ME 418
Lecture 4 - Air-Water Vapor Mixture Properties (Psychrometrics)
In-Class Notes for Fall 2024

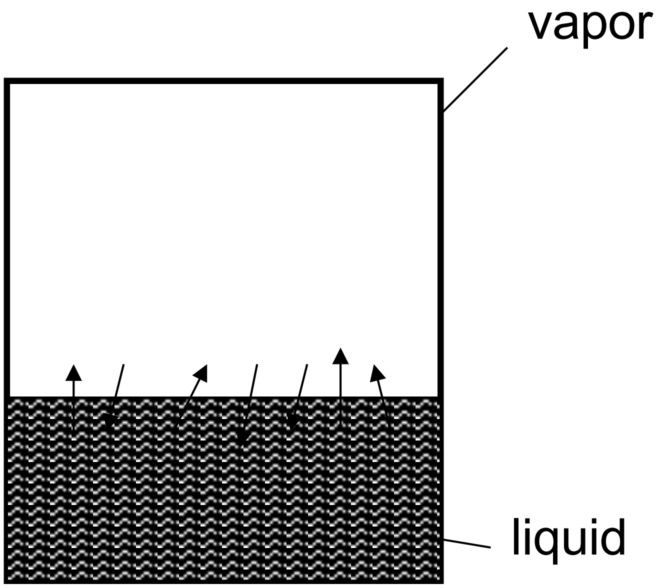
- Background
- Humidity Measures
- Psychrometric Chart
- Examples

Background

- Typical HVAC processes involve air in which moisture is added or removed, e.g., humidification, dehumidification, evaporative cooling
- Only mass flow rate of water vapor changes, mass flow rate of dry air is constant
- Specific moist air properties are given per unit mass of dry air
- Two assumptions are employed for calculating moist air properties from other measurements:
  - Dalton’s law for non-reacting mixtures (each component in the mixture occupies entire volume at mixture temperature and partial pressure)

    $P = P_a + P_v$
  - air and water vapor are a mixture of ideal gases

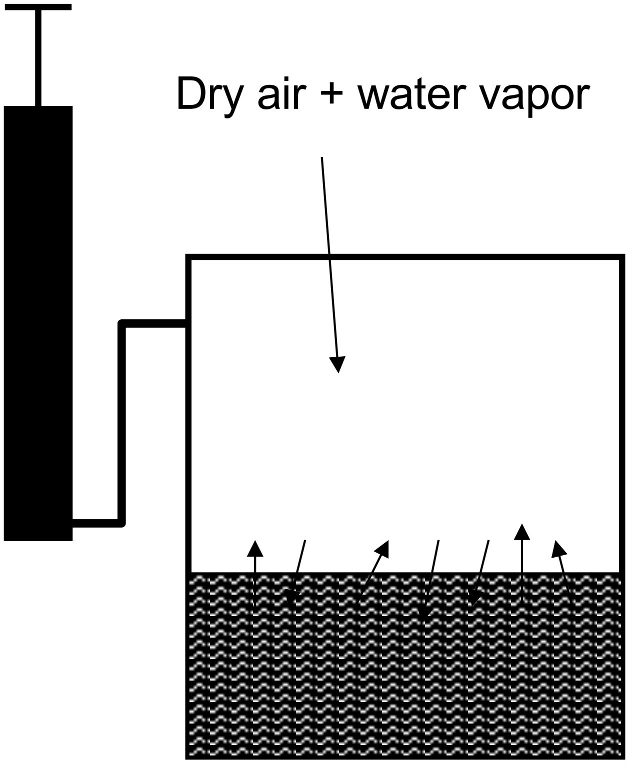
Start by considering pure liquid and vapor water in equilibrium at room temperature



<u>Questions</u>: What pressure is required for this two-phase mixture of water at 23 C?

Is ideal gas behavior an accurate model for the water vapor at these conditions? Why or why not?

Now, suppose a bicycle pump is used to introduce air into the space above the liquid.

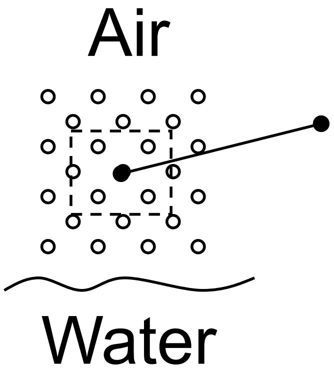


<u>Questions</u>: What happens to the total pressure?

What happens to the partial water vapor pressure if the temperature remains the same?

In the open atmosphere, is the water vapor in equilibrium with the liquid water on the surface, i.e. lakes and the ocean?

Humidity Measures



air & water vapor
@ given T & P

Need 3rd independent property to define the state

Humidity Ratio

$$\omega = \frac{\text{mass of water vapor}}{\text{mass of dry air}} = \frac{m_v}{m_a}$$

- measure of vapor concentration
- need ω for mass & energy balances

Using Humidity Ratio in Air-Water Vapor Mixtures:

Mixture mass: $m = m_a + m_v = m_a + \omega m_a = (1+\omega)m_a$

Mixture Enthalpy: $H = m_a h_a + m_v h_v = m_a\left(h_a + \omega h_v\right)$

Mixture Specific Enthalpy: $h = \dfrac{H}{m_a} = h_a + \omega h_v$

Evaluating ha and hv for constant specific heats

Ideal gas behavior is assumed so that *h = h(T)* only. Most HVAC applications involve relatively small temperature changes, such that specific heat can be assumed to be constant where

$$h_a = c_{p,a}\left(T - T_{ref.a}\right)$$

Dry air reference condition is vapor at 0 F or 0 C

$$h_v = c_{p,v}\left(T - T_{ref.v}\right) + h_{fg,ref}$$

Water reference is liquid at 32 F or 0 C

Then,

$$h = \left(c_{p,a} + \omega c_{p,v}\right)T + \omega\left(h_{fg,ref} - c_{p,v}T_{ref.v}\right)$$

