the same.
  - a. Determine the fin pitch that will produce about equal inside and outside resistances.
  - b. Determine the overall conductance  $U$  based on the inside surface areas.
  - c. For an air flow of 3000 L/s entering at  $20^\circ\text{C}$  and a water flow of 10 L/s entering at  $50^\circ\text{C}$ , determine the total length of tubing necessary to heat the air to  $35^\circ\text{C}$  in a two-row coil configuration.
  - d. Draw some conclusions from your analysis.
  
- 1.21 Design a coil made of copper tubes with steel fins in configuration C of Figure 1.5a to heat an air flow of 10,000 L/s from an entering temperature of  $15^\circ\text{C}$  to a leaving temperature of  $25^\circ\text{C}$ . Water is delivered from a boiler at a temperature of  $60^\circ\text{C}$  and drops  $5^\circ\text{C}$  through the coil.
  - a. Determine the heating capacity of the coil.
  - b. Determine the air-side area, heat transfer coefficient, velocity, overall thermal resistance, and pressure drop. It will help to curve fit the heat transfer relation from Figure 1.5a. This involves varying the number of rows and the face velocity so that the  $Ntu$  based on the air- and water-side resistances equals the  $Ntu$  required for the capacity. The air velocity should be in the range of 2 to 10 m/s.
  - c. Determine the water flow rate, heat transfer coefficient, and overall thermal resistance.
  - d. Determine the coil dimensions.
  - e. Draw some conclusions from your analysis.
  
- 1.22 Design a coil made of copper tubes with steel fins in configuration A of Figure 1.5a to heat an air flow of 10,000 L/s from an entering temperature of  $15^\circ\text{C}$  to a leaving temperature of  $25^\circ\text{C}$ . Water is delivered from a boiler at a temperature of  $60^\circ\text{C}$  and drops  $5^\circ\text{C}$  through the coil.
  - a. Determine the heating capacity of the coil
  - b. Determine the air-side area, heat transfer coefficient, velocity, overall thermal resistance, and pressure drop. It will help to curve fit the heat transfer relation from Figure 1.5a. This involves varying the number of rows and the face velocity so that the

$Ntu$  based on the air and water-side resistances equals the  $Ntu$  required for the capacity.

The air velocity should be in the range of 2 to 10 m/s.

- c. Determine the water flow rate, heat transfer coefficient, and overall thermal resistance.
- d. Determine the coil dimensions.
- e. Determine the capacity of the coil, the outlet air temperature, and the airside pressure drop as the airflow is varied from 2000 to 15,000 L/s
- f. Draw some conclusions from your analysis.

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