Or introducing a mixture specific heat (c_{pm})

$$h = c_{p,m}T + \omega\left(h_{fg,ref} - c_{p,v}T_{ref.v}\right)$$

Air-Water Vapor Flow Streams

Mixture mass flow rate: $\dot{m} = \dot{m}_a + \dot{m}_v = (1+\omega)\dot{m}_a$

Mixture flow enthalpy: $\dot{m}_a h_a + \dot{m}_v h_v = \dot{m}_a\left(h_a + \omega h_v\right)$

$$= \dot{m}_a h$$

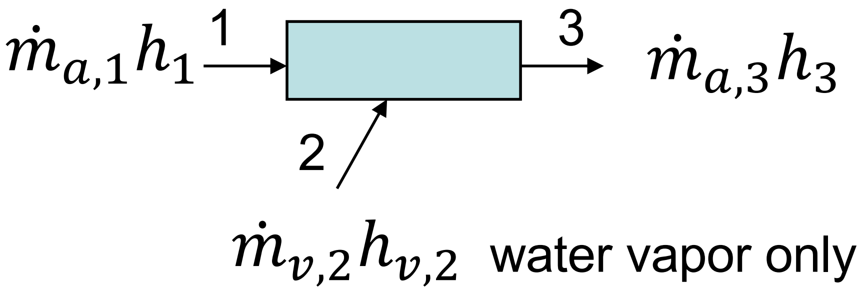
Mixture specific volume (v_m) is useful for converting mixture velocity or volumetric flow rate to dry air mass flow rate

$$\dot{m}_a = \frac{A_c V}{v_m} = \frac{\dot{V}}{v_m}$$

where

$$v_m = \frac{R_a T}{P_a} = \frac{R_a T}{(P - P_v)}$$

Mass and energy balance example for continuous addition of moisture (water vapor)



Dry air mass balance: $\dot{m}_{a,1} = \dot{m}_{a,3} = \dot{m}_a$

Water mass balance: $\omega_1 \dot{m}_a + \dot{m}_{v,2} = \omega_3 \dot{m}_a$

$$\rightarrow \dot{m}_{v,2} = \dot{m}_a(\omega_3 - \omega_1)$$

Energy balance: $\dot{m}_a h_3 = \dot{m}_a h_1 + \dot{m}_{v,2} h_{v,2}$

$$\rightarrow h_3 = h_1 + (\omega_3 - \omega_1) h_{v,2}$$

Relating Humidity Ratio to Partial Pressures:

For an ideal gas mixture,

$$\omega = \frac{m_v}{m_a} = \frac{P_v V / R_v T}{P_a V / R_a T} = \frac{P_v / R_v}{P_a / R_a}$$

$$\text{but } R = \frac{R_u}{M} \Rightarrow \omega = \frac{M_v P_v}{M_a P_a} = 0.622 \frac{P_v}{P_a}$$

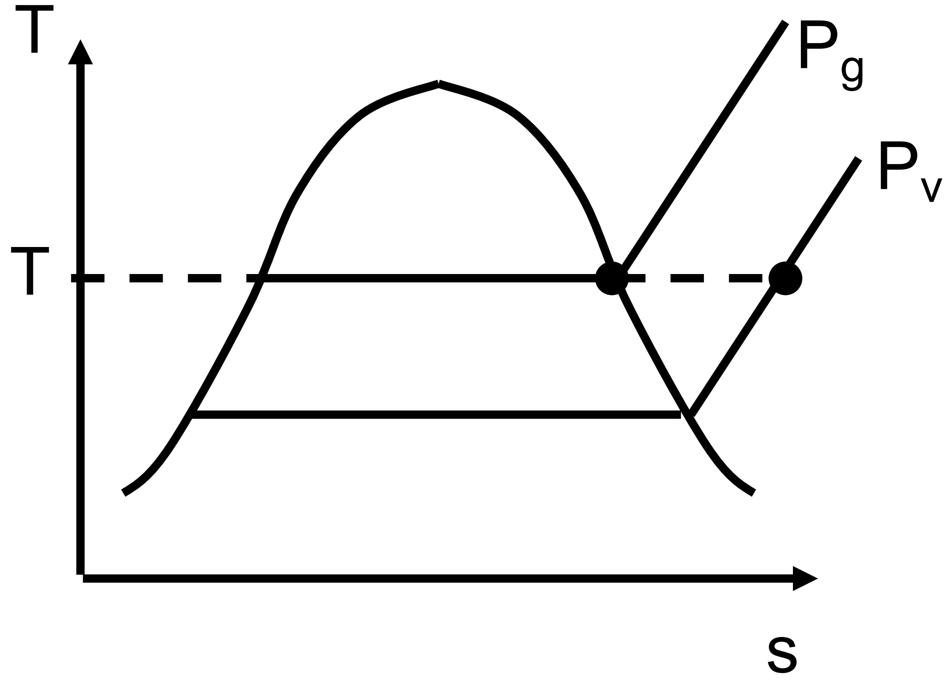
$$\text{also } P = P_a + P_v \Rightarrow \omega = 0.622 \frac{P_v}{(P - P_v)}$$

How to Determine Humidity Ratio from Measurements

- Not possible to directly measure water vapor pressure
- Could collect air-water vapor samples, remove moisture (condense or adsorb), and then weigh → not practical in general
- Need to relate humidity ratio to other low-cost measurements that depend on humidity

Relative Humidity

$$\phi = \frac{partial\ P\ of\ water\ vapor\ in\ mixture}{saturated\ P\ of\ water\ vapor\ at\ same\ T} = \frac{P_v}{P_g}$$



Given measurements of P, T, and ϕ, then

$$P_g = P_{sat}(T) \Rightarrow P_v = \phi \cdot P_g$$

$$\omega = 0.622 \frac{P_v}{P - P_v}$$

Also

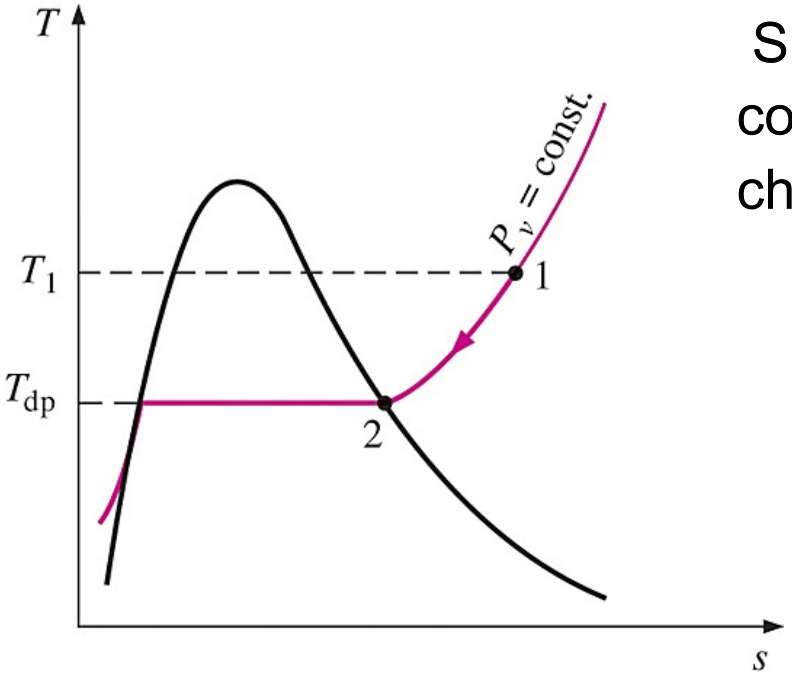
$$h = h_a(T) + \omega h_v(T)$$

How to Measure Relative Humidity

- Electrical properties (resistance or capacitance) of several materials depend uniquely on relative humidity
- Several low-cost relative humidity transducers on the market

Dew Point Temperature

Temperature at which vapor condenses if mixture is cooled at constant pressure.



Since the moisture composition does not change from 1 to 2:

$$P_V = P_{sat}(T_{dp})$$

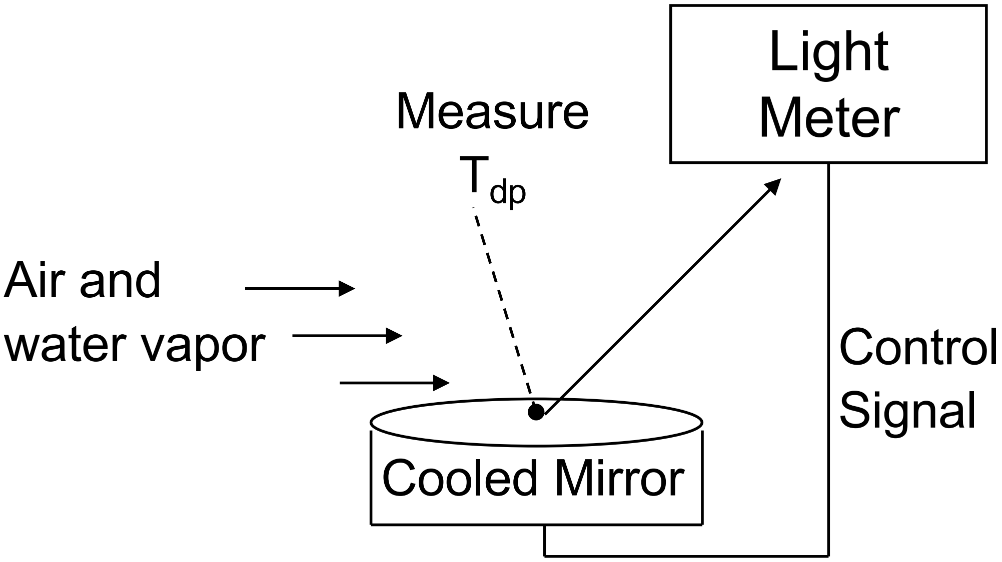
Given measurements of P, T, and T_{dp}, then

$$P_v = P_{sat}(T_{dp}) \Rightarrow \omega = 0.622\frac{P_v}{P - P_v}$$

How to Measure Dew Point

- Cool a surface to the point where moisture just condenses and measure surface temperature
- Chilled mirror devices exist on the market for directly measuring dew point temperature

Chilled Mirror Dew Point Device



Light Meter

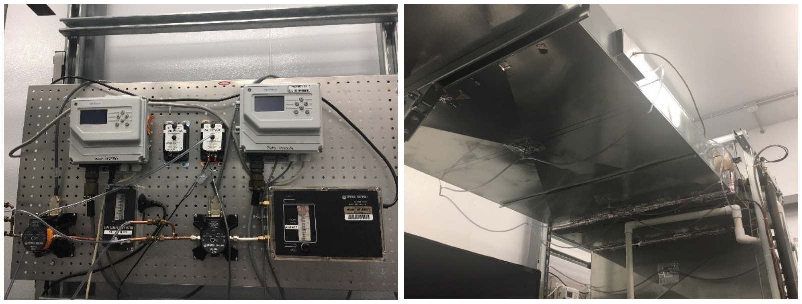
Measure T_{dp}

Air and water vapor

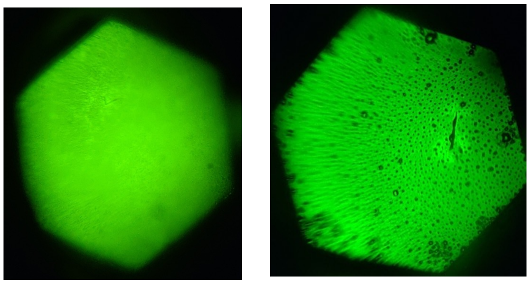
Control Signal

Cooled Mirror

Example: GE General Eastern OPTISONDE (~$4,500)

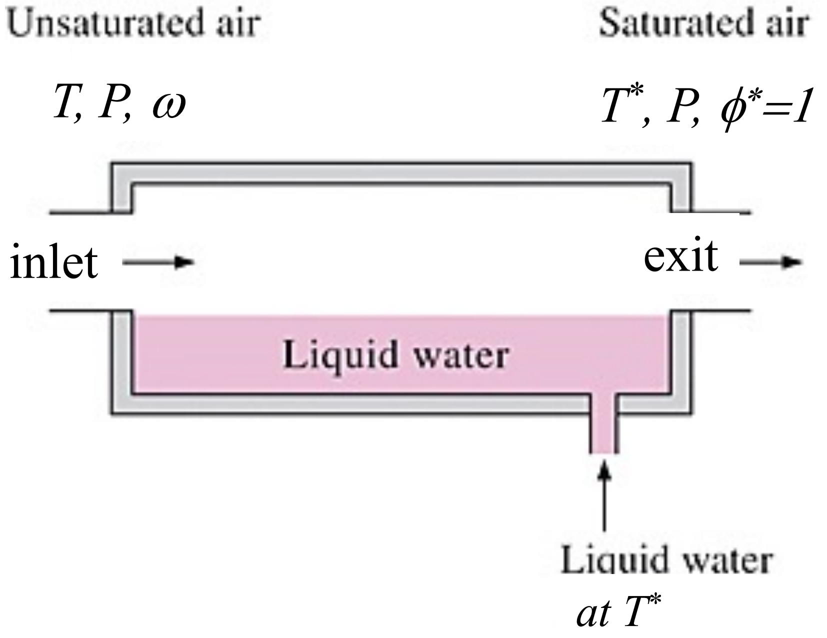


Surface of the mirror must be kept clean



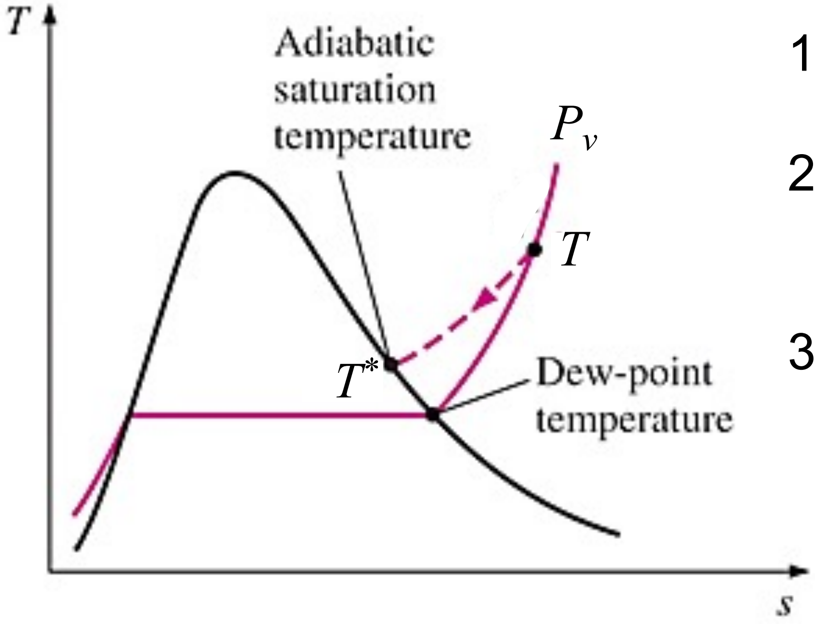
Adiabatic Saturation Temperature

Temperature that air reaches when allowed to evaporate liquid water in an adiabatic, constant pressure device



Concept

1. Allow air sample to evaporate water
2. Directly measure P, T, and T^*
3. Calculate ω from mass and energy balances



Notes

1. Air is cooled due to water evaporation
2. T^* for this device is called the adiabatic saturation temperature
3. Ideally add makeup water at T^* to avoid influencing adiabatic saturation temperature

Determining ω from P, T, and T^*

- Apply mass and energy balances to a control volume that includes air and water
- Assumptions: SSSF, negligible effects of kinetic and potential energy, constant pressure process, no heat transfer to or from the device, air and water vapor are ideal gases, liquid water is incompressible, constant specific heats

Applying separate steady flow mass balances to the dry air and water within the adiabatic saturator

$$\dot{m}_{a,inlet} = \dot{m}_{a,exit} = \dot{m}_a$$

$$\omega \cdot \dot{m}_a + \dot{m}_w = \omega^* \cdot \dot{m}_a$$

Then, the required makeup water flow can determined as

$$\dot{m}_w = \dot{m}_a(\omega^* - \omega)$$

Applying a steady-state energy balance to the adiabatic saturator

$$0 = \dot{m}_a h + \dot{m}_w h_w - \dot{m}_a h^*$$

where

$$h^* = h_a^* + \omega^* h_v^* \qquad h = h_a + \omega h_v$$

Then, solving these equations for ω

$$\omega = \frac{(h_a^* - h_a) + \omega^*(h_v^* - h_w)}{(h_v - h_w)}$$

Calculation Steps assuming constant C_p's:

1. Measure T, T^*, and total pressure P

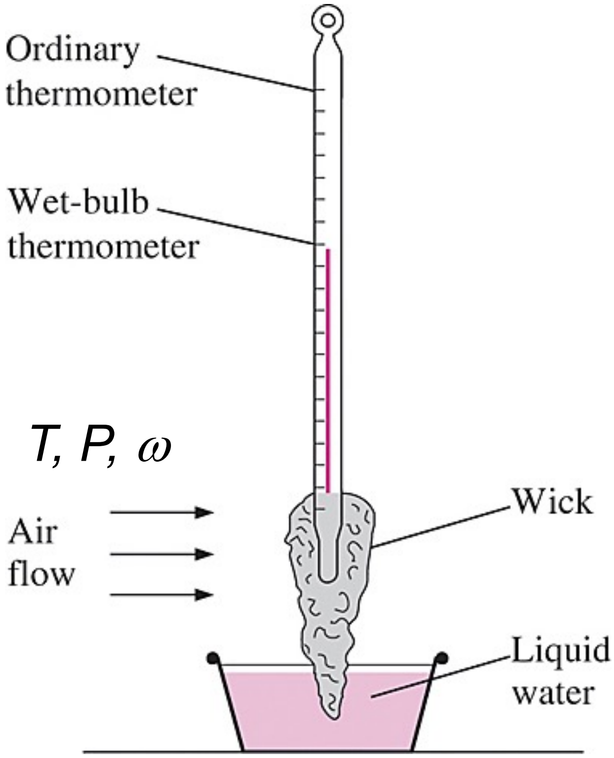
2. Calculate ω^* for saturated state point 2:
   $\omega^* = 0.622\, P_{sat}(T^*) / (P - P_{sat}(T^*))$

3. Get dry air and water properties:

$$h_a = c_{p,a}\left(T - T_{ref,a}\right)$$
$$h_a^* = c_{p,a}\left(T^* - T_{ref,a}\right)$$
$$h_v = c_{p,v}\left(T - T_{ref,v}\right)$$
$$h_v^* = c_{p,v}\left(T^* - T_{ref,v}\right)$$
$$h_w = c_{p,w}\left(T^* - T_{ref,w}\right)$$

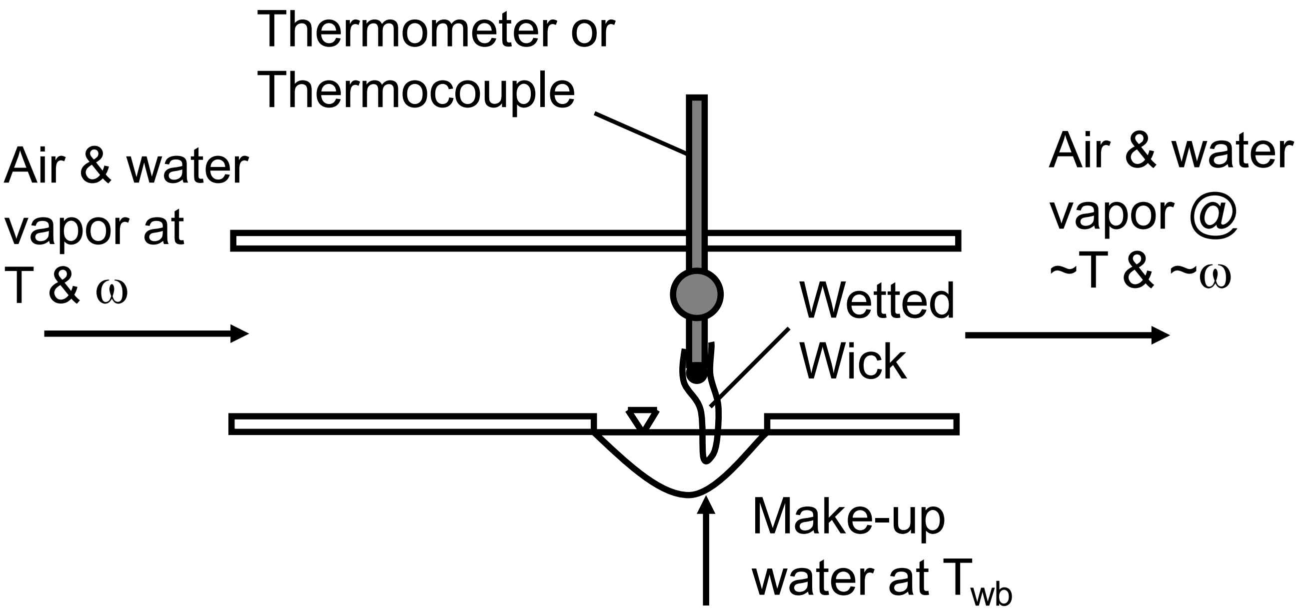
4. Calculate ω based on above relationship

Wet Bulb Temperature



» Use wet-bulb temperature, T_{wb}, as approximate measure of adiabatic saturation temperature

» Use T_{wb} in place of T^* to determine ω in adiabatic saturator relationship



Thermometer or Thermocouple

Air & water vapor at T & ω

Air & water vapor @ ~T & ~ω

Wetted Wick

Make-up water at T_{wb}

Common Properties for Moist Air Calculations

	IP	SI
Universal gas constant, R_u	1.986 Btu/(lbmol-R)	8.314 kJ/(kmol-K)
Air		
R_a	0.06856 Btu/(lbm-R)	0.2870 kJ/(kg-K)
M_a	28.97 lbm/lbmol	28.97 kg/kmol
$T_{\mathrm{ref,a}}$	0°F	0°C
$c_{p,a}$	0.2396 Btu/(lbm-R)	1.004 kJ/(kg-K)
Water		
R_v	0.1102 Btu/(lbm-R)	0.4615 kJ/(kg-K)
M_a	18.02 lbm/lbmol	18.02 kg/kmol
$T_{\mathrm{ref},v}$	32°F	0°C
$c_{p,v}$	0.446 Btu/(lbm-R)	1.868 kJ/(kg-K)
$h_{\mathrm{fg,ref}}$	1075 Btu/lbm	2501 kJ/kg
$T_{\mathrm{ref},w}$	32°F	0°C
$c_{p,w}$	1.01 Btu/(lbm-R)	4.18 kJ/(kg-K)

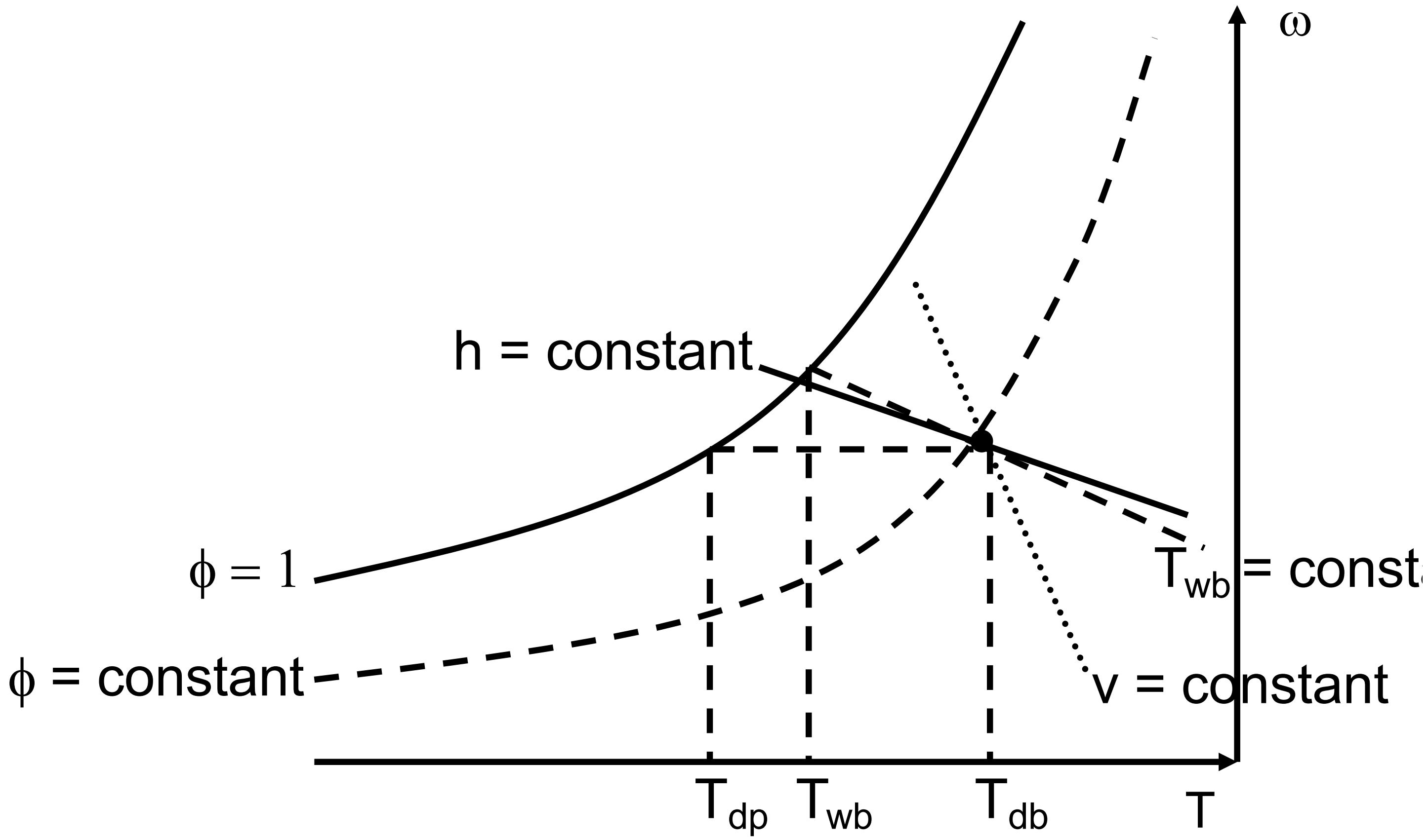
Psychrometric Charts

3 Ways to Obtain Air-Water Vapor Mixture Properties

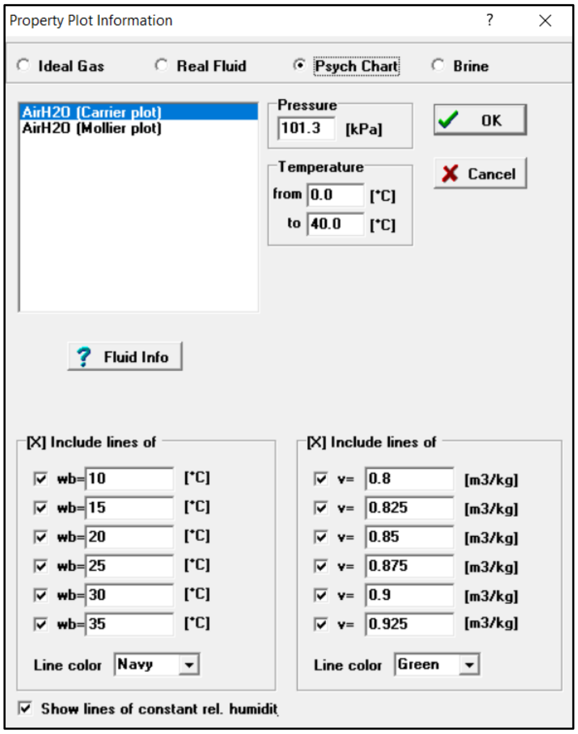
- Ideal gas mixture theory using air and water vapor properties (could use EES to solve equations)
- Use EES AirH2O Function
- Psychrometric Chart (typically for 1 atm)

Psychrometric Chart:

- graphical relationship between T_{db}, T_{wb}, T_{dp}, ω, ϕ, and h
- valid only for one constant pressure
- useful for visualizing processes



Psych Chart in EES



○ Ideal Gas ○ Real Fluid ● Psych Chart ○ Brine

AirH2O (Carrier plot)
AirH2O (Mollier plot)

Pressure: 101.3 [kPa]

Temperature
from 0.0 [°C]
to 40.0 [°C]

[X] Include lines of

☑ wb= 10 [°C]
☑ wb= 15 [°C]
☑ wb= 20 [°C]
☑ wb= 25 [°C]
☑ wb= 30 [°C]
☑ wb= 35 [°C]

Line color: Navy

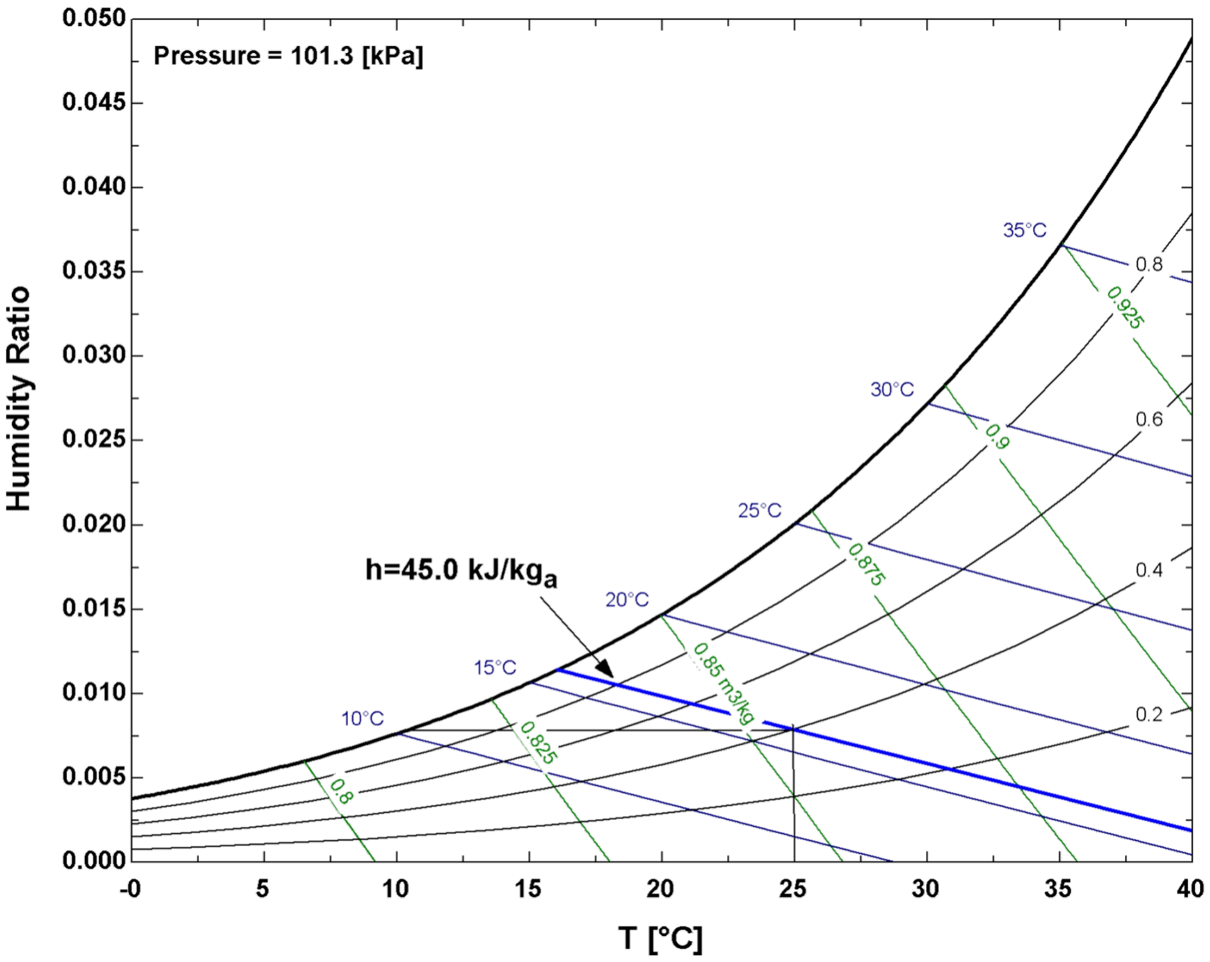
[X] Include lines of

☑ v= 0.8 [m3/kg]
☑ v= 0.825 [m3/kg]
☑ v= 0.85 [m3/kg]
☑ v= 0.875 [m3/kg]
☑ v= 0.9 [m3/kg]
☑ v= 0.925 [m3/kg]

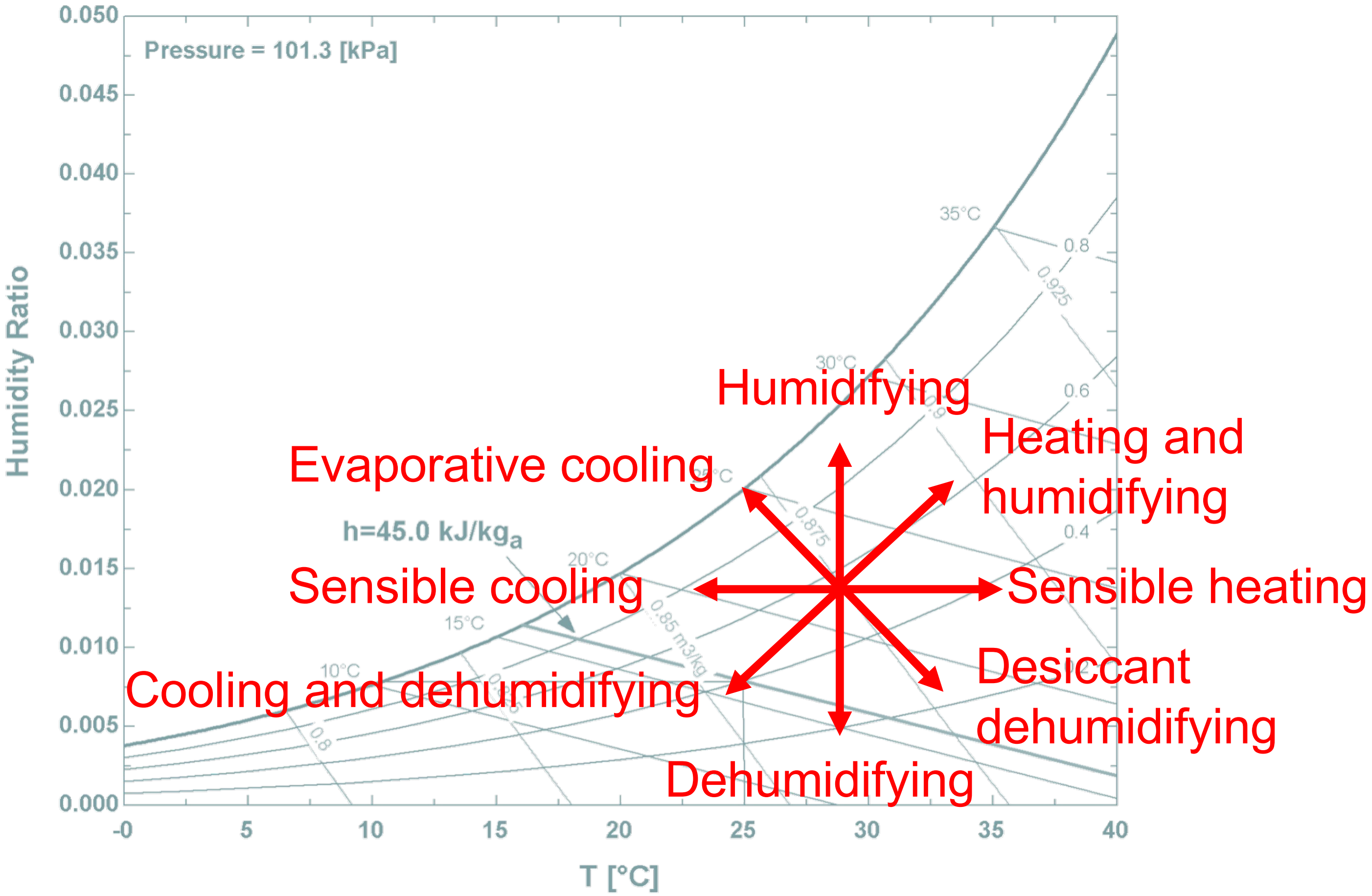
Line color: Green

☑ Show lines of constant rel. humidit

- Can generate Psych chart for any pressure
- Can overlay operating conditions for visualizing processes



Process Directions on a Psych Chart



Evaluating Mixture Properties in EES

h = enthalpy(AirH2O,P=?,T=?,R=?)

w = humrat(AirH2O,P=?,T=?,D=?)

R = relhum(AirH2O,P=?,T=?,W=?)

C_pm = specheat(AirH2O,P=?,T=?,B=?)

v_m = volume(AirH2O,P=?,T=?,R=?)

where R is relative humidity, w is absolute humidity, D is dewpoint, B is wetbulb

Property Evaluation Examples

Consider air at atmospheric pressure (14.7 psia), a temperature of 90 F, and relative humidity of 70%. Evaluate and compare air-water mixture properties for dewpoint, humidity ratio, wetbulb, enthalpy, and mixture specific volume using the following 3 different methods:

1. ideal gas mixture with constant specific heats;
2. EES AirH20 functions;
3. psychrometric chart

PSYCHROMETRIC CHART - US and SI Units

SEA LEVEL

Barometric Pressure: 29.921 Inches of Mercury (101.04 kPa)